

Cycle and Path Related Graphs on L – Cordial Labeling

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Abstract :- In this work we establish TL_n , DL_n , Q_n , DQ_n , IT_n , $K_1+K_{1,n}$, The generalized antiprism A_n^m , P_n , ΘK_1 , H_n , $H_n \Theta \overline{K_{1,m}}$, Duplication of all edges of the H_n , Braid graph $B(n)$, $Z-P_n$, The duplication of every edge by a vertex in C_n are L – Cordial.

Keyword:- L – Cordial Labeling (LCL), L – Cordial Graphs (LCG), Ladder, Snake, Path and Corona Graphs.
AMS Classifications: 05C78

I. INTRODUCTION

L – Cordial Labeling (LCL) was introduced in [7]. In [8,9] they discussed LCL behaviour of some standard graphs. Prime cordial, Cube difference and Square difference labeling of H- related graphs has been studied in [1,4,12]. 4 – Cordiality of path related graphs is investigated in [10]. Pairsum labeling of star and cycle related graphs, Prime labeling of duplication graphs, Difference Cordiality of ladder and snake related graphs, Super Mean labeling of antiprism and some more graphs have been proved in [5,6,11,13]. For this study we use the graph $G=(p,q)$ which are finite, simple and undirected. A detailed survey of graph labeling is given in [3]. Terms and results follow from [2]. In this work we study some standard and special graphs are LCG.

➤ Definition 1.1[7]

Graph $G(V,E)$ has L-cordial labeling if there is a bijection function $f:E(G) \rightarrow \{1,2,\dots,|E|\}$. Thus the vertex label is induced as 0 if the biggest label on the incident edges is even and is induced as 1, if it is odd. The condition is satisfied further by $V_f(0)$ which number of vertices labeled with 0 and $V_f(1)$ which is the number of vertices labeled with 1, and follows the condition that $|V_f(1) - V_f(0)| \leq 1$. Isolated vertices are not included for labeling here. A L-cordial graph is a graph which admits the above labeling.

➤ Definition 1.2[11]

The triangular ladder (TL_n) is obtain from a ladder by including the edges $v_i u_{i+1}$ for $i=1, 2, \dots, n-1$ with $2n$ vertices and $4n-3$ edges.

➤ Definition 1.3[11]

A diagonal ladder (DL_n) is a graph formed by adding the vertex of v_i with u_{i+1} and u_i with v_{i+1} for $1 \leq i \leq n-1$.

➤ Definition 1.4[9]

Q_n is said to be quadrilateral snake if each edge of the path P_n is replaced by a cycle C_4 .

➤ Definition 1.5[9]

A double quadrilateral snake DQ_n consist of two quadrilateral snake that have a common path.

➤ Definition 1.6[9]

The irregular triangular snake IT_n is derived from the path by replacing the alternate pair of vertices with C_3 .

➤ Definition 1.7[5]

A_n^m is the generalized antiprism formed by generalized prism $C_n \times P_m$ by adding the edges $v_i^j v_i^{j+1}$ for $1 \leq i \leq n$ and $1 \leq j \leq m-1$.

➤ Definition 1.8[1]

H_n -graph obtained from two copies of path with vertices a_1, a_2, \dots, a_n and b_1, b_2, \dots, b_n by connecting the vertices $a_{\frac{n+1}{2}}$ and $b_{\frac{n+1}{2}}$ if n is odd and $b_{\frac{n}{2}+1}$ and $a_{\frac{n}{2}}$ is joined if n is even.

➤ Definition 1.9[3]

The corona $G_1 \circ G_2$ is defined as the graph G obtained by taking one copies of G_1 (which has p points) p copies of G_2 and then joining the i^{th} point of G_1 to every point in the i^{th} copy of G_2 .

➤ Definition 1.10[13]

Duplication of an edge $e = xy$ of a graph G produces a new graph G' by adding an edge $e' = x'y'$ such that $N(x') = N(x) \cup (y') - \{y\}$ and $N(y') = N(y) \cup (x') - \{x\}$.

➤ **Definition 1.11[6]**

$G + H$ is the joining of two graphs G and H with vertex and edge set $V(G + H) = V(G) \cup V(H)$ and $E(G + H) = E(G) \cup E(H)$ respectively.

