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Comparison of Continuum Constitutive Hyperelastic Models based on Exponential Forms

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Abstract:- This work focuses on the comparison of the strain energy density functions for rubber-like materials based on exponential form. The Treloar experimental data had been used in the present paper. The optimal method for nonlinear parameter identification is developed. Indeed, a comparison between the Treloar data and the analytical solution is approached in order to identify a good hyperelastic parameters of the models. By processing simple tension, pure shear and equibiaxial tension curves showed the good agreement between the model and experimental data.

Keywords:- *Exponential forms, Rubber-like Materials, Approach-In-Stages, Identification Process, Material parameters.*

I. INTRODUCTION

Nowadays, the mechanical of rubber-like materials has a very active field of research due to their use in the many areas of applications. Since the fortieth century, some experimental and theoretical models have already been proposed by Mooney [1], Kuhn and Grün [2], Treloar [3, 4, 5] and James [6]. Long before, the multitude of phenomenological constitutive hyperelastic models has been proposed in attempts to describe the deformation modes of elastomeric polymers. Ones of them based on strain-invariants, Rivlin [7], Pucci and Saccomandi [8], Isihara [9], James [6], Yeoh [10], Lion [11], Haupt and Sedlan [12], Boyce-Arruda [13], Beda [14], Nunes [15], and Carroll [16] and the others on principal stretches Valanis [17], Peng [18] and recently Ogden [19], [20]. Even so, few of the following models have the accuracy, the efficiency, the ability and the capacity to reproduce all the deformation modes like simple, equal-biaxial tension and pure shear tests. Among these hyperelastic models, many have been proposed with an exponential dependence on the strain invariants form. In this context, it is worth mentioning above Hart-Smith that associated the exponential and the logarithmic form [21], Fung [22], Alexender [23], Gornet-Marckmann [24] and Beda [25] generalized the Hart-Smith model. In the litterature, many authors compared some hyperelastic models, M. García [26] presented a review of the application of the hyperelastic models to the analysis of fabrics using finite element method in 2006, Marckmann [27] compared twenty hyperelastic using genetic algorithm, Chagnon [28] compared the Hart-Smith with Arruda-Boyce. The good phenomenological hyperelastic model must be able to fit the Treloar experimental data [4].

The outline of this work compares the strain energy density functions based on the Hart-Smith first part and the general model combining the models has been proposed. In addition to that, this article presents a strategy of hyperelastic identification parameters that provide the optimal parameters based on Treloar experimental data.

II. KINEMATIC STRESS TENSORS IN MATERIAL FORM

According to the continuum mechanics theories, there exists the strain energy density function W, which is the property of rubber-like materials. The first Piola-Kirchhoff stress tensor P which is used in the study of the large deformation analysis can be expressed as the derivative of the variable W and the right Cauchy-Green tensor C [22, 29]. The general constitutive equation is given by the following:

$$P = 2.\frac{\partial W}{\partial C} \tag{1}$$

Then, considering that the right Cauchy-Green tensor C is symmetric, the first relationship can be expressed as:

$$\frac{\partial W}{\partial C} = \frac{\partial W}{\partial I_1}\frac{\partial I_1}{\partial C} + \frac{\partial W}{\partial I_2}\frac{\partial I_2}{\partial C} + \frac{\partial W}{\partial I_3}\frac{\partial I_3}{\partial C} \tag{2}$$

That equivalent to:

$$\frac{\partial W}{\partial C} = \left(\frac{\partial W}{\partial I_1} + \frac{\partial W}{\partial I_2} I_1\right) I - \frac{\partial W}{\partial I_2} C + \frac{\partial W}{\partial I_3} I_3 C^{-1} \tag{3}$$

Then, we can write:

$$\frac{\partial W}{\partial C} = \tau_0 I + \tau_1 C + \tau_2 C^{-1} \quad (4)$$

Where $\tau_0 = \frac{\partial W}{\partial I_1} + \frac{\partial W}{\partial I_2} I_1$, $\tau_0 = -\frac{\partial W}{\partial I_2}$, $\tau_2 = \frac{\partial W}{\partial I_3} I_3$ and $C = F^T F$, F is the deformation gradient matrix.

