

A Study on Operations Research and Application of Game Theory In War and Defense

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ABSTRACT

This paper looks forward to understanding the history or Operations Research in war, its importance in the field, its future in the field, and the application of Game theory in war/military operations/ defense. The paper will discuss game theory and its real life application in a war like situation. The game theory analysis will be limited to a two person game and a zero sum game with same saddle points. To truly understand how OR is used in war, a detailed literature is provided on its history and importance and its future. The paper concludes by several examples and situations that show the application of game theory. The paper overall looks to give a thorough understanding of operations research, its origin, and its applicability in war.

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CHAPTER 1: HOW HAS OR IN WAR EVOLVED SINCE THE PAST

Aurther C. Clarke, a British Science fiction writer once said that 'OR is the art of winning wars without actually fighting the war'.

The term 'Operations Research' was coined during the Second World War, when a group of scientists were called upon by the British Military Management to apply a scientific approach in the study of military operations to win the war. The objective was to discover the most effective usage of limited military resources by the application of quantitative techniques. This originated in the British military in the second half of the 1930s as a response to the mobilization of the German air force. Hence as a formal discipline OR originated from the efforts of the military advisors during the late 1930s. The tasks of these groups were to discover optimal ways to use current military forces, weapons, and additional equipment. These teams had various scientists from biologists, physicists, chemists to even engineers, and mathematicians, and they worked along with the military operations, assembling data and analyzing what went on.

The British held a massive air defense exercise in 1938 which helped them discover that the radar system could be used to spot aircrafts, but there were several problems related to the actual use of radar. Superintendent Albert Percival Rowe at Bawdsey Manor then made teams of scientists and engineers to do research into the operational aspects of the use of radar. These groups are generally recognized as the first Operation Research sections. The success of these operational researchers led to the creation of OR teams in other parts of the Royal Air Force, the Navy and the Army. Physicist Patrick Maynard Stuart Blackett formed the operations research team known as "Blackett's Circus" to investigate the use of radar in anti-aircraft gunnery in 1940. It comprised of three physiologists, two mathematicians, two mathematical physicists, one astrophysicist, one Army officer, one surveyor, and one physicist. Blackett is often credited as the most significant individual for the progress of operations research in Britain. In the beginning the OR groups primarily worked with problems related to the use of radar in anti-aircraft and anti-submarine warfare but their work area was gradually expanded to include other problems including strategy and logistics.

A. The Use of Operations Research in World War II Involved Analyzation and Creation Of:

➤ Intelligence Networks: Intelligence Operations is the process by which the government and military groups methodically accumulate and analyze information in order to discover the intentions of their enemies, protecting themselves from their enemies and taking advantage of their rivals' weaknesses.

B. The Creation of an Intelligence Network involved:

- 1. Collection of data through Human Intelligence
- 2. Signal Collection by tapping phone lines and monitoring radio signal Collection of Photographic evidence using satellite imagery and air photography.

Evaluation by combining raw intelligence with relevant data. They were fitted together to build patterns and then were secretly transmitted to allied officials through these networks made using techniques of Operations Research.

> Transportation and Supply Networks: These networks were used to supply military personnel with petroleum, food, ammunition and more soldiers if needed. The transportation model and the assignment model of Operations Research helped to minimise the cost and time of distribution of these supplies from various sources to various destinations and the allocation of the various resources to the various activities on one to one basis. They even help manage problems under barriers like a poor existing road structure.

On the other hand in the United States Vannevar Bush's vision was to develop an organizational structure that would make scientific research in warfare, defense, and development of new weapons more effective. 1940 National Defense Research Committee (NDRC) established under President Roosevelt with Bush as Chairman.

