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Theories of Open Systems: Realities and Perspectives

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***Abstract* — Problems of creation theories of open systems are analyzed. Methods of analytical mechanics, nonequilibrium thermodynamics and theory of information were used for the resolution of this problem. Mathematical aspects of creation theory of open systems are discussed. Polymetric analysis is represented as variant of creation universal open system.**

Keywords* — *open systems, action law, enthropy, polymetric analysis, thermodynamics, cybernetics, computer science, information, de Broglie

I. Introduction

The problem of creation the theory of open systems (TOS) originated from the beginning of science [1-8]. It is appropriate to recall the Archimedes phrase "Give me a fulcrum and I will move the world" [1]. In science, beginning with Descartes [1] and Newton [1; 9] it is important to choose the best system analysis and synthesis of knowledge that would most adequately described the relevant processes and phenomena.

At the dawn of modern science issue was not raised on open systems, everything was the opposite, as do closed system that could consistently explain, for example, earthly and celestial mechanics or electrodynamics [1]. In modern theoretical physics elements of the theory of open systems are included in Born (probabilistic) interpretation of quantum mechanics (expansion coefficients of wave function are the basis characteristics of the "thermostat", and the basis functions – internal system) [10].

In modern science, there is a problem regarding the relevance or applicability of these theories to explain new phenomena and processes and the issue of new science that would describe these phenomena. In particular case, in the twentieth century next synthetic sciences were created: cybernetics [3], the theory of electroweak interactions, grand unified theory, etc. [11-13]. With the development of computing a problem formalization of nonverbal knowledge is one of central problem of this science. The problems of formalization linguistics, art, mythology are urgent too [1].

So the purpose of the paperis slightly wider analysis than in physics problems ofthe theory of open systems.

Roughly speaking the development of modern science is realization R. Bacon – Descartes thesis "Science is as much science as it is mathematics". This thesis is basic for various scientific systems from W. Leubniz [14] to N. R. Campbell [15, 16]. These systems are divided philosophy and other science, verbal and nonverbal systems of knowledge.

But development of modern cybernetics and computing science is caused the necessity of creation unified system of science and knowledge. The science and knowledge are open systems. Therefore this universal system must be open. Main motto of modern computing science according by A. Ershov is famous phrase of Canadian philosopher L. Hall "Everything that go from head is reasonable" [1].

II. Physics and Mathematics of Open Systems

In modern physics and mathematics problem of relation between closed and open systems was formulated in Newtonian four rules of conclusions in physics []. It is first optimal method of creation of science in modern sense.

Aspects of the open systems in physical theories are connected with idea of expansion theory on other types of interaction (perturbation theory, symmetry breakdown).

So roughly speaking, modern theories of unified physical theories according J. D. Barrow may be represented as expanded formalization R. Boshkovich concept $1/r^2$ [11].

In physical theory two principles: action and entropy have unified nature [1, 17, 18]. Therefore theory of open systems may be based on these principles.

Problem of opening system is included in next physical notions: Maxwell devil, Schrödinger cat etc. These problems are connected with sizes of little point in mechanics of condensed matter. Basic chapters of modern theoretical physics are development and application of Newtonian mechanics and mechanics of condensed matter for resolution of the problems electrodynamics, atomic and nuclear physics and astrophysics. This fact caused the appearance of uncertainty principle, concept of derivative measurement (including procedure of quantum mechanical measurement) and some paradoxes of modern physics. So, L. Mandelstam was called Quantum Mechanics as science of derivative measurements [19].

"Thermodynamic" theory of open systems was developed by Yu. Klimontovich [4-6]. According this concept, the physics of open systems is understood non-equilibrium thermodynamics and its "micro expansion" - statistical theory of open systems and Synergetics [4-6;20]. The main objective of Synergetics is identifying common ideas, techniques and patterns in the processes of self-organization in various fields of science and sociology [20].

The emergence of the theory of self-organization (Synergetics) was prepared of researches of many scientists [4-6, 20], including L. Boltzmann and Poincare – founders, respectively, the statistical description of statistica and dynamic representations of complex movements; A. Lyapunov – one of the founders of the theory of stability of motion, the underlying theory of self-organization; Kolmogorov, who determined the metric entropy concept, which has an essential role in the theory of dynamical systems; D. Birkhoff, V. Volterra, L. Mandelstam, A. Andronov, N. Krylov, L. Landau, Ya. Zeldovich and many others. It is worth noting the role of V. Vernadsky [4-6]

According to H. Haken Synergetics [20] is a theory of self-consistent action. Although, according Yu. L Klimontovich Synerhetics hasn't single answer to the question: what is the self-organization [4-6] ? This, however, is not necessary. Need another – establishing criteria for the relative degree of order, organization of various nonequilibrium states of open systems. Without such criteria we can not give answer on the most basic questions, including capture the existence of self-organization.

The main objectives of TOS are and identify the relationship between dynamic and statistical description of complex movements. [4-6].

Essential role played by the concept of chaos and order. In physics, the concept of chaos, chaotic movement, fundamental order are introduced, but it not clearly defined.

In fact, starting with classic works of Maxwell, Boltzmann, Gibbs motion of atoms in a state of statistical equilibrium is called chaotic [4-6; 20]. Also movements are called chaotic and in states far from equilibrium, such as noise generators, turbulent flows a.o.