Based on isotropic and incompressible hyperelastic materials law, the strain energy function admits the three principal invariants of C denoted I_1, I_2 and I_3 these are written as :

$$\begin{cases} I_1 = trC = \sum_{i=1}^{3} \lambda_i^2 \\ I_2 = \frac{1}{2}\left[(trC)^2 - trC^2\right] = \sum_{i,j=1}^{3} \left(\lambda_i \lambda_j\right)^2, i \neq j \\ I_3 = \det C = \prod_{i=1}^{3} \lambda_i^2 \end{cases} \quad (5)$$

Generally, a constitutive relation of Cauchy-Green for hyperelastic properties is defined by:

$$\sigma = f(C) \quad (6)$$

Therefore, in the large deformation domain, the constitutive relationship between the Cauchy stress tensor and the strain energy density function is giving by the following relation:

$$\sigma = 2J^{-1} F \frac{\partial W}{\partial C} F^T \quad (7)$$

Taking into account the condition of the incompressibility and the isotropic of rubber-like materials, the relation (7) can be rewritten as:

$$\sigma = -pI + 2\frac{\partial W}{\partial I_1} C + -2\frac{\partial W}{\partial I_2} C^{-1} \quad (8)$$

Where *p* is the hydrostatic pressure.

III. NONLINEAR DEFORMATION BEHAVIOR

The relationship that describes the behavior of the elastomeric polymer is given by the relation (8).

- *For Simple tension:*

According to uniaxial tension and based on incompressible and isotropic conditions, $\lambda_1 = \lambda$ and $\lambda_2 = \lambda_3 = \lambda^{-1}$. The two invariants for this test are:

$$\begin{cases} I_1 = \lambda^2 + \frac{2}{\lambda} \\ I_2 = 2\lambda + \frac{1}{\lambda^2} \end{cases} \quad (9)$$

Considering equations (5), and (6), the stress tensor relation in the terms of invariants tensor can be expressed by:

$$\sigma = 2\left(\lambda - \lambda^{-2}\right)\left(\frac{\partial W}{\partial I_1} + \lambda^{-1}\frac{\partial W}{\partial I_2}\right) \quad (10)$$

- *For Equibiaxial tension:*

Considering equibiaxial tension test: $\lambda_1 = \lambda_2 = \lambda$ and $\lambda_3 = \lambda^{-2}$. Thus, the two strain invariants tension are:

$$\begin{cases} I_1 = 2\lambda^2 + \frac{1}{\lambda^4} \\ I_2 = \lambda^4 + \frac{2}{\lambda^2} \end{cases} \quad (11)$$

According to the stress tensor function based on relations (5) and (7) becomes:

$$\sigma = 2\left(\lambda - \lambda^{-5}\right)\left(\frac{\partial W}{\partial I_1} + \lambda^2 \frac{\partial W}{\partial I_2}\right) \quad (12)$$

- *Pure shear*

This deformation test, $\lambda_1 = \lambda$, $\lambda_2 = 1$ and $\lambda_3 = \lambda^{-2}$. Thus, the two invariants deformation are the same.

$$I_1 = I_2 = \lambda^2 + \frac{1}{\lambda^2} + 1 \quad (13)$$

The stress tensor could be also expressed as:

$$\sigma = 2\left(\lambda - \lambda^{-3}\right)\left(\frac{\partial W}{\partial I_1} + \frac{\partial W}{\partial I_2}\right) \quad (14)$$

IV. THE DIFFERENCE HYPERELASTIC MODELS

a) RIVLIN R. S. hyperelastic model

The strain energy density $W(I_1, I_2)$ which characterizes the mechanical response of an isotropic, incompressible and hyperelastic like-rubber materials expressed in the terms of the invariants tensor or immediately in the terms of principal stretches like $W(\lambda_1, \lambda_2, \lambda_3)$. At the first, Rivlin expressed the strain energy function like an infinite convergent power series based on invariants of Green deformation tensor. That material must be homogenous, isotropic and incompressible.

Many hyperelastic models had been built from the first term of the classical work of Rivlin [7] given by the following relation :

$$W_{Riv} = \sum_{m,n=0}^{\infty} C_{mn}\left(I_1-3\right)^m\left(I_2-3\right)^n \qquad (15)$$

b) HART-SMITH model

Hart-Smith modified the Gent-Thomas model by substituting the first part by an exponential function power of the first invariant I_1. Thus, based on the two first invariants I_1 and I_2, He proposed the following hyperelastic relationship for the strain energy:

$$W_{H-S} = C_1\int \exp\left(\beta\left(I_1-3\right)^2\right)dI_1 + C_2\ln\frac{I_2}{3} \qquad (16)$$

c) VERONDA-WESTMANN model

In 1970, Veronda-Westmann [30] proposed a biological model to study in uniaxial tests the skin of cats. This derives from the Fung model. Veronda prolonged just the Fung strain energy density by adding the second term. The whole hyperelastic model expressed by the following expression:

$$W_{V-W} = C_1\left[\exp\left(\beta\left(I_1-3\right)\right)-1\right] - C_2\left(I_2-3\right) \qquad (17)$$

d) YEOH-modified model

The Yeoh modified model [31] derives from the Yeoh 1990 [10] hyperelastic model that extended by adding the exponential form.