1941 Bush became the leader of the Office of Scientific Research and Development (OSRD).

1943 Operations research was imported from Great Britain and integrated into OSRD as Office of Field Service.

Bush and OSRD have been named the key players in the promotion of operations research in the USA during World War II. American scientists and mathematicians came to play decisive roles in the Second World War as OSRD proved it to be a very effective and successful organization. This structure was indeed unsuited with operations research that was developed in Great Britain exactly to function as a facilitating link between the creators of new technologies and the users of these new tools.

Philip Morse's Antisubmarine Warfare Operations Research Group was established within the Anti-Submarine Warfare Unit in the US Navy in 1942 with Captain Wilder D. Baker as the chief. The group was expanded into the Operations Research Group during the war, and afterwards it was renamed to the Navy's Operations Evaluation Group. The group started by developing a search theory for planes patrolling waters to locate rival submarines. The group encountered a major obstacle which was lack of relevant data, specifically enumerated data. Hence, the scientists went to anti-submarine bases, join some of the flights, and accumulated the required data. One more change of practice that the group suggested after their studies of operations was the setting of the depth charges the aircrafts dropped when they attacked the submarines. The usually depth setting for the charges which was seventy-five feet, was changed to thirty feet. As a result, the number of submarines sunk improved significantly.

Origin of Linear Programming: Logistic Planning in the Air Force came out of work done primarily by the mathematician George Bernard Dantzig, who was employed by the Army Air Force's Combat Analysis Branch of Statistical Control in to work on what they called "programming planning methods" in the US Air Force during and after the war. An Air Force program was a timetable for activities, it was an enormous logistic plan that was based on figures of flown sorties, dropped bombs etc. Which were then used in organizing armed forces and equipment, arranging training, providing logistic support for activities etc. But it took about seven months to calculate such a program. A small group had become convinced that mathematical techniques, backed up by extensive electronic computers, were required in order to tackle the programming problem by 1947. A no suitable computer existed, the Air Force had to fund the development of the essential equipment. In October 1948 Project SCOOP was established (Scientific Computing of Optimum Programs). From 1946 to 1952 Dantzig worked on the assignment developing a mathematical linear model for the Air Force programming problem based on Wassily Leontief's economical Inter-industry Input-Output Model. The prospect of comparing consistent programs in order to choose for example the cheapest one became an option after the advent of the computer. The objective now was to determine that program which will, most closely achieve objectives without going above the specified resource limitations.

Dantzig ended up with linear programming which is identical to the formulation of a linear programming problem given in most workbooks nowadays. He also developed the simplex algorithm for solving linear programming problems while he worked for the Air Force.

CHAPTER 2: IMPORTANCE OF OPERATION RESEARCH IN WAR

As the name indicates the word Operation is used to refer to the 'problems of military' and the word Research is use for 'inventing new method'.

A research design to determine most efficient way to do something new in war. It assists high level decision makers with analyses that could be used to support planning for strategic ground, air, and maritime operations during war.

It helps during war by analyzation and network building. It helps in:

- Intelligence
- Transportation
- Supply

Intelligence Operations: Process by which the government and military. Groups of organization systematically collect and evaluate info for the process of discovering the intentions of their rival and exploiting their rival's weakness. Operation research is a 'war baby'. It is because the first problem attempted to solve in a systematic way was concerned with how to set time fuse bomb to be dropped from an aircraft on to a submarine.

Operation research can help the military in various ways during war. Some of the important areas in military OR are discussed in subsequent sections:-

- 1. Selection and Acquisition of Weapons- The problem of evaluation of weapon efficiencies, or weapon economics for brevity, arises at all levels. One may ask: Is it an advantage to change from say Rifle A to Rifle B, and if so what will be the relative advantage gained? Is it worthwhile, and if so, to what extent, to replace TNT with RDX in shells? OR also helps in the evaluation of guns and rockets on the Basis of a, measure of effectiveness (MOE)' and the total cost of the system to be inducted.
- **2.** *Evaluation of Tactical Plans-* Is the existing deployment of a weapon most effective in achieving its intended mission? Can the effectiveness of an operation be improved through scientific analysis?

