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Numerical Simulation of Vortex Flows in Cylindrical Geometries

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Abstract—In this work, a numerical model has been developed to study about the steady, three dimensional vortex breakdown in a closed cylindrical container with a rotating end wall. The velocity fields are obtained by solving numerically the three dimensional Navier-Stokes equation. The attraction of swirling flow generated inside an enclosed cylinder by the rotation of one of its end wall is mainly due to the axial vortex breakdown which is characterized by four variables such as the height, radius of the cylinder, the constant angular speed of the end wall and the kinematic viscosity. These four variables are used to define two non-dimensional parameters which completely satisfy the flow conditions, they are the aspect ratio and the rotational Reynolds number. The results of the study has been presented and discussed in this paper.

Keywords—Finite Volume Method, Reynolds Number, Vortex Breakdown

I. INTRODUCTION

Swirling flows are observed in variety of natural phenomenon such as tornadoes and typhoons, and its importance has been experience in a number of technical applications, such as aeronautics, heat exchanger, spray drying, separation, combustion, etc. In combustion systems, such as in gas turbine engines, diesel engines, industrial burners, boilers, swirling flows were originally used to improve and control the mixing rate between fuel and oxidant streams in order to achieve flame geometries and heat release rate appropriate to the particular process application. Swirling flows are generated in a cylindrical container with one rotating end wall give rise to axial vortex breakdown, even under steady, laminar, and axisymmetric conditions. The basic configuration consists of a closed cylindrical container with one rotating end wall. It gives rise to only two dimensionless parameters, viz., a Reynolds number Re based on the angular velocity and the radius R of the rotating disk that forms the end wall of the container, and aspect ratio (H/R), where H denotes the container height.

The motion of a viscous fluid contained in a closed cylinder, with a rotating disk lid which recirculates the flow inside the container, poses an attractive example of confined swirling flow. For certain combinations of (aspect ratio and Reynolds number), the confined vortex undergo breakdown. One of the particular interests is that vortex breakdown may be observed under laminar and steady conditions, without the influence of turbulence. When increasing the swirl, a strong coupling develops between axial and tangential velocity components. A point is reached, when the adverse pressure gradient along the jet axis cannot be further overcome by the kinetic energy of the fluid particles flowing in the axial direction, and a recirculation flow is set up in the central portion of the jet. The formation of the recirculation flow zone, forms the vortex breakdown. It is an abrupt change of flow structure that occurs in swirling flows. Experimental investigations (e.g.Cullen, Escudier 1996) [1] have demonstrated that for Re > 1000, the flow is steady and that the secondary meridional flow has streamlines with monotonic curvature. In cylindrical co-ordinate system with the origin at the center of the rotating endwall, this means that the azimuthal component of vorticity, associated with the simple overturning meridional flow is negative. Above a critical Re, which depends on aspect ratio, the experiments have shown that the stream tubes become wavy and at higher Re the flow on the axis stagnates and leads to the formation of one or more steady axisymmetric vortex breakdown bubbles. Lopez(1988) [2] obtained numerical solutions of the axisymmetric Navier-Stokes equations for this flow. These show that the transition to breakdown in the flow is associated with the generation of a positive component of vorticity through the tilting and stretching of the vorticity associated with the central vortex. This vorticity vector field primarily consists of a negative component in the vertical direction, due to the spiraling motion down towards the Ekman layer, and a negative azimuthal component due to the meridional overturning flow. In Brown and Lopez(1990) [3] this vortex deformation mechanism was investigated analytically by considering an inviscid swirling flow. It was found that for such a flow, wavy solutions, and hence vortex breakdown, are possible as a result of vortex deformation when the helix angle of the velocity vector is greater than that of the vorticity vector, the helix angle of a vector being the ratio of its azimuthal to vertical components.