➤ **Definition 1.12[10]**

$Z-P_n$ is obtained from the pair of path P_n' and P_n'' by joining i^{th} vertex of P_n' with $(i+1)^{th}$ vertex of P_n'' .

➤ **Definition 1.13[10]**

The Braid graph $B(n)$ is formed by the pair of path P_n' and P_n'' by joining i^{th} vertex of P_n' with $(i+1)^{th}$ vertex P_n'' and i^{th} vertex of P_n'' with $(i+2)^{th}$ vertex of P_n' with the new edges.

II. MAIN RESULTS

➤ **Theorem 2.1:**

TL_n is L -cordial for $(n \geq 2)$.

Proof:

Let $G = TL_n$ be a triangular ladder then $G = (2n, 4n-3)$. We define vertex and edge sets as $V(G) = \{u_i, v_i / i = 1, 2, \dots, n\}$ and $E(G) = E_1 \cup E_2$

Where $E_1 = \{u_i v_i / i = 1, 2, \dots, n\}$

$E_2 = \{u_i u_{i+1}, v_i v_{i+1}, u_{i+1} v_i / 1 \leq i \leq n-1\}$

Then the bijection f is defined as follows

- For $i = 1, 2, \dots, n-1$
- $f(u_i u_{i+1}) = 2n + 2i - 2$
- $f(v_i v_{i+1}) = 2n + 2i - 1$
- $f(v_i u_{i+1}) = 2i$
- $f(u_i v_i) = 2i - 1 ; i = 1, 2, \dots, n.$

In view of the above labeling, we have $V_f(0) = V_f(1)$. Hence TL_n admits LCL.

• **Example 1**

LCL of TL_5 is given in figure 1

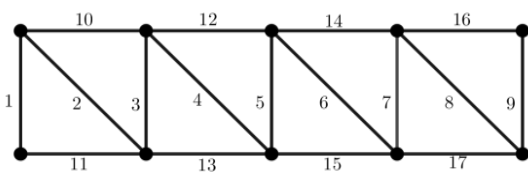


Fig 1

➤ **Theorem 2.2:**

DL_n are L -cordial graphs if $n \geq 2$.

Proof:

Consider $G = DL_n$, and we define $V(G)$ and $E(G)$ as in theorem 2.1 additionally with edges $u_j v_{j+1} ; 1 \leq j \leq n-1$. Then the bijective function is defined as follows

When $n \equiv 1 \pmod{2}$

For $j = 1, 2, \dots, n-1$

- $f(u_j u_{j+1}) = p + 3j - 1$
- $f(v_j v_{j+1}) = p + 3j - 3$
- $f(v_j u_{j+1}) = 2j$
- $f(u_j v_{j+1}) = p + 3j - 2$
- For $1 \leq j \leq n$
- $f(u_j v_j) = 2j - 1.$

When $n \equiv 0 \pmod{2}$

- For $1 \leq j \leq n-1$
- $f(u_j u_{j+1}) = 2j$
- $f(v_j v_{j+1}) = 2j - 1$
- $f(v_j u_{j+1}) = p + 2 + j$
- $f(u_j v_{j+1}) = 2p + j - 3$
- For $i = 1, 2, \dots, n$
- $f(u_i v_i) = p + j - 2.$

Hence DL_n admits L -Cordial labeling with $V_f(0) = V_f(1)$ Therefore DL_n L -Cordial graph. LCL of DL_3 is given in figure 2.

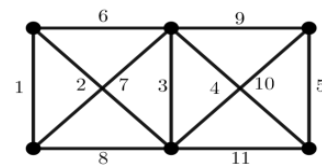


Fig 2

➤ **Theorem 2.3:**

Q_n admits L -Cordial labeling.

Proof:

Let $G = Q_n$ with $V(Q_n) = \{u_i, v_j, w_j / 1 \leq i \leq n + 1, 1 \leq j \leq n\}$ and $E(Q_n) = \{u_i v_i, v_i w_i, u_i u_{i+1}, u_{i+1} w_i : 1 \leq i \leq n\}$ then $|V(Q_n)| = 3n + 1$ and $|E(Q_n)| = 4n$.