It helps in the deployment patterns of various weapons for defense of some vulnerable areas, search schemes for an area, etc.

- 3. System Modification and Improvement- It also helps in comparing the cost of indigenous production vs license production vs import of weapon system and specifying the requirement of spares over the life cycle of weapon systems being acquired. Within the scope of logistics management, OR studies have also been undertaken to improve the movement transportation of forces in narrow mountainous terrains, transshipment of supplies from base to forward units, positioning of communication radio sets in order to ensure a sufficient signal strength for communication in unfavorable terrain, and scales for authorization of these equipment to Army units.
- 4. Design and Development of weapon system- The design and development activity of the weapons in the MOD is initiated by the Services by defining the General Staff Qualitative Requirements (gsqrs) which specify the operational characteristics that the users (Services) desire to have in the proposed weapon system. The work in a large system is usually divided into subsystems and each of these may need the help 'of other R&D agencies, universities and other public and private organizations.
- 5. War Games- Training of Service officers to appreciate a threat situation and plan for remedial measures within the available resources effectively is an important activity in defense. These are necessary in order to reevaluate and ascertain the effectiveness of the existing manpower and weapon systems in a conflict scenario.

CHAPTER 3: WHERE IS OR USED IN WAR?

The application of operations research in war goes back to hundreds of years of implementation and the follow ups thereafter to work on the areas gone wrong. Originally the operations research process was used to study the complexities involves in dealing with various sciences to take decisions in the matters of men, machines and materials allocation and usage during war.

The premise for anticipating an attack or the probable threats is considering the occurrences and frequencies of the impacts of an occurred attack in the past and putting the same quantified figure in a simulation model that simulates that this event could have happened several times and output results that the simulation tool gives you an advance look at something you can't collect data over time

In the modern warfare system where every country is majorly focused on fortifying its army, the apply chain optimization helps the entity in making the best rational decision in terms of resource availability and economic viability as far as the material acquisition and army strength is concerned. This tool directs the forces to what to buy, when to buy and at what prices to buy to give them the edge of economies of scale and external market conditions.

According to the extended study of stochastic training profile model, something as simple as sending your tanks to Iraq requires the forces to assess the terrain and topological conditions of the land so that the tank being send is built with the appropriate suspension and aero-dynamics so that it could fit in the terrain models.

CHAPTER 4: ANALYSIS OF GAME THEORY IN OR

The theory of game provide a mathematical model that can be useful in decision acing concept – in this case in war.

Game Theory is the study of various situations where completion exists. If there exists two or more parties battling one another, game theory maps out possible strategies and methods they may use to come out on top. Strategies may be of a kind where competitors use influence to their advantage.

Key words:

- Situation the game that is taking place
- Players the parties involved
- Pay off what each player received from the outcome of a situation
- Strategy the path or plan of action that each party decides for every step of the game
- Tactics the parties' respective line to attack or to defend

A necessary part of the situation is the need for conflict. For game theory to work the relationship between the parties must be hostile and a conflict needs to exist in the situation. At the end of the situation, a fact that needs to be advocated is that one player must get benefits that the other doesn't. The payoff of one party will be greater than that of the other party.

Two person game – game with 2 players

Zero sum game – game where one party loses, one party wins

2 person game + zero sum game = base for games with many players

In war we see a direct replica of game theory. There exists a conflicts between the two (or more) players, and the game turns out to be of type zero sum – one party wins, the other loses. The parties attack and defend back and forth until either there exists complete defeat or mutual agreement to end. This is just like having a game end.

The game comes to an end following the rules of engagement. The rules of engagement are the rules that dictate what the party can do in the next stage. For example, in Ludo – a party may only come onto the board if they get a 6. Once the rules of engagement say that following this move the other party must back off, the war ends.