$$W_Y = \sum_{i=1}^{3} C_i\left(I_1-3\right)^i + \frac{\alpha}{\beta}\left[1-exp\left(\beta\left(I_1-3\right)\right)\right] \qquad (18)$$

e) LAMBERT-DIANI model

In 1999, Lambert-Diani and Rey [32], after using a multistage procedure, get a generic hyperelastic model given by the following strain energy:

$$W_{L-D} = \int \exp\left(\sum_{i=0}^{n} a_i\left(I_1-3\right)^i\right)dI_1 + \int \exp\left(\sum_{j=0}^{m} b_j\left(\ln I_2\right)^j\right)dI_2 \qquad (19)$$

f) L. GORNET-MARCKMANN model

Gornet and Marckmann [24] developed a new constitutive hyperelastic model on static stiffness modeling of rubber-like materials for multiaxial. Their strain energy based also on Hart-Smith model by replacing the second term in I_2 :

$$W_{G-M} = C_1\int \exp\left(\beta\left(I_1-3\right)\right)dI_1 + C_2\int\frac{1}{\sqrt{I_2}}dI_2 \qquad (20)$$

g) T. BEDA model

The model of Beda [33] generalizes the Hart-Smith strain energy [21] who takes a unique value of the hyperelastic parameter $\alpha = 2$. Beda considers that α is a variable. The form of the hyperelastic model is given by the following relationship:

$$W_{Be} = C_1\int \exp\left(\beta\left(I_1-3\right)^\alpha\right)dI_1 + C_2\ln\frac{I_2}{3} \qquad (21)$$

V. METHOD OF IDENTIFICATION OF THE OPTIMAL AND ACCURATE HYPERELASTIC PARAMETERS

The approach-in-stage method will be used for identifying all model parameters in the present paper. The technique consists to identify step by step the generating function [34, 35, 36, 37, 25, 38, 33]. The method supposes to approximate the function $y(x)$ that corresponds to the function $A\varphi(x)$, that corresponds to $y(x) = A\varphi(x)$. This strategy consists to plot $y(x)$ versus $\varphi(x)$ and ought to be linear, with slope equal to *A*. In other cases, Beda showed that, the curve of $y(x)$ versus $\left[\varphi(x)\right]^\alpha$ should be convex and rising if $\alpha > 1$, concave if $0 < \alpha < 1$ convex and falling if $\alpha < 0$ [35]. In this paper, all the hyperelastic parameters of the material in this work will be determined by the approach-in-stage method.

Base on deformation modes, the Hart-Simth model, the Gornet-Marckmann model and the Beda model, the stress tensor can be respectively rewritten for:

➢ *Simple tension mode:*

✓ The Hart-Smith model

$$\sigma = 2\left(\lambda-\lambda^{-2}\right)\left(C_1e^{\beta\left(I_1-3\right)^2} + \frac{C_2}{\lambda I_2}\right) \qquad (22)$$

✓ The Gornet-Marckmann model

$$\sigma = 2\left(\lambda-\lambda^{-2}\right)\left(C_1e^{\beta\left(I_1-3\right)^2} + \frac{C_2}{\lambda\sqrt{I_2}}\right) \qquad (23)$$

✓ The Beda model

$$\sigma = 2\left(\lambda-\lambda^{-2}\right)\left(C_1e^{\beta\left(I_1-3\right)^2} + \frac{C_2}{\lambda I_2}\right) \qquad (24)$$

➢ *Equibiaxial tension mode:*

✓ The Hart-Smith model

$$\sigma = 2\left(\lambda-\lambda^{-5}\right)\left(C_1e^{\beta\left(I_1-3\right)^2} + \lambda^2\frac{C_2}{I_2}\right) \qquad (25)$$

✓ The Gornet-Marckmann model

$$\sigma = 2\left(\lambda - \lambda^{-5}\right)\left(C_1 e^{\beta(I_1-3)^2} + \lambda^2 \frac{C_2}{\sqrt{I_2}}\right) \quad (26)$$