We define $g : E \rightarrow q$ by

- For $i = 1, 2, \dots, n$
- $f(u_i v_i) = 2i - 1$
- $f(u_{i+1} w_i) = 2i$
- $f(v_i w_i) = 2n + i$
- $f(u_i u_{i+1}) = 3n + i.$

Hence for n odd we have $V_g(0) = V_g(1) = \frac{3n+1}{2}$ and

$V_g(0) = \left\lceil \frac{3n+1}{2} \right\rceil, V_g(1) = \left\lfloor \frac{3n+1}{2} \right\rfloor$ for n even. Therefore Q_n is L -Cordial graph.

• **Example 2**

LCL of Q_3 is shown in figure 3.

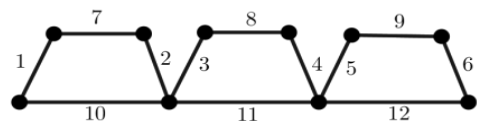


Fig 3

➤ **Theorem 2.4:**

DQ_n admit L -Cordial labeling.

Proof:

Let $V(G) = \{u_i, v_j, w_j, x_j, y_j / 1 \leq i \leq n, 1 \leq j \leq n - 1\}$ and $E(G) = \{u_i u_{i+1}, u_{i+1} w_i, v_i w_i, u_i v_i, u_i x_i, x_i y_i, y_i u_{i+1} / 1 \leq i \leq n - 1\}$. Then $|V(G)| = 5n - 4$ and $|E(G)| = 7n - 7$. Define a map $g : E \rightarrow \{1, 2, \dots, q\}$ as follows:

Case (i) When n is even

For $i = 1, 2, \dots, n-1$
 $g(u_i u_{i+1}) = i$
 $g(u_i v_i) = n + i - 1$
 $g(u_{i+1} w_i) = 2(n - 1) + i$
 $g(u_i x_i) = 3(n - 1) + i$
 $g(x_i y_i) = 6n - 6 + i$
 $g(v_i w_i) = 5n - 5 + i$
 $g(y_i u_{i+1}) = 4n - 4 + i$.

Case (ii) When n is odd

For $1 \leq i \leq n - 1$
 $g(u_i v_i) = 2i - 1$
 $g(u_{i+1} w_i) = 2i$
 $g(u_i x_i) = 2(n - 1) + i$
 $g(y_i u_{i+1}) = 2(n + i - 1)$
 $g(u_i u_{i+1}) = 4n - 4 + i$
 $g(v_i w_i) = 5n - 5 + i$
 $g(x_i y_i) = 6(n - 1) + i$.

Therefore $V_g(0) = V_g(1)$ when $n \equiv 0(mod 2)$ and $V_g(0) + 1 = V_g(1)$ when $n \equiv 1(mod 2)$, Thus DQ_n is a LCG.

• **Example 3**

Illustration of LCL of DQ_5 is given in figure 4.

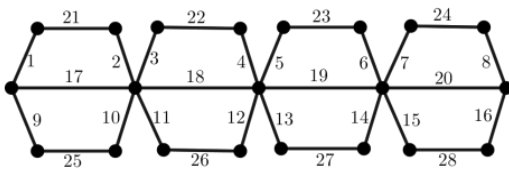


Fig 4

➤ **Theorem 2.5:**

Irregular triangular snake (IT_n) admits L-Cordial labelling.

Proof:

Let $V(IT_n) = \{u_i, v_j : 1 \leq i \leq n, 1 \leq j \leq n - 2\}$, $E(IT_n) = \{u_i u_{i+1}, u_i v_j, v_j u_{i+2} : 1 \leq i, j \leq n - 1\}$ with $2n-2$ and $3n-5$ vertices and edges respectively.

Then $f : E(IT_n) \rightarrow \{1, 2, \dots, q\}$ is as follows

For $1 \leq i \leq n - 1$
 $f(u_i u_{i+1}) = \begin{cases} 2i - 1, n \equiv 1(mod 2) \\ i, n \equiv 0(mod 2) \end{cases}$

For $1 \leq i, j \leq n - 2$
 $f(u_i v_j) = 2n + i - 3$ for all n

$$f(v_j u_{i+2}) = \begin{cases} 2i, n \equiv 1(mod 2) \\ 2n - 2 - i, n \equiv 0(mod 2) \end{cases}$$

By the above labeling, it is clear that the vertex label is distributed equally whenever n is odd or even. Hence IT_n is L-Cordial graph.