Strategies = circumstantial depending on the point the game is in

Different situations = different strategies

Different tactics = different strategies

For example -

Whilst in war, one of the party's strategy is to use the tactic of staying hidden till nightfall and then attack. They do this repeatedly and their pattern of playing will become detectable by the other party.

An alternative strategy is one that is used when a party's original strategy exhausts and becomes very evident. An alternative strategy in the case above could be that the party decides to hide in trees and attack the other party in the afternoon as they won't be expecting it.

A strategy gives the parties leverage to decide on a move seeing what others have done. There can be multiple stages in a game and war, with multiple moves.

Key words in the solving game theory:

- Pay off matrix matrix in which the game analysis occurs
- Game tree extensive form of analysis using a diagram to analyse the result
- Saddle point equilibrium point point that's minimum in the row and maximum in the column
- Pure Strategy when both players use the same strategy
- Mixed Strategy when both players guess on the call of action and use probability to measure pure strategies
- Value of the game final gains and losses by both parties, the pay offs

The rows shows attacks and the columns show defends.

All methods of Game theory used in War:

- Two person
- Zero sums
- Graphical solution of an xn or mx2 game
- The min-max theorem for matrix games
- Linear programming solution for matrix games
- The smow Shapley theorem for matrix games

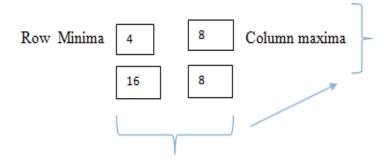
A. Solving of Game Theory In A Basic Sum

If a payoff matrix of a war move is as below:

		В	
		Ι	II
A	Ι	4	12
	II	16	8

To solve:

		В	
		Ι	II
A	Ι	4	12
	II	16	8



Saddle = 8 (because maximum and minimum are same)

Strategy by A – Strategy II

Strategy by B – Strategy II

Value of game = 4

2 countries are at war. Currently both countries armies have set up 2 tents. Party A is going to attack one of the tent. Tent one is worth 20 units, while Tent 2 is worth 10 units. If Party A attacks a tent that is defended, he loses the battle and gets nothing at all. If Party A attacks a tent that is not defended, he gets the value of that tent too and Party B gets the tent taken away from him – he loses their respective value.

Payoff matrix:

		Party B	
		Defend Tent 1 (D1)	Defend Tent 2 (D2)
Party A	Attack Tent 1 (A1)	0	20
	Attack Tent 2 (A2)	10	0

Here the act of probability comes to play. Since Tent 1 is worth 2 times the value of Tent 2, chances that B will defend Tent 1 are higher than them defending Tent 2. The probability of A attacking Tent 1 is twice as much as of Tent 1.

Therefore, best strategy for B is to defend Tent 1 with 2/3 probability and

Tent 1 with 1/3probabiltiy. Vice versa goes for A attacking Tent 1 and 2.

These probabilities create a mixed strategy.

When A plays:

B defends Tent 1 - 0*1/3 + 10*2/3 = 6.67

B defends Tent 2 - 20*1/3 + 0*2/3 = 6.67

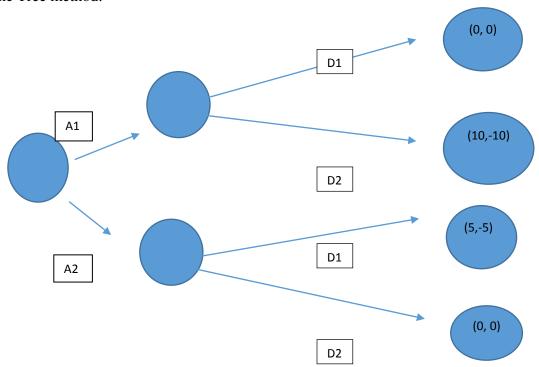
When B plays:

A attacks Tent 1 - 0*2/3 + 20*1/3 = 6.67

A attacks Tent 2 - 10*1/3 + 0*2/3 = 6.67

The value of the game is 6.67 units as A's strategy allows them to gain minimum 6.67 units and B's strategy makes sure that he doesn't lose more than 6.67 units.