✓ The Beda model

$$\sigma = 2\left(\lambda - \lambda^{-5}\right)\left(C_1 e^{\beta(I_1-3)^2} + \lambda^2 \frac{C_2}{I_2}\right) \quad (27)$$

➢ *Pure shear mode*

✓ *The Hart-Smith model*

$$\sigma = 2\left(\lambda - \lambda^{-3}\right)\left(C_1 e^{\beta(I_1-3)^2} + \frac{C_2}{I_2}\right) \quad (28)$$

✓ *The Gornet-Marckmann model*

$$\sigma = 2\left(\lambda - \lambda^{-3}\right)\left(C_1 e^{\beta(I_1-3)^2} + \frac{C_2}{\sqrt{I_2}}\right) \quad (29)$$

✓ *The Beda model*

$$\sigma = 2\left(\lambda - \lambda^{-3}\right)\left(C_1 e^{\beta(I_1-3)^2} + \frac{C_2}{I_2}\right) \quad (30)$$

VI. EVALUATION OF THE HYPERELASTIC PARAMETERS THE MODELS

According to the relations (23) and (24), the reduced stress can be respectively written like:

$$\frac{\phi}{2} = \frac{\sigma}{\lambda - \lambda^{-2}} = C_1 e^{\beta(I_1-3)^2} + \frac{C_2}{\lambda\sqrt{I_2}} \quad (31)$$

And

$$\frac{\phi}{2} = \frac{\sigma}{\lambda - \lambda^{-2}} = C_1 e^{\beta(I_1-3)^\alpha} + \frac{C_2}{\lambda I_2} \quad (32)$$

Through both relationships, the hyperelastic parameters estimated by using the approach-in-stage method [37]. In the first stage, the evaluation of C_2 consists to plot $\phi/2$ versus $\left(\lambda\sqrt{I_2}\right)^{-1}$ and the linear segment permits to estimate the slope C_2, displayed in figure **1**. The relation (32) can be rewritten like:

$$\frac{\phi}{2} = \frac{\partial W}{\partial I_1} + \frac{\partial W}{\partial I_2} \quad (33)$$

This equation equivalent to:

$$\frac{\partial W}{\partial I_1} = \frac{\phi}{2} - \frac{\partial W}{\partial I_2} = C_1 e^{\beta(I_1-3)^\alpha} \quad (34)$$

The previous equation allows us to evaluate C_1, β and α according to the Beda model by taking the logarithm form [33], the results are displayed in figures **2** and **5**. The figure **1** shows the identification of the parameter C_2 at the first partial solution. Both other hyperelastic parameters C_1 and β of the Gornet-Marckmann model are deeply evaluated by the method [38].