Significant prevalent has term dynamic chaos. He describes a complex movements of low-dimensionality ("simple") nonlinear dissipative dynamic systems. A classic example of such a system is Lorenz equations in the theory of thermal convection [4-6]. So by the same period of chaos characterize various types of complex movements. It is caused to the need for the introduction of criteria relative measure of randomness or order. These criteria include: *K*-entropy (Krylov-Kolmogorov-Sinay entropy), Lyapunov indexes, determined in different ways the dimension of the phase space of the complex motion [4-6].

The literature focused on the criteria that are based on a comparison renormalized to a given value of the average values of energy efficient Boltzmann-Gibbs-Shannon entropy [17; 18], as well as comparing the values of entropy stable and unstable states of open systems [4-6].

Selecting of controlling parameters in the process of self-organization, usually either based on existing information about the system, or through additional studies, such as bifurcation diagrams. It is possible, of course, and errors as a criterion relative degree of order and should provide the ability to control the right choice. Control parameters can serve a variety of characteristics. Here are a few examples. In classical and quantum generators it is feedback, pump or external power. In multystable systems choice of proper stationary state may be due to changes in initial conditions. Control parameters may be a "slow" time, such as time monitoring of the patient's health. In hydrodynamics, according to depending on the nature of fluid flow, the role of control parameters are Reynolds, Rayleigh and Taylor numbers [1, 13]. In the presence of several control parameters search of the most effective ways of self-organization.is possible.

Now we consider the concept of physical chaos [4-6]. Denote $a=(a_1...a_n)$ the set of selected controlling parameters. We distinguish two states with the values $a=a_0$, $a=a_0-\Delta a.$ The changes $\Delta a>0$ are corresponded to the process of self-organization meet changing conditions. This process of self-organization may be written as

$$a=a_0+\Delta a,\quad \Delta a_i>0\quad (i=1,2,...n).$$

Condition $a=a_0$ is called the state of physical chaos. Accept it for the "starting point" when comparing the degree of order. Word "physical chaos" was introduced to emphasize the physical nature of these criteria. Note that the state of physical chaos can be significantly nonequilibrium..

Now we consider the relationship between evolution and self-organization. The concept of dynamic evolution is dynamic, but has a very general nature. In physics considered, such as the evolution to equilibrium state for closed systems and the the evolution to the stationary states for open systems are analysed in physics. ,Evolution can be seen as a sequence of formation of new structures. In biology, according to Darwin, formation of new structures is due to natural selection [4-6].

What is the relationship between the concepts evolution and self-organization? Speaking of self-organization processes, we mean the process by which (according to the above criteria) appear complex and sophisticated structures. In this approach, the question arises: Is there any evolutionary process is a process of self-organization? The answer, of course, is no, because neither physical nor even biological systems are not founded "inner desire" to self-organization. In fact, evolution can lead to degradation [4-6]. In physics, the example of this concept is a transition to the equilibrium state, which has most chaotic structure according to Boltzmann and Gibbs. Degradation of structures is possible, of course, and in biology, for example mutations. Thus, self-organization is just one of the possible variants of evolution [4-7; 20].

Now we are analyzing criteria of evolution and self-organization in physical systems.

Firstly, we consider the H-theorem Boltzmann and Gibbs theorem [4-7; 20]. The name "H-theorem" the letter H comes from the English word Heat. This emphasizes that it is the evolution of entropy in the process of establishing thermal equilibrium. Proof of the H-theorem is based on propertty Boltzmann collision integral $I_p:\int\varphi(p)I_p dp\geq\varphi$ with $\varphi=-k\ln f$ and condition, that the gas particles do not go beyond the system. This result is usually represented as:

$$\frac{dS}{dt}\geq 0, \qquad (1)$$

where entropy $S(t)=-k\int f\ln fdrdp.$

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Sign of equality refers to equilibrium state, when f_0 – Maxwell distribution.

Further Yu. Klimontovich connects entropy with Lyapunov functional, so that the H-theorem formulated somewhat differently [4-6]. It also follows from (1), but including several other properties in the collision integral $\int \phi(p) I_B dp = 0$ for $\varphi = 1,\ p,\ \frac{p^2}{2m}$, and reduced to the claim that is Lyapunov functional $S(t)$ is existed with properties [4-6]:

$$\Lambda_s \equiv S_0 - S(t) = k\int \ln\frac{f}{f_0}\cdot f\mathrm{d}r\mathrm{d}p \ge 0,$$
$$\frac{\mathrm{d}(S_0 - S)}{\mathrm{d}t} \le 0, \langle H\rangle = \left\langle \frac{p^2}{2m}\right\rangle = \mathrm{const}. \tag{2}$$

This implies that the equilibrium is stable and corresponds to maximum entropy S_0. In (2) we noted that in the process of evolution to equilibrium average of the energy of rarefied gas remains unchanged. In this case, it is not an additional condition, it is property of Boltzmann equation. However, it is because of this Lyapunov functional is entropy, and not any other characteristics of system.

We now consider an arbitrary system with a Hamiltonian function $H(x)$ Equilibrium state is characterized by Gibbs canonical distribution:

$$f_0(x) = \exp\frac{F_0 - H(x)}{kT}, \quad \int f_0(x)dx = 1, \tag{3}$$

where F_0 – Helmholtz potential (free energy).

Let $f(x, t)$ is random distribution of the same regulation, but with one limitation: the average value Hamiltonian function for distributions f_0 and f is the same, that is:

$$\int Hf_0(x)dx = \int Hf(x,t)dx. \tag{4}$$

The entropies, which are corresponded to distributions f_0 and f, were denoting as S_0, S. Then, according to Gibbs theorem, we have:

$$S_0 - S = k\int f\ln\frac{f}{f_0}dx \ge 0. \tag{5}$$

So, if the average energy is stable, an entropy has maximum in equilibrium state. There are no restrictions on the interaction of particles in the system. In this case Gibbs result is more general than H-theorem [4-6]. Here, however, is not considered the question of the temporal evolution of the functions $S_0 - S(t)$ at relaxation to equilibrium state.