• **Example 4**

LCL of IT_5 is given in below figure 5.

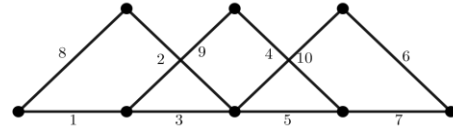


Fig 5

➤ **Theorem 2.6:**

$K_1 + K_{1,n}$ graph admits L-Cordial labeling.

Proof:

Let $G = K_1 + K_{1,n}$ with $V(G) = \{x, y\} \cup \{w_j / 1 \leq j \leq n\}$ and $E(G) = \{xy, xw_j, yw_j / j = 1, 2, \dots, n\}$.

Define a bijection $g : E(G) \rightarrow \{1, 2, \dots, 2n + 1\}$ as follows

For $1 \leq j \leq n$

$$f(xy) = 1$$

$$f(xw_j) = 2j$$

$$f(yw_j) = 2n - 2j + 3.$$

Thus the vertex labeling is distributed as $V_g(0) = V_g(1) + 1$ for n odd and $V_g(0) = V_g(1)$ for n even. Hence $K_1 + K_{1,n}$ is a LCG. Illustration of above labeling of $K_1 + K_{1,5}$ is shown in figure 6.

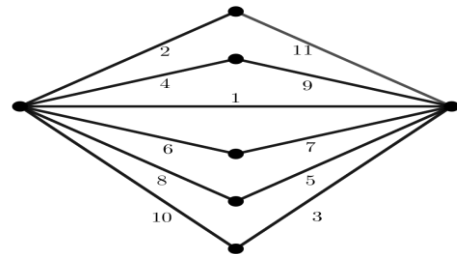


Fig 6

➤ **Theorem 2.7:**

The generalized antiprism A_n^m is L-Cordial if $m \geq 2, n \geq 3$.

Proof:

Consider $G(V, E) = A_n^m$. Let $V(A_n^m) = \{v_i^j / 1 \leq i \leq n, 1 \leq j \leq m\}$ and

$$E(A_n^m) = \{v_i^j v_{i+1}^j, v_i^j v_i^{j+1}, v_i^j v_{i+1}^{j+1} / 1 \leq i \leq n - 1, 1 \leq j \leq m - 1\}.$$

The labeling $f : E(G) \rightarrow \{1, 2, \dots, 3nm - 2n\}$ defined as follows:

For $j = 1, 2, \dots, m$

$$f(v_1^j v_n^j) = \begin{cases} nj & , n \equiv 0(\text{mod}2) \\ n(3m - 3j + 1), & n \equiv 1(\text{mod}2) \end{cases}$$

For $i = 1, 2, \dots, n, j = 1, 2, \dots, m - 1$

$$f(v_i^j v_i^{j+1}) = \begin{cases} n(3m - 3j - 1) + i, & n \equiv 1(\text{mod}2) \\ 2nm + nj - 2n + i, & n \equiv 0(\text{mod}2) \end{cases}$$

For $1 \leq i \leq n - 1, 1 \leq j \leq m$

$$f(v_i^j v_{i+1}^j) = \begin{cases} 3nm + i - 3nj & , n \equiv 1(\text{mod}2) \\ nj - n + i & , n \equiv 0(\text{mod}2) \end{cases}$$

For $1 \leq i \leq n - 1, 1 \leq j \leq m - 1$

$$f(v_i^j v_{i+1}^{j+1}) = \begin{cases} n(3m - 3j - 2) + i, & n \equiv 1(\text{mod}2) \\ mn + nj - n + i, & n \equiv 0(\text{mod}2) \end{cases}$$

Thus we have the vertex distribution as $V_f(0) = V_f(1) + 1$, when is odd and $V_f(0) = V_f(1)$ when n is even, hence it follows that A_n^m is L -Cordial graph.

• Example 5

Illustration of LCL of A_4^3 and A_5^3 is given in figure 7, 8.