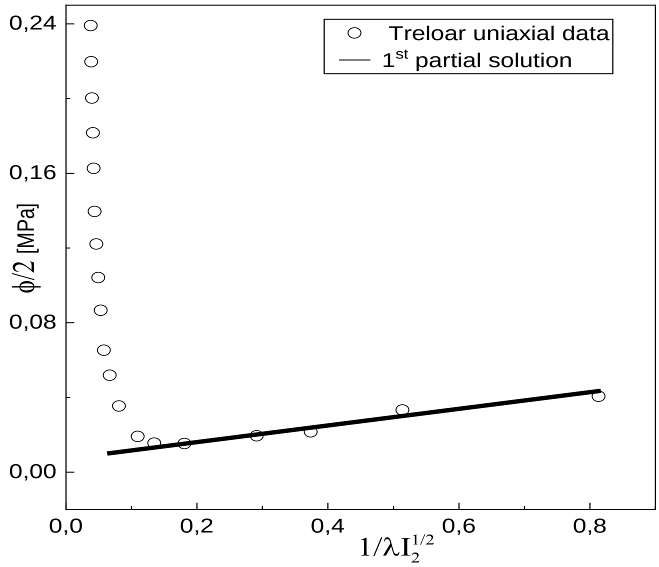

Fig. 1 :- Identification of the parameter C_2

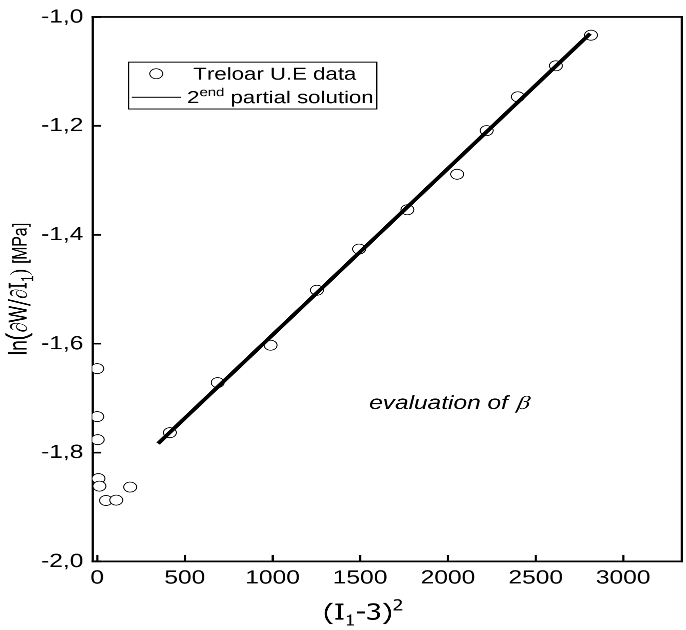

Fig. 2 :- Evaluation of the parameter β for the Gornet-Marckmann model

The comportment of $\partial W/\partial I_1$ is widely explained in the large deformation domain [4 21, 32, 33]. Some else considered as a power function or as an infinite series [29, 25]. Let's us consider $\partial W/\partial I_1$ as an exponential form, then one plots, $\ln(\partial W/\partial I_1)$ versus $(I_1-3)^\alpha$ and varies α, the hyperelastic parameter until to get the linear segment, see in the figures **2** and **5**. Taking account the previous explanation, the relation (34) can be expressed like:

$$\ln\left(\frac{\partial W}{\partial I_1}\right)=\beta\left(I_1-3\right)^\alpha+l \qquad (35)$$

β and l are the hyperelastic parameters. Considering the equation (35), one assumes that:

$$\frac{\partial W}{\partial I_1}=C_1 e^{\beta(I_1-3)^\alpha} \qquad (36)$$

The equation (36) is evaluated at the third stage to deduce the constant of the parameter C_1, see in the figures **3** and **6.**