Lopez (1989) [4], which compares an experimental visualization image of vortex breakdown bubbles with the streamlines obtained from their axisymmetric flow computations. The computations capture correctly the shape, size and location of the bubbles but, as one would anticipate, fail to reproduce the distinct asymmetric folds observed in the laboratory images at the downstream end of the first bubble. The basic features that have emerged from the experiments are: (1) abrupt and drastic structural changes occur in a vortex breakdown, (2) axial flow in the core decelerates, sometimes resulting in stagnation and reversal of flow, (3) the flow is unsteady within the breakdown structure and turbulent downstream, (4) axisymmetric (or bubble) breakdown is characterized by slow oscillations, i.e., flow is nearly steady, (5) the bubble structure is very complex consisting of two recirculating regions and four stagnation points, and (6) the breakdown itself is not a result of instability but a sudden and finite transition from one state to the other. Blackburn and Lopez (2000) [5], and Marques and Lopez (2001) this symmetry breaking of the flow is attributed to an inflectional instability of the swirling jet produced by the turning of the Ekman layer on the stationary vertical sidewall. This result looks to be confirmed by a recent stability analysis of Blackburn (2002) carried out in a cavity of aspect ratio=2.5 and at Re=4000. The symmetry breaking was recently explored experimentally and numerically by Lopez et al. (2004) [6], in an extended problem of flow in a cylinder of length 2H, but driven by two co-rotating rigid end walls. In this configuration the mid-plane is a reflection boundary that corresponds to a flat, stress-free interface. Their study showed that for a shallow system (aspect ratio = 0.26), the flat stress-free model fails to capture the primary instability by imposing a hidden symmetry condition on the numerical solutions, restricting the solution to an even z-parity subspace. Nevertheless in the deep system (aspect ratio= 2), they obtained good agreement between experiments and computations and so the imperfections in the physical free-surface experiment do not qualitatively change the dynamics. In the present work, an attempt has been made to develop a numerical model for the axial vortex breakdown in cylinder with one of its endwalls is rotating. In the entire case studied in this paper, Re is the rotational Reynolds number, R is the radius of the cylinder and H is the height of the cylinder. A numerical code has been developed in FORTRAN, numerical simulation were carried out and the results are interpreted using PARAVIEW.

II. MODELLING

The geometry of problem selected for the investigation has the shape of a cylinder as shown in fig.1.The top end wall of the cylinder is rotating with a constant angular speed and the other parts are stationary. Meshing has been done such that close grid lines have been placed near the axis of the cylinder.

Total number of cells used is and 789800 number of nodes used is 772299.

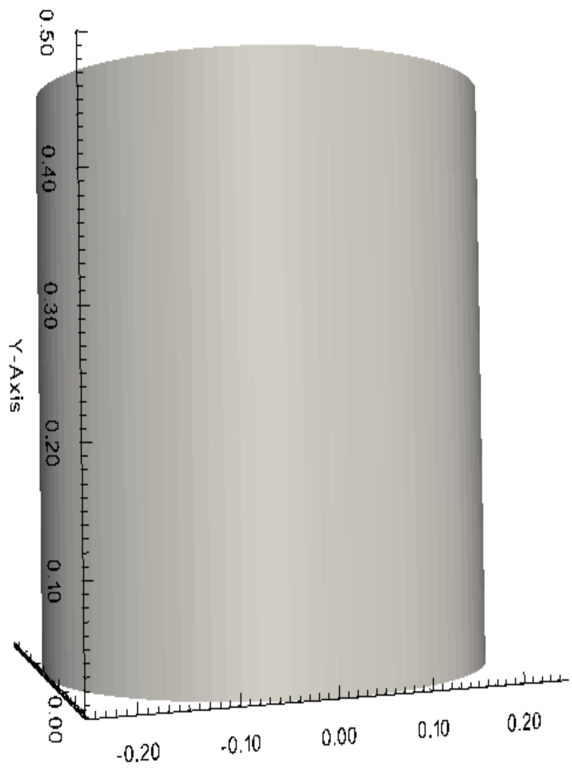

Fig. 1.Geometry of the Cylinder.

III. MATHEMATICAL MODELLING

The cylinder has a height-to-radius ratio, called aspect ratio of 2 and a rotational Reynolds number (Re□□r^2/□).The partial differential equations governing the flow are written in Cartesian coordinates such as the momentum and continuity equations are given in equations (1-2). Velocity components are evaluated by solving equation (1) which is a pressure linked momentum equation.