We consider now the H-theorem in the theory of Brownian motion. Give a simple case where Brownian particles uniformly distributed in space. Then the velocity distribution function at linear friction satisfies the Fokker-Planck equation [4-8]:

$$\frac{\partial f(\vartheta,t)}{\partial t} = D\frac{\partial^2 f(\vartheta,t)}{\partial\vartheta^2} + \frac{\partial(\gamma\vartheta f)}{\partial\vartheta},$$
$$D = \gamma\frac{kT}{m}, \quad \int fd\vartheta = 1. \tag{6}$$

Equilibrium solution is the Maxwell distribution of, and may be written in the next form:

$$f_0 = \exp\frac{F_0 - H}{kT}, \quad H = \frac{m\vartheta^2}{2}. \tag{7}$$

For a system in a thermostat free energy may determine for non-equilibrium state too:

$$F(t) = U(t) - TS(t), \quad U(t) = \langle H\rangle,$$
$$S(t) = -k\int f\ln fd\vartheta. \tag{8}$$

The difference of free energies $F(t) - F_0$ is Lyapunov functional [4-6]. In this case next inequalities are true:

$$\Lambda_F = F(t) - F_0 = kT\int f\ln\frac{f}{f_0}d\vartheta \ge 0,$$
$$\frac{d(F - F_0)}{dt} \le 0. \tag{9}$$

In the theory of Markov processes Lyapunov functional Λ_F is called "Kullback entropy", in control theory – Krasovsky functional [4-6].

We come, therefore, to a result similar to the Boltzmann H-theorem with substantial difference that in Brownian motion role of Lyapunov function has free energy of nonequilibrium state [4-6]. But free energy, unlike entropy, can't be defined for any non-equilibrium condition and hasn't a full set of properties, whicr are required for use as a measure of uncertainty in the time of statistical description.

Lyapunov functional has role in the Brownian motion as entropy. For this, however, we must renormalized the solution of equation (1) to a given value of the average energy (compared to the conditions (2), (4)). If after this we mark entropy, which is caused by renormalization distribution, as $\tilde{S}(t)$ the Lyapunov functional while satisfying inequality (2). For equation (6) results can be represented explicitly:

$$S_0 - \tilde{S}(t) = \frac{3}{2}k\ln\frac{T(t)}{T_0} \ge 0,$$
$$\frac{d(S_0 - \tilde{S}(t))}{dt} = -3k\gamma\frac{T(t) - T_0}{T_0}e^{-2\gamma t} \ge 0. \tag{10}$$

A correlation between initial T_0, traveling $T(t)$ and the "final" (thermostat) temperature T may be obtained from additional conditions and has the form:

$$\frac{m\langle\vartheta\rangle^2}{2} + \frac{3}{2}kT(t) = \frac{3}{2}kT, \quad T_0 \le T(t) \le T. \tag{11}$$

In the theory of Brownian motion carried renormalization is possible only in a limited area of initial velocities, as seen example is certainly illustrative. He, however, is useful because it shows the fundamental possibility of using entropy Lyapunov functional for system in a thermostat too. In formulating the criteria of the relative degree of order – criteria for self–organization – this possibility is constructive.

Now we consider a more complex example of Brownian motion in open system – generator van der Pol. Lyapunov functional is determined as the difference of free energies:

$$\Lambda_F = F(t) - F_0 = D\int f(E,t)\ln\frac{f(E,t)}{f_0}dE \geq 0,$$
$$\frac{d\left|F(t)-F_0\right|}{dt} \leq 0. \quad (12)$$

We show that the H-theorem can be formulated and for entropy too. Again, procedure of renormalization may be used, but now for given value function $H(E)$:

$$\int H(E)\tilde{f}(E,t)dE = \int H(E)f_0(E)dE. \quad (13)$$

From this equation we find renormalized noise intensity $\tilde{D}\{\tilde{f}\}$. It depends functionally from the distribution $\tilde{f}$, that satisfies the nonlinear Fokker-Planck equation with diffusion coefficient *D*. Lyapunov functional is determined as the difference of entropies S (t) - S_0 and satisfies the inequality:

$$\tilde{\Lambda}_S = S_0 - \tilde{S}(t) = \int \tilde{f}(E,t)\ln\frac{\tilde{f}(E,t)}{f_0}dE \geq 0,$$
$$\frac{d\left(S_0 - \tilde{S}(t)\right)}{dt} \leq 0, \quad \langle H(E)\rangle = \text{const}. \quad (14)$$

So, in the process of evolution to the stationary state with distribution $f_0(E)$, renormalized entropy S (t) increases and remains constant when reaching stationary state.

Note again that the use of entropy as Lyapunov function gives certain advantages. First, the entropy can be expressed through the distribution function for arbitrary nonequilibrium state for which can be determined the distribution function. Secondly, free energy is not unlike entropy properties by which it can be taken as a measure of uncertainty, randomness.

The growth of entropy (or decrease of free energy) in open systems in the evolution to stationary state is possible because the given parameters a, b, D is sufficient to determine only the stationary state, and, therefore we can select the initial distribution $f(E, t = 0)$ [4-6].