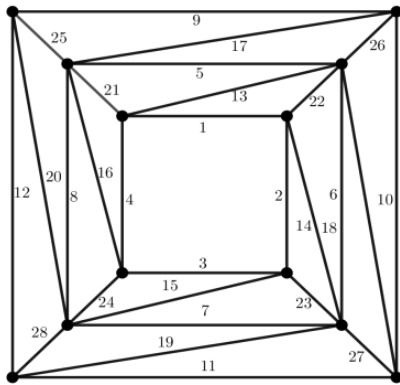


Fig 7

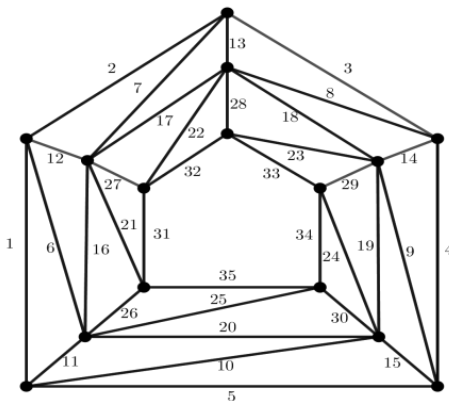


Fig 8

➤ Theorem 2.8:
 $P_n \circ K_1$ is L -Cordial .

Proof:

Consider a graph $G = P_n \circ K_1$ where G has $2n$ vertices and $2n-1$ edges. Let $V(G) = \{x_j y_j, j = 1, 2, \dots, n\}$ and $E(G) = E_1 \cup E_2$ where $E_1 = \{x_j x_{j+1} / 1 \leq j \leq n - 1\}$

$E_2 = \{x_j y_j / j = 1, 2, \dots, n\}$
Define a bijection $f: E \rightarrow \{1, 2, \dots, 2n - 1\}$ as two cases

Case (i)

When n is even
 $f(x_j x_{j+1}) = j, 1 \leq j \leq n - 1$
 $f(x_j y_j) = n + j - 1, 1 \leq j \leq n.$

Case(ii)

When n is odd
 $f(x_1 x_2) = 1$
 $f(x_1 y_1) = 2$
 $f(x_{j+1} x_{j+2}) = 2j + 2, 1 \leq j \leq n - 2$
 $f(x_{j+1} y_{j+1}) = 2j + 1, 1 \leq j \leq n - 1.$

Hence $P_n \circ K_1$ is L -Cordial graph. Since it satisfies the condition of LCL .

➤ Theorem 2.9:
The graph $H_n (n \geq 3)$ is L -Cordial graph.

Proof :

Let $G=(V,E)$ be the H_n graph with the vertex set $V=\{u_j, v_j / 1 \leq j \leq n\}$ and the edge set

$$E = \begin{cases} u_j u_{j+1}, v_j v_{j+1}, & j = 1, 2, \dots, n \\ \frac{u_{n+1} v_{n+1}}{2}, & n - \text{odd} \\ \frac{u_{\frac{n}{2}+1} v_{\frac{n}{2}}}{2}, & n - \text{even} \end{cases}$$

Let f be a bijective function from $f: E \rightarrow q$ defined as follows for $j=1, 2, \dots, n-1$

$$f(u_j u_{j+1}) = 2j$$

$$f(v_j v_{j+1}) = 2j + 1$$

$$f\left(\frac{u_{n+1} v_{n+1}}{2}\right) = 1, \text{ when } n \text{ odd}$$

$$f\left(\frac{u_{\frac{n}{2}+1} v_{\frac{n}{2}}}{2}\right) = 1, \text{ when } n \text{ even.}$$

Therefore the above labelling satisfies the condition of L -Cordial labelling for all $(n \geq 3)$. Thus H_n is L -Cordial graph.

➤ Theorem 2.10:
The $H_n \circ \overline{K_{1,m}}$ is L -Cordial graph, for $n \geq 3, m \geq 2$.