Game Tree method:



CHAPTER 5: CONCLUSION

Our view is that the results of OR practice, especially in long-range planning studies, are often now well

regarded or used by decision makers. There are serious concerns about the quality of OR practice. It is no secret

that the two major operations research/management science societies in the world experienced losses rather than

growth in membership in previous years.

Here are some research directions that, if pursued, may provide a much needed focus for the field of operations

research.

• Versatility: A planning objective for the practice of OR

Versatility means ability to adapt or be adapted to many different functions and activities. Over the past 10

years there has been a marked increase in the criticism of OR practice both from clients and within the

discipline. The criticism is diverse: quality is poor; there is too much optimization; the results of which are

usually irrelevant for decision making; the theory is not implemented correctly; there is too much planning

analysis and not enough operations analysis. The new OR plans must be made in such a way so that they can be

applied to each and every situation.

For example: In the late 1950's and early 1960's, emphasis in US defense analysis shifted from short-term

operational problem solving to broader, mid-to long range planning issues such as weapon and force

developments. The planning, programming, budgeting (PPB) system was initiated in 1961 along with the

requirement for cost effectiveness analysis to justify military request for resources. In the 17 years since the

establishment of the PPB systems, the cost effectiveness philosophy and procedures have, at least formally,

become an integral part of the defense planning process.

This had been developed by decision theorists and macroeconomists for many years before dud's requirement

for them, and these structures have been extended by operations researchers since then. They contain a number

of decision elements such as alternative courses of action, future states, state probabilities, effectiveness

measure and cost measure. OR plans must be versatile as well as efficient in case of costs, quality, timeliness

etc.

• Research on the practice of OR

The results of OR practice are not well thought of by decision makers, and the quality of the practice is questioned by many. It is no secret that some practitioners might be deemed inadequately prepared by their peers. Although practitioners rarely fail to inform educators of their critical views regarding the education of new analysts, a large share of the criticism also comes from academicians involved with the process.

Summary

If current trends continue, OR may lose its identity as a distinct activity and be assimilated into other fields of endeavor.

Although, the versatility ideas are clearly still imprecise, and methods are not available to assist in their implementation, we can and we should pursue the spirit of the concept in our planning and analysis support to decision-makers and managers.

Conscious efforts should be initiated to develop an empirically based operational science describing the important operational and management processes that are the basis and unique content of OR. It is time to stop the continual controversy between or practitioners and mathematical theorists regarding the nature of OR, since neither activity by itself is sufficient. OR eventually must involve all three areas- OR practice, Operational science and mathematical OR methodology- synergistically, with primary emphasis on the first two.

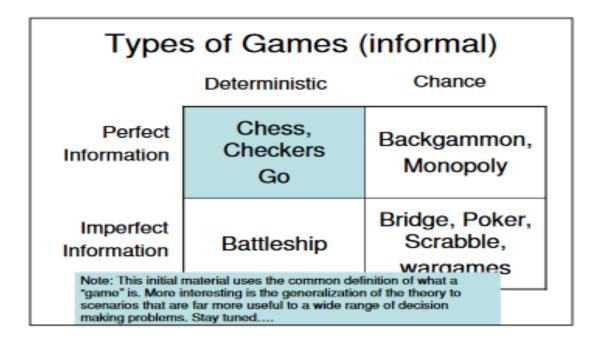
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APPENDIX A: TYPES OF GAMES



Types of Games (informal)			
	Deterministic	Chance	
Perfect Information	Chess, Checkers Go	Backgammon, Monopoly	
Imperfect Information	Battleship	Bridge, Poker, Scrabble, wargames	
		•	

Definitions

- Two-player game: Player A and B. Player A starts.
- Deterministic: None of the moves/states are subject to chance (no random draws).
- Perfect information: Both players see all the states and decisions. Each decision is made sequentially.
- Zero-sum: Player's A gain is exactly equal to player B's loss. One of the player's must win or there is a draw (both gains are equal).