We now turn to one of the major problems of statistical theory of open systems – the establishment of criteria for the relative degree of order in nonequilibrium states for control parameter space [4-6]. So, we select control parameters. Among these parameters can be included and "slow" time, describing, for example, the process of recovery. For van der Pol generator feedback (pumping) a_H may be selectes as control parameters [4-6].

We assume control parameters change so slowly that time to set a stationary state in each intermediate state, that characterized, for example, for the generator with help distribution $f_0(E, a)$. For these conditions we can talk about the evolution of stationary states in the control parameter space *t* [4-6].

For this case procedure of creation the criteria is the same as for the problem without control parameters. Initial value of control parameter is included averaged Hamiltonian function.

The main criteria here is next, to the addition to formulas of type (5), ratio

$$\frac{d\Lambda_S}{d\Delta a} \geq 0. \quad (15)$$

If there is inequality (15), and signs of inequalities (4) and (15) are the same, the change Δa is controlling. And this change can be arbitrarily small.

So, the relative degree of order the nonequilibrium states can be performed with help of values renormalized entropy. This criterion is called S-theorem (from word Self-organization) [4-6]. This emphasizes that this is a self-organization criteria.

We must distinguish dynamic and statistical description of open systems. We distinguish two classes of systems: dynamic and statistical (stochastic). This division is not accepted. Indeed, famous work Sinai [4-6] is called "Stohastycality of dynamic systems." The term chaotic motion is used for deterministic dynamical systems also.

We introduce the property of reproducibility the motion given initial conditions as basis of classification. Then, by definition, dynamic systems are reproducible, and the stochastic systems are irreproducible dissipative systems.

For this case reproducibility of solution is depended at the structure of a mathematical model. If equation is included random sources, the process is playable and dynamic movement, although it can be unpredictable because of its complexity.

In the physics of open systems can be seen in terms of Synergetics as a branch of modern control theory - the theory of autonomous systems. This branch of physics enriches and modern control theory. Along with functionals Lagrange, Hamilton, Jacobi, Ito, Wiener the Lyapunov functionals are developing too [3-5]. Chapter of Synergetics (gradient system) is a catastrophe theory, which gives quite simply describe critical phenomena [1]. But modern control theory has long moved beyond physics and used to solve a variety of problems in various fields of science, culture and technology.

Now we represented de Broglie formula [18]:

$$\frac{S_e}{k_B} = \frac{S_a}{\hbar}, \quad (16)$$

which was obtained from the analysis of thermodynamics point [1], a measure of disordered physical information (number of photons) equally structured information (where S_a – action; $\hbar$ – Planck constant, S_e – entropy, k_B – Boltzmann constant). Through (16) the ratio of the increase of entropy for nonequilibrium processes (open systems) can be expanded at the action, that is, in other words, the physics of open systems can be built and action functional .

Then this principle would be:

$$dS_a \geq 0. \tag{16a}$$

In this case we have for the equals sign, roughly speaking, the action principle, to mark the event more – physics of open systems.

The "mathematical" way of creation theory of open systems is connected with foundations of mathematics [21, 22]. This problem has very old history. It may be begin from famous Pythagorean phrase "Numbers rule of the World" [1]. Later [1], Plato created three types of numbers: arithmetical (pure mathematics); sensitive (applied mathematics) and ideal (other chapters of knowledge).

In our time Russel and Whitehead, Hilbert and Brauer created three "structural" concepts of foundation mathematics: logical, formal (axiomatic) and intuitionistic or pragmatic according to Poincare [1, 21, 22]. First two are based on closed systems axioms: logical or other (arithmetic, geometrical etc.). These concepts couldn't be foundations of mathematics because closed system axioms can't be used for the creation new systems with various hierarchy. In mathematical sense it is result of famous Gedel incompleteness theorem [21-24]. Intuitionistic (pragmatic) concept gives unlimited number of systems [1].

The one of authors of logical concept A. N. Whitehead refused from this concept. He gives one's view about necessity of organism concept of foundations of mathematics [23]. According [1] it may be concept, which is based on nature of mathematics. But nature of mathematics is analysis, synthesis and formalization of all possible knowledge [21].

On basis this analysis we can formulate conditions for theory of open systems: 1. This theory must include minimal necessary fundamental notions and principles. 2. It must be theory with variable hierarchy. 3. It must include "opening" principles and parameters.

III. Polymetrical Analysis as Universal Theory of Open Systems

Polymetric analysis (PA) was created as alternative optimal concept to logical, formal and constructive conceptions of modern mathematics and mor general theory of information [1] and is corresponded the basic notions of the universal theory of open systems. This concept is based on the idea of triple minimum: mathematical, methodological and concrete scientific [1, 25-27].

However, one of the main tasks of polymetric analysis is the problem of simplicity-complexity that arises when creating or solving a particular problem or science.

In methodological sense, PA is the synthesis of Archimedes thesis: "Give me a fulcrum and I will move the world", and S.Beer idea about what complexity is a problem of century in cybernetics, in one system. And as cybernetics is a synthetic science, the problem should be transferred and for all of modern science. Basic elements of this theory and their bonds with other science are represented in Figure 1 [1].

Basic concept of creation PA is concept of triple optimum (minimum): mathematical, methodological and concrete scientific [1, 25-27].

The polymetric analysis may be represented as universal theory of synthesis in Descartian sense. For resolution of this problem we must select basic notions and concepts, which are corresponded to optimal basic three directions of Figure 1.