Proof:

Consider $G = H_n \circ \overline{K_{1,m}}$ with $V(G) = \{x_i, y_i, x_{i,j}, y_{i,j} / 1 \leq i \leq n, 1 \leq j \leq m\}$ and $E(G) = \{x_i x_{i+1}, y_i y_{i+1} / 1 \leq i \leq n - 1\} \cup \{x_{\frac{n+1}{2}} y_{\frac{n+1}{2}} \text{ if } n \text{ is odd, } x_{\frac{n}{2}+1} y_{\frac{n}{2}} \text{ if } n \text{ is even}\} \cup \{x_i x_{i,j}, y_i y_{i,j} / 1 \leq i \leq n, 1 \leq j \leq m\}$. Then the graph $H_n \circ \overline{K_{1,m}}$ has $2n(1+m)$ vertices and $2n(m+1)-1$ edges.

Define a bijective function $f: E \rightarrow \{1, 2, \dots, |E|\}$ as follows

For $i=1, 2, \dots, n-1$
 $f(x_i x_{i+1}) = 2i$
 $f(y_i y_{i+1}) = 2i + 1$
For $i=1, 2, \dots, n, j=1, 2, \dots, m$

$$f(x_i x_{i,j}) = 2n + 2(j - 1) + 4(i - 1) + 2(m - 2)(i - 1)$$

$$f(y_i y_{i,j}) = 2n + 1 + 2(j - 1) + 4(i - 1) + 2(m - 2)(i - 1)$$

$$f\left(\frac{x_{n+1} y_{n+1}}{2}\right) = 1, \text{ when } n \text{ odd}$$

$$f\left(\frac{x_{\frac{n}{2}+1} y_{\frac{n}{2}}}{2}\right) = 1, \text{ when } n \text{ even.}$$

Then it is easily observe that the above labeling function satisfies the condition of *L-Cordial labeling*. Thus $H_n \circ \overline{K_{1,m}}$ admits *L-Cordial labeling*.

• *Example 6*

LCL of $H_3 \circ \overline{K_{1,3}}$ shown in figure 9.

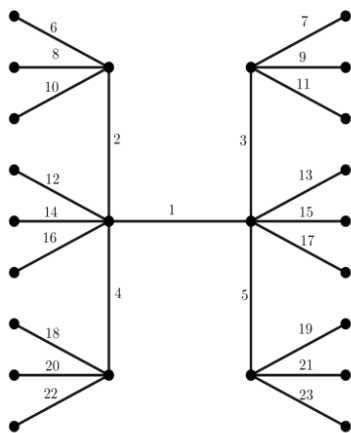


Fig 9

➤ *Theorem 2.11:*

Duplication of all edges of the H_n ($n \geq 3$). Graph admits *L-Cordial labeling*.

Proof:

Let G be the graph obtained by duplicating all edges of $u_i u_{i+1}$ and $v_i v_{i+1}$ by new vertices u'_i, v'_i for $i=1,2,\dots,n-1$ respectively. Let w be a new apex vertex obtained by duplicating the edge $\frac{u_{n+1} v_{n+1}}{2}, n - \text{odd}$ (or) $\frac{u_{\frac{n}{2}+1} v_{\frac{n}{2}}}{2}, n - \text{even}$. Then we observe that $|V(G)| = 4n - 1$ and $|E(G)| = 6n - 3$, we define a bijection $g : E \rightarrow \{1, 2, \dots, q\}$ as follows

when n odd

$$g\left(\frac{u_{n+1} v_{n+1}}{2}\right) = 1$$

$$g\left(\frac{u_{n+1} w}{2}\right) = 2$$

$$g\left(\frac{v_{n+1} w}{2}\right) = 3.$$

when n even

$$g\left(\frac{u_{\frac{n}{2}+1} v_{\frac{n}{2}}}{2}\right) = 1$$

$$g\left(\frac{u_{\frac{n}{2}+1} w}{2}\right) = 2$$

$$g\left(\frac{v_{\frac{n}{2}} w}{2}\right) = 3.$$

For $i=1,2,\dots,n-1$

$$g(u_i u'_i) = 2n + 4i - 2$$

$$g(u'_i u_{i+1}) = 2n + 4i$$

$$g(u_i u_{i+1}) = 2i + 2$$

$$g(v_i v'_i) = 2n + 4i - 1$$

$$g(v'_i v_{i+1}) = 2n + 4i + 1$$

$$g(v_i v_{i+1}) = 2i + 1.$$

From the above labeling it is clear that $V_f(0) = V_f(1) + 1$ for all ($n \geq 3$). Thus duplication of H_n is *L-Cordial graph*.