The equations are given in the following:

$$\frac{\partial u}{\partial t} + u.\nabla u = -\frac{1}{\rho}\nabla P + \vartheta\nabla^2 u \quad (1)$$

$$\frac{\partial u}{\partial t} + \nabla.u = 0 \quad (2)$$

Numerical method for the solution is discussed in the following section.

IV. BOUNDARY CONDITIONS

The top lid of the cylinder rotates with a constant speed and the bottom lid is stationary.

- At top lid, $u = -\omega * z, v = 0, w = \omega * x$
- Over other walls, $u = 0, v = 0, w = 0$

V. NUMERICAL METHODOLOGY

The governing equations (1) to (2) are non linear coupled partial differential equations. These equations are discretised using a finite volume method on a, structured and collocated grid. A projection method [14] is employed for decoupling velocity and pressure. Projection methods are fractional step methods based on decomposition of velocity field into an intermediate velocity and gradient of a scalar field. Fractional step methods integrate the Navier-Stokes equations in time, at each time-step the momentum equation is solved without considering the pressure field to yield an intermediate velocity field that does not satisfy the continuity equation. The pressure Poisson equation is discretised on the general control volumes using central difference scheme. The resulting discretisation equation of pressure is a symmetric set of equations, which is solved using conjugate gradient method. The divergent free velocity components are obtained by correcting the intermediate velocity using the gradient of pressure. In collocated grid arrangement, the velocity components are evaluated for the centers of the control volume. In addition to the velocity components cell surface normal velocities are also evaluated. The surface normal velocities and the cell centre velocities are corrected separately. An in house code has been developed in FORTRAN. The numerical results are visualized using open-source visualization software, ParaView.

VI. RESULT AND DISCUSSIONS

The in-house code developed using the above method has been validated by solving the flow in a lid driven cavity [13].The results are found to be match closely. Now the code has been applied for solving the flow in a circular cylinder with a rotating top wall. The aim of this work is to study the resulting axial vortex breakdown. Finally developed code can be applied for simulation of swirling flows of vortex engines. The aspect ratio defined as the ratio of height to radius is taken as 2 (radius r = 0.25m, height = 0.50m).The viscosity taken is 0.000197368 m2/sec. Here the top wall is rotating with a constant angular velocity corresponding to the required value of rotational Reynolds number.

As the code is written in cartesian co-ordinate system, velocity of the fluid below the no slip rotating wall is tangential to concentric circles or the plane area just below the rotating lid and this tangential velocity is resolved along x and y direction and is prescribed as boundary condition for velocity. Computations are performed on grid (100 X 100 X 78) consisting of a total of 789800 cells.

Initially the fluid which fills the cylindrical container is at rest. At time t=0, the top wall (top plate) is made to rotate at particular rpm corresponding to respective rotational Reynolds number (Re). When the top wall is rotating an additional boundary layer is developed on the rotating wall. The rotating end wall acts as a pump, which draws the fluid axially towards it and drives outwards. The fluid swirls along the cylindrical wall, spirals in across the fixed end wall and then again turns into the axial direction towards the rotating end wall resulting in a concentrated vortex core along the axis. The inward spiraling motion results in an initial increase in swirl velocity, due to conservation of angular momentum and so the creation of a concentrated vortex. For certain combination of aspect ratio (H/R) and Reynolds number this vortex undergoes breakdown, ie a stagnation point followed by a recirculation zone of limited extent appears on the cylinder axis. Results of two Reynolds numbers are explained below. Fig. 2 shows the computational grid used for the present investigation.