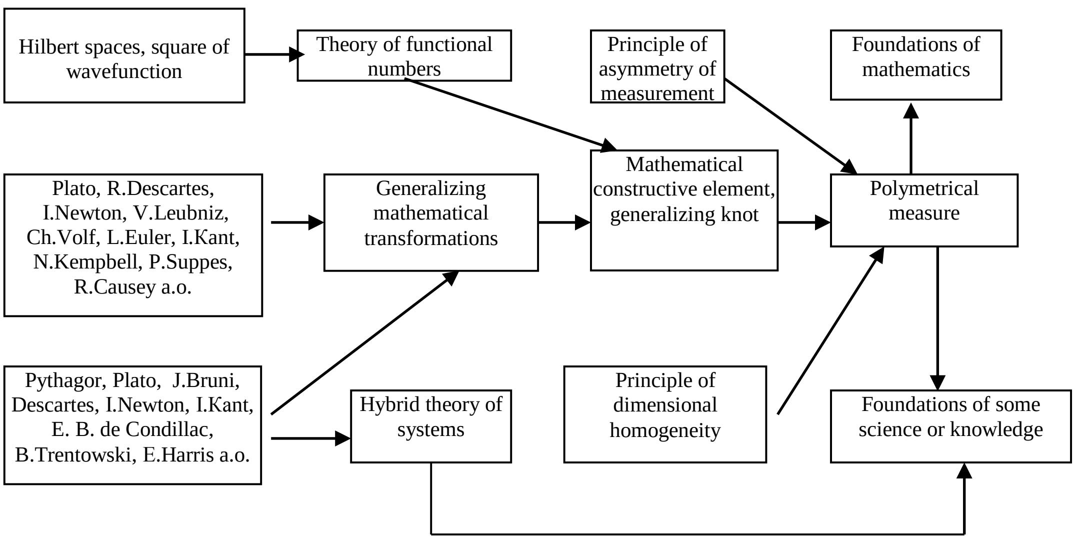

Figure 1. Schema of polymetric method and its place in modern science [1, 25, 26]

The universal simple value is unit symbol, but this symbol must be connected with calculation. Therefore it must be number. For the compositions of these symbols (numbers) in one system we must use system control and operations (mathematical operations or transformations). After this procedure we received the proper measure, which is corresponding system of knowledge and science.

Roughly speaking the basic peculiarity of polymetric analysis is the realization of Plato concepts of three types of numbers with computational point of view [1].

The basic questions, which must be resolve polymetric analysis are:

1) creation united system of optimal formalization the knowledge;
2) creation natural concept of foundations of mathemnatics, which is based on nature of mathematics,
3) creation universal theory of open system.

Therefore the basic axiomatic of the polymetric analysis was selected in the next form [1, 25-27]. This form is corresponded to schema of Figure 1.

Definition 1. **Mathematical construction** is called set all possible elements, operations and transformations for resolution corresponding problem. The basic functional elements of this construction are called constructive elements.

Definition 2. The mathematical constructive elements $N_{x_{ij}}$ are called **the functional parameters**

$$N_{x_{ij}} = x_i \cdot \overline{x}_j , \qquad (17)$$

where x_i , $\overline{x_j}$ – the straight and opposite parameters, respectively; · – respective mathematical operation.

Definition 3. The mathematical constructive elements $N_{\varphi_{ij}}$ are called the **functional numbers**

$$N_{\varphi_{ij}} = \varphi_i \circ \overline{\varphi}_j . \qquad (18)$$

Where $\varphi_i\left(x_1,...x_n,\overline{x_1}...\overline{x_m},...N_{x_{ij}},\ldots\right)$, $\overline{\varphi_j}\left(x_1,...x_n,\overline{x_1}...\overline{x_m},...N_{x_{ij}},...\right)$ are the straight and opposite functions, respectively; ∘ – respective mathematical operation.

Remark 1. Functions φ_I , $\overline{\varphi}_j$ may be have different nature: mathematical, linguistic and other.

Definition 4. The mathematical constructive elements $N^d_{x_{ij}}$ are called the **diagonal functional parameters**

$$N^d_{x_{ij}} = \delta_{ij} N_{x_{ij}} . \qquad (19)$$

Where δ_{ij} is Cronecker symbol.

Definition 5. The mathematical constructive elements $N^d_{\varphi_{ij}}$ are called the **diagonal functional numbers**

$$N^d_{\varphi_{ij}} = \delta_{ij} N_{\varphi_{ij}} . \qquad (20)$$

Example 1. If $x_i = x^i$, $\overline{x}_j = x^{-j}$ and max*{i}* = max*{j}* = *m*, then $\{N^d_{\varphi_{ij}}\}$ is diagonal single matrix.

Another example may be the orthogonal eigenfunctions of the Hermitian operator.

Remark 2. This example illustrate why quantities (17) – (2o) are called the parameters and numbers. Practically it is the simple formalization the measurable procedure in Fig.1. The straight functions correspond the “straight” observation and measurement and opposite functions correspond the “opposite” observation and measurement. This procedure is included in quantum mechanics the Hilbert’s spaces and Hermitian operators.

The theory of generalizing mathematical transformations is created for works on functional numbers [1].

Definition 6. **Qualitative transformations** on functional numbers $N_{\varphi_{ij}}$ (straight A_i and opposite $\overline{A}_j$) are called the next transformations. The straight qualitative transformations are reduced the dimension $N_{\varphi_{ij}}$ on *i* units for straight parameters, and the opposite qualitative transformations are reduced the dimension $N_{\varphi_{ij}}$ on *j* units for opposite parameters.