• *Example 7*

L-Cordial labeling of duplication of edge of H_3 is explained in the figure 10.

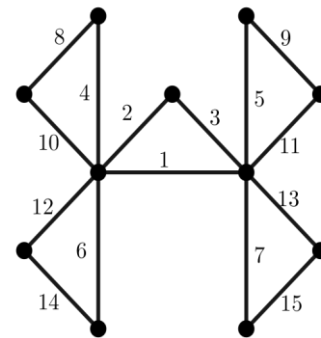


Fig 10

➤ *Theorem 2.12:*

The Braid graph $B(n)$ admits *L-Cordial* for all n .

Proof:

Consider $G = B(n)$ as the Braid graph. Let u_1, u_2, \dots, u_n be the vertices of path P'_n and v_1, v_2, \dots, v_n be the vertices of path P''_n . Thus $B(n)$ is a graph obtain by joining j^{th} vertex of P'_n with $(j + 1)^{th}$ vertex of P''_n and j^{th} vertex of P''_n with $(j + 2)^{th}$ vertex of P'_n with new edges.

Define *L-Cordial labeling* $f: E \rightarrow \{1, 2, \dots, 4n - 5\}$ as follows

For $1 \leq j \leq n - 1$

$$f(u_j u_{j+1}) = 2n + 2j - 4$$

$$f(v_j v_{j+1}) = 2n + 2j - 3$$

$$f(u_j v_{j+1}) = j.$$

For $1 \leq j \leq n - 2$

$$f(v_j u_{j+2}) = n + j - 1$$

Thus for all n $V_f(0) = V_f(1)$. Therefore $B(n)$ is *L-Cordial graph*.

• *Example 8*

LCL of $B(5)$ is shown in below figure 11.

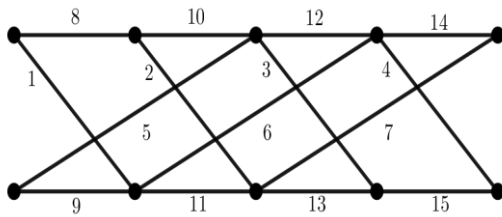


Fig 11

➤ *Theorem 2.13:*

The graph $Z-P_n$ is L -Cordial for all n .

Proof:

Let $V = \{u_i, v_i; 1 \leq i \leq n\}$ and $E = \{u_i u_{i+1}, v_i v_{i+1}, v_i u_{i+1} / i = 1, 2, \dots, n-1\}$ be a graph G with $|V(G)| = 2n$ and $|E(G)| = 3n - 3$.

We define bijection $f : E \rightarrow \{1, 2, \dots, 3n - 3\}$ as follows

For $i = 1, 2, \dots, n - 1$

$$f(u_i u_{i+1}) = \begin{cases} 3i - 2, & \text{when } n \text{ odd} \\ 3i - 1, & \text{when } n \text{ even} \end{cases}$$

$$f(v_i u_{i+1}) = \begin{cases} 3i - 1, & \text{when } n \text{ odd} \\ 3i - 2, & \text{when } n \text{ even} \end{cases}$$

$$f(v_i v_{i+1}) = 3i \text{ for all } n$$

Hence the vertex label is distributed evenly for all n . Therefore $Z-P_n$ admits LCL . Thus $Z-P_n$ is a L -Cordial graph.

• *Example 9*

Illustration of LCL of $Z-P_4$ is given in figure 12.

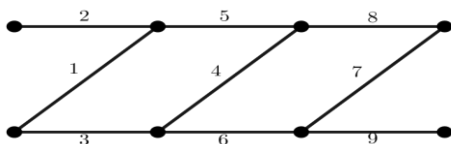


Fig 12

➤ *Theorem 2.14:*

The duplication of every edge by a vertex in C_n admits L -Cordial labeling.