APPENDIX B: SUMS IN GAME THEORY

Sum 1:

2.1.2 Two Players, Zero-sum

A game is called **zero-sum** if the sum of payoffs equals zero for any outcome. That means that the winnings of the winning players are paid by the losses of the losing players.

For zero-sum two-player games, the bimatrix representation of the game can be simplified: the payoff of the second player doesn't have to be displayed, since it is the negative of the payoff of the first player.

Example 2 Assume we are playing ROCK-SCISSORS-PAPER for one dollar. Then the payoff matrix is

	Rock	Scissors	Paper
Rock	0	1	-1
Scissors	-1	0	1
Paper	1	-1	0

The first cell says "0", which stands for "0, 0" a payoff of 0 for both players. The second cell entry of "1" should be read as "1, -1", a payoff of 1 for Ann which has to be paid by Beth, therefore a payoff of -1 for Beth.

Sum 2:

2.1.3 Three or More Players

If we have more than two players, we need another systematic way to generate the needed $k_1 \cdot k_2 \cdot \cdots \cdot k_n$ cells corresponding to the different outcomes, into which we write the n payoffs for the n players. Here is an example:

Example 3 LEGISLATORS' VOTE: Three legislators vote whether they allow themselves a raise in salary of \$2000 per year. Since voters are observing the vote, there is some loss of face for a legislator to vote for a raise. Let's assume that the legislators estimate that loss of face is worth \$1000 per year. What happens if all three vote at the same time? (This game is a variant of the game described in [K2007]).

This is a simultaneous three-player game. It is best visualized with two matrices. Player A chooses the matrix, B chooses the row, and C chooses the column. The payoffs (in thousands of dollars) are

	C votes for raise	C votes against it	
B votes for raise	1, 1, 1	1, 1, 2	
B votes against	1, 2, 1	-1, 0, 0	

A vo	tes against a C votes for raise	C votes against it
B votes for raise	2, 1, 1	0, -1, 0
B votes against	0, 0, -1	0, 0, 0

APPENDIX C: GAME THEORY USE IN MILITARY

A study by RAND Project AIR FORCE (PAF) shows how military planners can use game theory to understand the effects of U.S. strategy and capabilities on the enemy in TCT operations. Game theory uses mathematics to model human decisionmaking in competitive situations. It is ideally suited for analyzing military situations because it depicts the realistic situation in which both sides are free to choose their best "moves" and to adjust their strategy over time. Military planners can apply these principles to TCT operations through game theoretic analysis. The method consists of the following steps:

- Determine the tactical options available to each side. For example, in a simple SEAD operation, the attacker can choose to
 fly a strike aircraft or a SEAD aircraft. The defender may choose to activate SAMs or to leave them inactive.
- Assign a numerical value to each possible outcome. Analysts represent commanders in the field by judging the potential
 gain or loss of an exchange. These numbers reflect real-world measurements such as the strength of a weapon system and
 the probability of hitting a target.
- Calculate all possible strategies and their outcomes. Intelligent opponents vary their tactics in order to appear
 unpredictable to the enemy. Thus, a combatant's overall strategy is determined by how often he chooses one tactical option
 over another. Analysts calculate all possible strategies and the net gain or loss to each side.
- Find each side's optimum strategy. Experience teaches that as opponents in a game adjust to each other's actions, each
 player will eventually settle on an optimum strategy. In military terms, the optimum strategy is not necessarily the most
 desirable outcome (i.e., winning the exchange), but the best that one can do against an opponent of given strength.
- Determine the expected result of the game. Having found each side's optimum strategy, analysts check to see whether the
 outcome of the encounter favors the attacker or the defender.