Definition 7. **Quantitative (calculative) transformations** on functional numbers $N_{\varphi_{ij}}$ (straight O_k and opposite $\overline{O}_p$) are called the next transformations. The straight calculative trasformations are reduced $N_{\varphi_{ij}}$ or corresponding mathematical constructive element on *k* units its measure. The opposite quantitative transformations are increased $N_{\varphi_{ij}}$ or corresponding mathematical constructive element on *l* units its measure, i.e.

$$O_k \overline{O}_l N_{\varphi_{ij}} = N_{\varphi_{ij}} - k \oplus l . \qquad (21)$$

Definition 8. **Left and right transformations** are called transformations which act on left or right part of functional number respectively.

Definition 9. The maximal possible number corresponding transformations is called **the rang of this transformation**

$$rang(A_i \overline{A}_j N_{\varphi_{ij}}) = \max(i, j) ; \qquad (22)$$

$$rang(O_k \overline{O}_p N_{\varphi_{ij}}) = \max(k, p) . \qquad (23)$$

Remark 3. The indexes *i,j, k,p* are called **the steps of the corresponding transformations.**

For this case we have finite number of generalizing transformations.

The basic types of generalizing mathematical transformations are represented in Table 1 [1, 24, 25].

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Table 1. *The basic types of generalizing mathematical transformations.*

№	Tronsfor-mation	1	2	3	4	5	6	7	8	9	10	11	12	Representation		
														S	O	M
1	full straight	+	+	+	+	+	+	+	+	+	+	+	+	+	–	–
2	full opposite	+	+	+	+	+	+	+	+	+	+	+	+	-	+	–
3	full mixed	+	+	+	+	+	+	+	+	+	+	+	+	-	–	+
4	left full straight	+	-	+	–	+	-	+	–	+	–	+	–	–	+	–
5	right full opposite	–	+	–	+	–	+	–	+	–	+	–	+	+	–	–
6	left straight	+	–	+	–	+	–	+	–	+	–	+	–	+	–	–
7	right opposite	–	+	–	+	–	+	–	+	–	+	–	+	–	+	–
8	mixed full straight	+	-	+	–	+	–	+	–	+	–	+	–	–	–	+
9	mixed full opposite	–	+	–	+	–	+	–	+	–	+	–	+	–	–	+
10	left half-straight	+	–	–	+	–	+	+	–	–	+	–	+	–	+	–
11	mixed half-straight	+	–	–	+	–	+	+	–	–	–	–	–	–	–	+
12	right semi-opposite	–	+	+	–	+	–	–	+	+	–	+	–	+	–	–
13	mixed semi-opposite	–	+	+	-	+	–	–	+	+	–	+	–	–	–	+
14	mixed straight	+	–	+	–	+	–	+	+	+	+	+	+	–	–	+
15	mixed opposite	–	+	–	+	–	+	+	+	+	+	+	+	–	–	+

Remarks to Table 1. S – straight; O – opposite; M – мixed; 1 – A_i; 2 – $\bar{A}_j$; 3 – A^r; 4 – $\bar{A}^r$; 5 – A^l; 6 – $\bar{A}^l$; 7 – O_k; 8 – $\bar{O}_p$; 9 – O^r; 10 – $\bar{O}^r$; 11 – O^l; 12 – $\bar{O}^l$. In Table 1 sigh + (plus) is defined that action of corresponding transformation on $N_{\varphi_{ij}}$ is fully or particularly; sign (minus) – is absented.

Basic element of PA is the generalizing mathematical elements or its various presentations – informative knots [1 – 4]. Generalizing mathematical element is the composition of functional numbers (generalizing quadratic forms, including complex numbers and functions) and generalizing mathematical transformations, which are acted on these functional numbers in whole or its elements [1]. Roughly speaking these elements are elements of functional matrixes.

This element ${}^{stqo}_{nmab}M_{ijkp}$ may be represented in next form

$$ {}^{stqo}_{nmab}M_{ijkp} = A_i \bar{A}_j O_k \bar{O}_p A^r_s \bar{A}^r_t O^r_q \bar{O}^r_o A^l_n \bar{A}^l_m O^l_a \bar{O}^l_b N_{\varphi_{ij}} \, . \quad (8) $$

Where $N_{\varphi_{ij}}$ – functional number; $O_k, O^r_q, O^l_a, \bar{O}_p, \bar{O}^r_o, \bar{O}^l_b$; $A_i, A^r_s, A^l_n, \bar{A}_j, \bar{A}^r_t, \bar{A}^l_m$ are quantitative and qualitative transformations, straight and inverse (with tilde), (r) – right and (l) – left.

Polyfunctional matrix, which is constructed on elements (17) is called informative lattice. For this case generalizing mathematical element was called knot of informative lattice [1 – 4]. Informative lattice is basic set of theory of informative calculations. This theory was constructed analogously to the analytical mechanics [1].

Basic elements of this theory are [1, 25-27]:

1. Informative computability C is number of possible mathematical operations, which are required for the resolution of proper problem.

2. Technical informative computability $C_t = C \sum t_i$, where t_i – realization time of proper computation.

3. Generalizing technical informative computability $C_{t0} = k_{ac} C_t$, where k_{ac} – a coefficient of algorithmic complexity [1].

Basic principle of this theory is **the principle of optimal informative calculations** [1, 25-27]: any algebraic, including constructive, informative problem has optimal resolution for minimum informative computability C, technical informative computability C_t or generalizing technical informative computability C_{to}.

