New Approach to Solve Assignment Problem

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Abstract: - Assignment model comes under the class of linear programming model, which looks alike with transportation model with an objective function of minimizing the time or cost of manufacturing the products by allocating one job to one machine or one machine to one job or one destination to one origin or one origin to one destination only. Basically assignment model is a minimization model. . In this paper to introduce a new approach to solve Assignment problem namely alternate method for solving assignment problem. An example using Alternate methods and the existing Hungarian method has been solved and compared it.

Keywords: -Assignment Problem, Hungarian Algorithm for Assignment Problem, Alternate Method for Assignment Problem, Linear Integer Programming, Optimization.

I. INTRODUCTION

The Assignment problem is a special structure of Transportation Problem, in which number of jobs (task) is equal to number of persons (facilities).Thus the objective of the problem is how the assignment should be made to achieved allocation. In the assignment model worker represent source and jobs represent destination. The supply amount at each source exactly 1.for example if n=5 person can assigned to 5 jobs. Then the number of such possible ways is 5! =120.these allocation will take a large time. There are many methods to develop an easy computational technique for such problem. Hungarian method is one of them See [1, 2] for the history of this method. Various articles [3, 4, 5, and 6] have been published for solving these type of problem. In this paper we achieved exact optimal solution, which is same as that of Hungarian method.

II. FORMATION OF ASSIGNMENT PROBLEM

The Assignment problem can be stated in the form of n x n matrix, $[C_{ij}]$ called the cost matrix, where C_{ij} assigning i-th job to j-th person.

Person

		1	2			j			n
	1	C_{11}	C_{12}			C_{1j}			C_{1n}
	2	C_{21}	C_{22}			C_{2j}			C_{2n}
Job									
	i	C_{i1}	C_{i2}			C_{ij}			C_{in}
	n	C_{n1}	C_{n2}			C_{nj}			C_{nn}

III. METHOD FOR SOLVING AN ASSIGNMENT PROBLEM

Hungarian Method for solving a minimal assignment problem;

1. Subtract the minimum element of each row in the cost matrix [c_{ij}] from every element of the corresponding row.

2. Subtract the minimum element of each column in the reduced matrix obtained in the step 1from every element of the corresponding column.

3. (a) Starting with row 1 of the matrix obtained in stepII,examine rows successively until a row with exactly one zero element is found, mark [] at this zero , as an assignment will be made there. Mark[x] at all other zeros in the column to show that they cannot be used to make other assignments. Proceed in the way until the last row is examined.
   (b) After examining all the rows completely proceed similarly examining the columns, examine the columns starting with column 1 until a column containing exactly one unmarked zero is found, mark [] at this zero and cross at all zero of the row in which []is marked . Proceed in this way until the last column is examined
   (c) Continue these operations (a) and (b) successively until we reach to any of the two situations. (i) All the zeros are marked or crossed. Or (ii) the remaining zeros lies at least two in each row and column in case (i),we have a maximal assignment and in case (ii) still we have some zeros to be treated for which we use the trial and error method to avoid the use of highly complicated algorithm. Now there are two possibilities;

   (i) If it has an assignment in every row and every column, then the complete optimal assignment is obtained.
   (ii) If it does not contain assignment in every row and every column, one has to modify the cost matrix by adding or subtracting to create some more zeros in it. For this proceed to the next step IV.

4. When the matrix obtained in the step 3 does not contain assignment in every row and every column then we draw the minimum number of horizontal and vertical lines necessary to cover all zeros at least once. For this the following procedure is adopted:
   (i) Mark (√) all rows for which assignment has not been made.
   (ii) Mark (√) column which have zeros is marked rows.
   (iii) Mark (√) rows which have assignment in marked columns.
   (iv) Repeat step (ii) (iii) until the chain of marking ends.
   (v) Draw minimum number of lines through unmarked rows and through marked columns to cover all the zeros. This procedure will yield the minimum number of lines that will pass through all zeros.

Select the smallest of the elements that do not have a line through them subtract it from all the elements that do not have a line through them, add it to every element that lies at the intersection of two lines and leave the remaining elements of the matrix unchanged.

5. At the end of step of V numbers of zeros are increased in the matrix than that in step III and to obtain the desired solution.