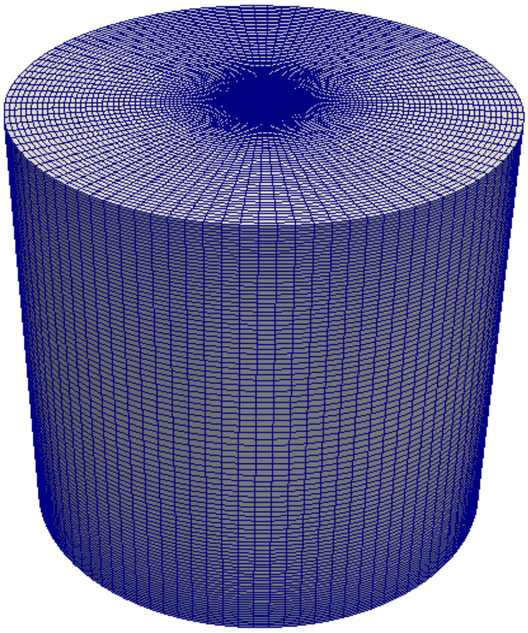

Fig.2. Physical Domain and Computational Grid

Results for Re = 1900 From fig. 3 it is clear that, there occurs a recirculation flow zone at the regions close to the rotating end wall. This formation of the recirculation flow zone causes the formation of axial vortex breakdown.

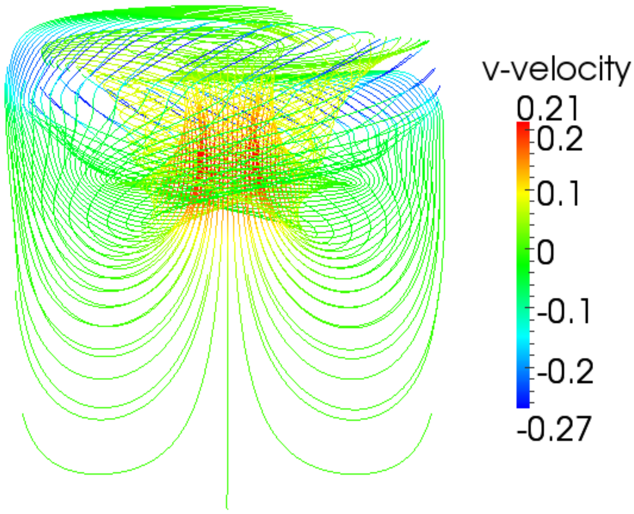

Fig.3. Streamlines Inside the Cylinder

Velocity plot (u, v, w) corresponds to this Reynolds number are shown in fig.4, 5 and 6 respectively. From these contours it is clear that all the flow features inside the cylinder is axisymmetric in nature.