The principle of optimal informative calculations is analogous to action and entropy (second law of thermodynamics) principles in physics. This fact is caused of formula (16). This formula may represented in next form

$$ \frac{S_e}{k_B} = \frac{S_a}{\hbar} = S_{dl} \, , \quad (16b) $$

where S_{dl} – a dimensional less system function of information. Roughly speaking it may be informative calculations.

The principle of optimal informative calculation is more general than **negentropic principle the theory of the information** and **Shennon theorem** [1, 17, 25-27]. This principle is law of the open systems or systems with variable hierarchy. The negenthropic principle and Shennon theorem are the principles of systems with constant hierarchy.

Idea of this principle of optimal informative calculation may be explained on the basis de Broglie formula (16) (equivalence of quantity of ordered and disorder information) [1]. Therefore we can go from dimensional quantities (action and entropy) to undimensional quantity – number of proper quanta or after generalization to number of mathematical operations. Thus, theory of informative calculations may be represented as numerical generalization of classical theory of information and analytical mechanics according to computational point of view [1].

For classification the computations on informative lattices hybrid theory of systems was created [1, 25-27]. This theory allow to analyze proper system with point of view of its complexity, The basic principles of hybrid theory of systems are next: 1) **the criterion of reciprocity;** 2) **the criterion of simplicity**.

The criterion of reciprocity is the principle of the creation the corresponding mathematical constructive system (informative lattice). The criterion of simplicity is the principle the optimization of this creation.

The basic axiomatic of hybrid theory of systems is represented below.

Definition 10. The set of functional numbers and generalizing transformations together with principles reciprocity and simplicity (informative lattice) is called **the hybrid theory of systems** (in more narrow sense the criterion of the reciprocity and principle of optimal informative calculations).

Criterion of the reciprocity for corresponding systems is signed the conservation in these systems the next categories:

1) the completeness;

2) the equilibrium;

3) the equality of the number epistemological equivalent known and unknown knotions.

Criterion of the simplicity for corresponding systems is signed the conservation in these systems the next categories:

1) the completeness;

2) the equilibrium;

3) the principle of the optimal calculative transformations.

Criterion of reciprocity is the principle of creation of proper informative lattice. Basic elements of principle reciprocity are various nuances of completeness. Criterion of the simplicity is the principle of the optimality of this creation.

For more full formalization the all famous regions of knowledge and science the **parameter of connectedness σ_t** was introduced. This parameter is meant the number of different bounds the one element of mathematical construction with other elements of this construction. For example, in classic mathematics $\sigma_t = 1$, in linguistics and semiotics $\sigma_t > 1$. The parameter of connectedness is the basic element for synthesis in one system of formalization the all famous regions of knowledge and science. It is one of the basic elements for creation the theory of functional logical automata too.

At help the criteria of reciprocity and simplicity and parameter of connectedness the basic famous parts of knowledge and science may be represent as next 10 types of hybrid systems [1, 25-27]:

1. The system with conservation all positions the criteria of reciprocity and simplicity for all elements of mathematical construction ($N_{\varphi_{ij}}$ and transformations) is called the *simple system.*

2. The system with conservation the criterion of simplicity only for $N_{\varphi_{ij}}$ is called the *parametric simple system.*

Remark 4. Further in this classification reminder of criteria of reciprocity and simplicity is absented. It means that these criteria for next types of hybrid systems are true.

3. The system with conservation the criterion of simplicity only for general mathematical transformations is called *functional simple system.*

4. The system with nonconservation the principle of optimal informative calculation and with $\sigma_t = 1$ is called the *semisimple system.*

5. The system with nonconservation the principle of optimal informative calculation only for $N_{\varphi_{ij}}$ and with $\sigma_t = 1$ is called the *parametric semisimple system.*

6. The system with nonconservation the principle of optimal informative calculation only for general mathematical transformations and with $\sigma_t = 1$ is called the *functional semisimple system.*

7. The system with nonconservation the principle of optimal informative calculation and with $\sigma_t \neq 1$ is called *complicated system.*

8. The system with nonconservation the principle of optimal informative calculation only for $N_{\varphi_{ij}}$ is called *parametric complicated system.*

9. The system with nonconservation the principle of optimal informative calculation only for general mathematical transformations and with $\sigma_t \neq 1$ is called *functional complicated system*.

10. The system with nonconservation the criteriums of reciprocity and simplicity and with $\sigma_t \neq 1$ is called *absolute complicated system*.

With taking into account 15 basic types of generalized mathematical transformations we have 150 types of hybrid systems; practically 150 types of the formalization and modeling of knowledge and science.

Only first four types of hybrid systems may be considered as mathematical, last four types are not mathematically. Therefore HTS may be describing all possible system of knowledge. Problem of verbal and nonverbal systems of knowledge is controlled with help of types the mathematical transformations and parameter connectedness [1, 25-27].

HTS may be used for the classification and creation old and new chapters of all science, including computing science.

HTS may be used for the represented of evolution of systems in two directions: 1) from simple system to complex system (example, from classic to quantum mechanics) and 2) conversely, from complex system to simple system (example, from formal logic to mathematical logic) [1].

Hybrid theory of systems is open theory. Parameters of openness are number of generalizing mathematical transformations and parameter of connectedness. Thereby we have finite number of types of systems, but number of systems may be infinite. Hybrid theory of systems allows considering verbal and nonverbal knowledge with one point of view [1, 25-27]. Therefore this theory may be represented as variant of resolution S. Beer centurial problem in cybernetics (problem of complexity) [1, 28].

HTS may be represented as application PA (HTS) to the problem of calculation [1< 25-27]. This theory was used for the problem of matrix computation and problem of arrays sorting [1].