A. *Numerical Example*

Five men are available to do five different jobs. From past records, the time (in hour) that each man takes to do each job is known and is given in the following table:

Job

Man

	A	B	C	D	E
1	1	3	2	3	6
2	2	4	3	1	5
3	5	6	3	4	6
4	3	1	4	2	2
5	1	5	6	5	4

Job

Man

	A	B	C	D	E
1	1	3	2	3	6
2	2	4	3	1	5
3	5	6	3	4	6
4	3	1	4	2	2
5	1	5	6	5	4

Solution:

Step 1. Subtract the smallest element of each row from every element of the corresponding now, we get the following matrix:

	A	B	C	D	E
1	0	2	1	2	5
2	1	3	2	0	4
3	2	3	0	1	3
4	2	0	3	1	1
5	0	4	5	4	3

Step 2.Subtract the smallest element of each column from every element of the corresponding column, we get the following matrix:

	A	B	C	D	E
1	0	2	1	2	4
2	1	3	2	0	3
3	2	3	0	1	2
4	2	0	3	1	0
5	0	4	5	4	2

Step3.Giving the zero assignment in usual manner and get reduced matrix.

	A	B	C	D	E
1	[0]	2	1	2	4
2	1	3	2	[0]	3
3	2	3	[0]	1	2
4	2	[0]	3	1	0
5	0	4	5	4	2

Since row 5 and column 5 have no assignment we proceed to the next step

Step 4.The minimum numbers of lines drawn in the usual manner are 4.

Step5. Now the smallest of the elements that don not contains line through them is 1. Subtracting this element 1 from the elements that do not have a line through them . adding to every elements that lies at the intersection of two lines and leaving the remaining elements unchanged,

	A	B	C	D	E
1	0	1	0	1	3
2	2	3	2	[0]	3
3	3	3	[0]	1	2
4	3	[0]	3	1	0
5	[0]	3	4	3	1

Step6. Again repeating the step 3 we make the zero assignments in matrix and see that even now the row 1 and column 5do not contain any assignments .Therefore we again repeat step 4 of drawing lines.

Step7. According to our usual manner the minimum number of lines drawn is 4

Step 8.Again repeating step5, we get following matrix.

	A	B	C	D	E
1	[0]	~~0~~	~~0~~	~~0~~	2
2	3	3	2	[0]	3
3	3	2	[0]	~~0~~	1
4	4	[0]	4	1	~~0~~
5	~~0~~	2	4	2	[0]

Step 9.Repeating the step 3 we make the zero assignments and get the following option assignments,

1→A,2→D,3→C,4→D,5→E.

So minimal assignment: 1+1+3+1+4=10.

IV.ALTERNATE METHOD FOR SOLVING ASSIGNMENT PROBLEM

This section presents an alternate method to solve the assignment problem which is different from the preceding method.

The new algorithm is as follows:

1. Subtract the smallest element of each row from every element of the corresponding row.
2. Subtract smallest element of each column from every element of the corresponding column.
3. Consider the location of zero at each row. If row contain only one zero then assign it for the corresponding row and delete the corresponding row and column after allocation. Otherwise read the location of zero below for further process.
4. If there is more than one zero than find the successor of zero and compare the maximum value and assign zero
5. Repeating (3), (4) and find the optimal solution.

A. Numerical Example Using Alternate Method

Job

Man

	A	B	C	D	E
1	1	3	2	3	6
2	2	4	3	1	5
3	5	6	3	4	6
4	3	1	4	2	2
5	1	5	6	5	4

Step1. Applying step 1, 2 to obtain following matrix

	A	B	C	D	E
1	0	2	1	2	4
2	1	3	2	0	3
3	2	3	0	1	2
4	2	0	3	1	0
5	0	4	5	4	2

Observe the location of zero

ROW	COLUMN
1	A
2	D
3	C
4	B,E
5	A

Assign 2→D and remove corresponding row and column of above matrix.

Step 2.Reduced matrix after row reduction:

	A	B	C	E
1	0	2	1	4
3	2	3	0	2
4	2	0	3	0
5	0	4	5	2

Again observe the location of zero

ROW	COLUMN
1	A
3	C
4	B,E
5	A

Assign 3→C and remove corresponding row and column of the above matrix.

Step 3.Reduced matrix after row reduction:

	A	B	E
1	0	2	4
4	2	0	0
5	0	4	2

Observe the location of zero

ROW	Column
1	A
4	B,E
5	A

Since row 1 and 5 have same number of zero then take any one.

Assign 1→A and remove corresponding row and column.

Step 4.Reduced matrix.

	B	E
4	0	0
5	4	2

In the 5 row there is no zero then subtract the smallest element 2 from all element of that row.

Step5.

	B	E
4	0	0
5	2	0

Assign 5→E and 4→B.

So minimal Assignment:1+3+1+4+1=10

Comparision

PROBLEM	HUNGARIAN METHOD	ALTERNATE METHOD
1.	10	10

V. CONCLUSION

This Paper seeks to solve a placement problem. It was observed that the solution is the minimum achievable result. The use of a scientific approach gives a systematic and transparent solution. The new alternate method can be applicable in all kind of assignment problem. It may benefit from the proposed approach for placement and selection. We get the optimal solution which is same as the optimal solutions of Hungarian-method. Therefore this paper introduces a different approach which is easy to solve Assignment problem.

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