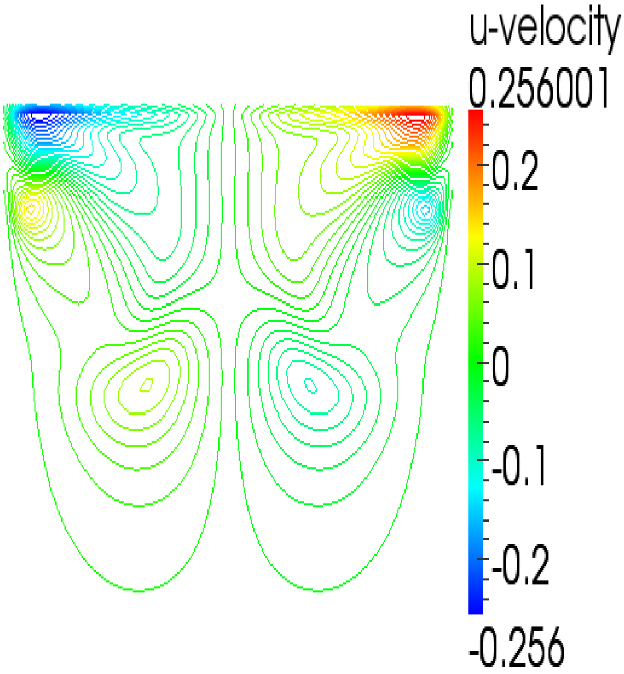

Fig. 4.U-Velocity Plot At XY Plane

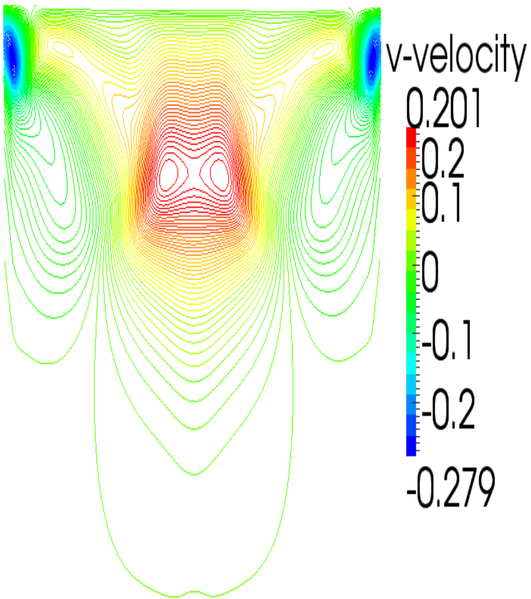

Fig. 5.V-Velocity Plot At XY Plane

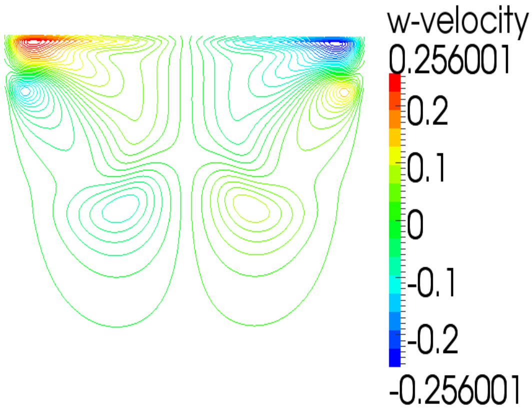

Fig. 6.W-Velocity Plot At XY Plane

These figures shows axial vortex breakdown close to the upper portion near the top rotating end wall. The axial vortex breakdown is also clear from figures 3 and 5. The contour plot of x and z components of velocity are identical because of the axisymmetric nature of the flow. Fig. 4 shows the y component of velocity, when upper wall of the cylinder is rotating, a strong coupling develops between the axial and tangential velocity components. A point is reached, when the adverse pressure gradient along the axis cannot be further overcome by the kinetic energy of the fluid particles flowing in the axial direction and a recirculation flow is set up in the central portion of the jet. This formation of the recirculation flow zone forms the axial vortex breakdown. From fig. 4, it is clear about the formation of recirculation zone and thus the formation of axial vortex breakdown.

Results for Re = 3800.

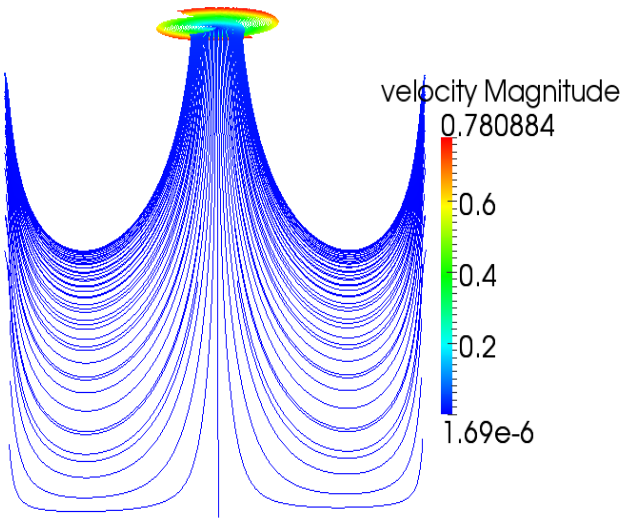

Fig. 7. Streamlines Inside the Cylinder

Fig. 7.Shows the streamlines inside the cylinder for Reynolds number of 3800. Here the solution is not converged. Till now there is no recirculation flow zone and so the axial vortex breakdown has not been predicted till 5 lakhs number of time steps. The solution for the case is going on.

VII. CONCLUSION

An in-house Fortran code for solving the unsteady Navier-Stokes equation was successfully validated by solving flow in a driven cavity at Re=1000 using a 643 non uniform grid. The cylindrical cavity with rotating end wall exhibits an axisymmetrical flow. An axial vortex breakdown is observed along the axis has been predicted for Re=1900.

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