HTS may be connected with problem of computational complexity. This problem was appeared in modern cybernetics for resolution of problem the transition from infinite (analytical) to discrete representation of computing procedures [1, 25-27]. In may be connected with 4 and 5 Smale problems [1, 29].

HTS may be used for the classification of knowledge and science with point of view of their complexity. These results may be represented as theorems [1].

Theorem 1. The classical mechanics is the simple system.

Proof. The classical mechanics is closed system therefore criteria of the reciprocity and simplicity are true. The action principle is the analogous the principle of optimal informative calculations. Parameter of connectedness is equal 1. But its definition of simple system and theorem is proved.

Theorem 2. The quantum mechanics is the semisimple system : 1) in Heisenberg's representation – parametric simple; 2) in Shrödinger's representation – functional simple; 3) in the representation of interaction – semisimple system.

Proof. The quantum mechanics is closed system as classical mechanics. But the criterion of the simplicity isn't true for operators (in Heisenberg's representation), for wave functions (in Shrödinger's representation) and for operators and wave functions (in representation of the interaction). Parameter of connectedness is equal 1 for all three representations. But it is the definitions of proper systems and theorem is proved.

Theorem 3. Logic is a simple system.

Proof. Logic is a closed system. Criteria for reciprocity and simplicity of the system implemented. Parameter connectivity is equaled one. Thus mathematical logic is a simple system.

Theorem 4. Linguistics is a complex system.

Proof. Linguistics is semi-closed or open system. Criteria reciprocity and simplicity may not come true. Parameter connectivity, typically, is greater than one. But it is the definition of a complex system.

These theorems practically are represented the system character of the theoretical (classic) and quantum mechanics in modern science.

Once again we return to the foundations of mathematics. Classical mathematics is characterized by parameter of connectedness that is equal to one. It means that quite complex and sophisticated mathematical system is not mathematical in the classical sense. But the foundations of mathematics we have a theory with a broader subject base as classical mathematics (including mathematical logic and set theory). This theory is in our view should include formalizing the procedure (functional numbers and criteria of reciprocity and simplicity), process analysis and synthesis (qualitative and quantitative transformation) and the problem of uniqueness (parameter of connectedness). This theory is also essential to have provisions that take into account its opening from the system point of view. In polymetric analysis meet this requirement parameter of connectedness and possible failure of certain provisions of criteria of reciprocity and simplicity. The theories of «structural lines» in the foundations of mathematics do not meet these requirements. This provision can be formulated as the following theorem.

Theorem 5. The theory of "structural lines" in the foundations of mathematics (logical, formal and intuitive) can not be extended to all mathematics.

Proof. The theory of «structural lines» is permanent measures, while mathematics because of its development has a variable structure, and each structural element has its own measure. But metamathematycal theory must to be theory with variable measure. We got a contradiction and thus proved the theorem.

If we consider polymetric conceptin terms of H. Kantor expression "The essence of mathematics lies in its freedom", this freedom is included in the variable measure.

Polymetric analysis may be represented as generalization of basic problem of cybernetics in Wiener sense "Cybernetics is the science of the Control and Communication in the Animal and the Machine" [30].

But PA is more general as cybernetics. It may be used as metascience and expert system for real systems and theory of formation of new scientific systems [1].

PA may be represented as "dynamical" expanding formalization of Errol E. Harris polyphasic concept of modern science [1, 31-33]. But Harris method is philosophical and "static", polymetric method is "dynamic". PA allows to select and change measure in inside of proper system and select and change the hierarchy of this system.

PA is universal system of synthesis of knowledge. But this synthesis is realized through measure (number). Each science or knowledge has own treasures and measure. Therefore problem of division science on philosophy and other sciences (see N. R. Campbell [15]) is very relative. The classification science and knowledge with help polymetric concept, according to simplicity-complexity of optimal formalization, is fuller and more corresponded of present state and development of science and knowledge.

Selection of quadratic forms as basic elements of PA is further development of direction of observation many scientists: Pythagor (Pythagorean theorem), Plato (three types of numbers), Descartes (using Pythagorean theorems for creation analytical geometry), B. Riemann (creation Euclidean space as generalizing of analytical geometry), D. Hilbert (Hilbertian spaces) etc.

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PA may be used as theoretical foundations of computer science too. It describes this science in your standing and development more simple, optimal and sufficient as logical or constructive concepts [1], and may be used as expert system for existinf sciences and instrument for creation of new sciences.

PA may be represented as more full formalization E. B. de Condillac "A treatise about systems" [34]. But in this case we have open systems, systems with variable measure and variable hierarchy. These both conditions are interconnected and therefore it represents the one universal system.

Thus, we show that Polymertric Analysis is general theory of open systems and may be used for the resolutions various problems of system type for many sciences.

IV. CONCLUSIONS

1. Basic concepts of theories of open systems are analyzed.
2. Methods of physical theories, which can be used for resolution this problem, are represented.
3. It was shown that de Broglie formula may be used for the unification physical optimality principle (action and entropy) and allow to create theory of informative calculations analogously to physics.
4. Basic concepts of foundation of mathematics and its difficulties are observed with point of view of open systems.
5. Basic necessary conditions for creations of theories of open systems were formulated.
6. Polymetric analysis as example of the universal system of formalization knowledge, which is based on nature of mathematics.
7. One of basic element of polymetric analysis, hybrid theory of systems, may be represented as theory of open systems (systems with variable hierarchy).
8. It was shown that problem of creation theory of open systems is connected with problem of complexity of modern knowledge and science.

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