Proof:

Let $a_j, j = 1, 2, \dots, n$ be the consecutive vertices of C_n . Let G be the graph obtained by duplication of all the edges $a_1 a_2, a_3 a_4, \dots, a_n a_1$ by new vertices $b_1, b_2, \dots, b_{n-1}, b_n$ respectively. Then the labeling $f : E(G) \rightarrow \{1, 2, \dots, 3n\}$ defined as follows

For all n

$$f(a_j b_j) = 3j - 2$$

For $1 \leq j \leq n - 1$

$$f(a_j a_{j+1}) = \begin{cases} 3j - 1, & n \equiv 0 \pmod{2} \\ 3j, & n \equiv 1 \pmod{2} \end{cases}$$

$$f(b_j a_{j+1}) = \begin{cases} 3j, & n \equiv 0 \pmod{2} \\ 3j - 1, & n \equiv 1 \pmod{2} \end{cases}$$

$$f(a_1 a_n) = \begin{cases} 3n - 1, & n \equiv 0 \pmod{2} \\ 3n, & n \equiv 1 \pmod{2} \end{cases}$$

$$f(a_1 b_n) = \begin{cases} 3n, & n \equiv 0 \pmod{2} \\ 3n - 1, & n \equiv 1 \pmod{2} \end{cases}$$

Clearly the above edge labeling is L -Cordial labeling. Hence G is a LCL .

• *Example 10*

L -Cordial labeling duplication of every edge by a vertex C_4 is shown in figure 13

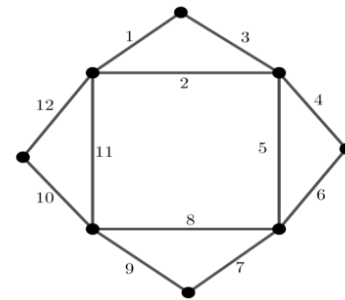


Fig 13

III. CONCLUSION

In this work we studied L -Cordial Labeling of some standard and their related graphs.

REFERENCES

- [1]. J.Arthy, K.Manimekalai and K.Ramanathan, "Cube Difference Labeling Of Some Special Graphs". AIP Conference Proceedings 2112, 020140 (2019).
- [2]. Frank Harary, Graph theory, Narosa Publishing House, 2001.
- [3]. J.A. Gallian, A dynamic survey of graph labeling, The electronics journal of Combinatorics, 17 (2010) #DS6.
- [4]. P.Jagadeeswari, K.Manimekalai and K.Ramanathan,, "Square Difference Labeling Of H -Graphs" AIP Conference Proceedings 2112, 020147 (2019).
- [5]. P.Jeyanthi and D.Ramya, "Super Mean Labeling Of Some Classes Of Graphs", International J.Math Combin.Vol.1(2012),83-91.
- [6]. Mukund V.Bapat, "Some Vertex Prime Graphs and A New Type Of Graph Labeling", International Journal Of Mathematics Trends and Technology (IJMTT),Vol 47, Number 1(2017),49-55.
- [7]. Mukund, V.Bapat, "A Note On L -Cordial Labeling Of Graphs", International Journal Of Mathematics And Its Applications, Vol 5,Issue 4-D(2017),457-460.
- [8]. Mukund V.Bapat, "New Results On L -Cordial Labeling Of Graphs", International Journal Of Statistics And Applied Mathematics,2018,3(2),641-644.
- [9]. R.Ponraj and S.Sathish Narayana, "Difference Cordiality Of Some Snake Graphs",J.Appl.Math & Informatics Vol.32(2014), No.3-4,pp.377-387.
- [10]. N.B .Rathod and K.K.Kanani, "4-Cordiality Of Some New Path Related Graphs", International Journal Of Mathematics Trends And Technology,Vol.34,1(2016), 5-8.

- [11]. M.A.Seoud and Shakir M.Salman, “On Difference Cordial Graphs”, Mathematics Aeterna Vol.5,2015 No.1,105-124.
- [12]. G.Subashini, K.Manimekalahi and K.Ramanathan, “Prime Cordial Labeling Of $H_n \circ K_2$ Graphs”, AIP Conference Proceedings 2112, 020128 (2019).
- [13]. S.K Vaidya and U.M.Prajapati, “Prime Labeling In The Context Of Duplication Of Graph Elements”, International Journal Of Mathematics And Soft Computing, Vol.3, No.1(2013),13-20.