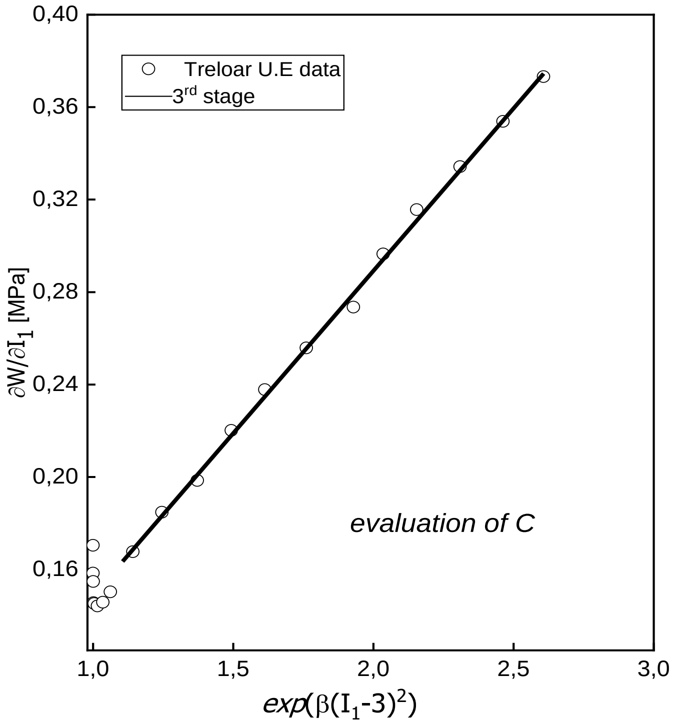

Fig. 3 :- Evaluation of C_1 for the Gornet-Marckmann model

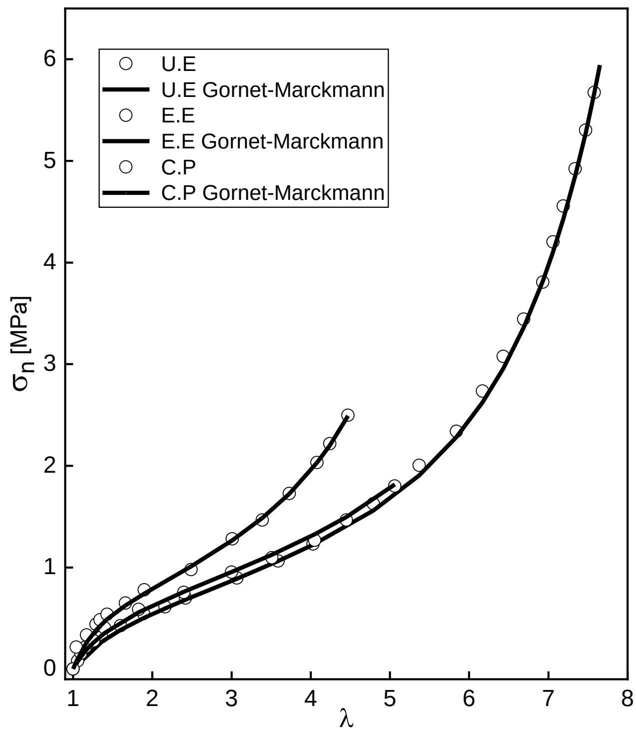

Fig. 4 :- Comparison of the Gornet-Marckmann model with Treloar data according to uniaxial extension versus equation (23), equibiaxial extension versus equation (26) and pure shear test versus equation (29).

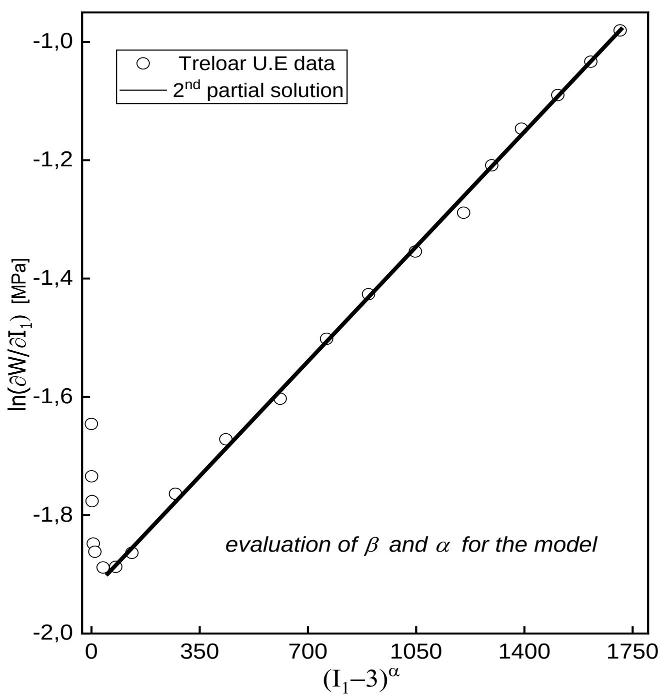

Fig. 5 :- Evaluation of the parameter β and α for the Beda model

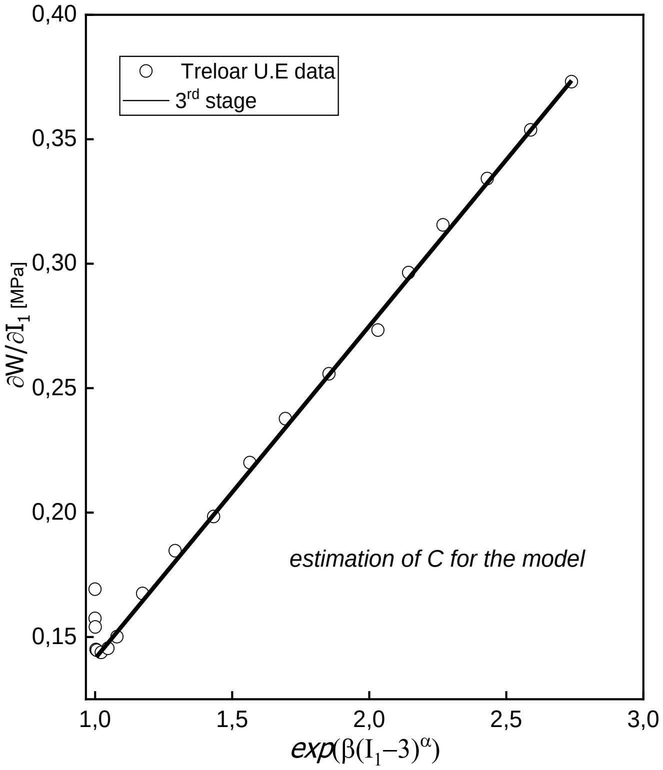

Fig. 6 :- Evaluation of C_1 for the Beda model

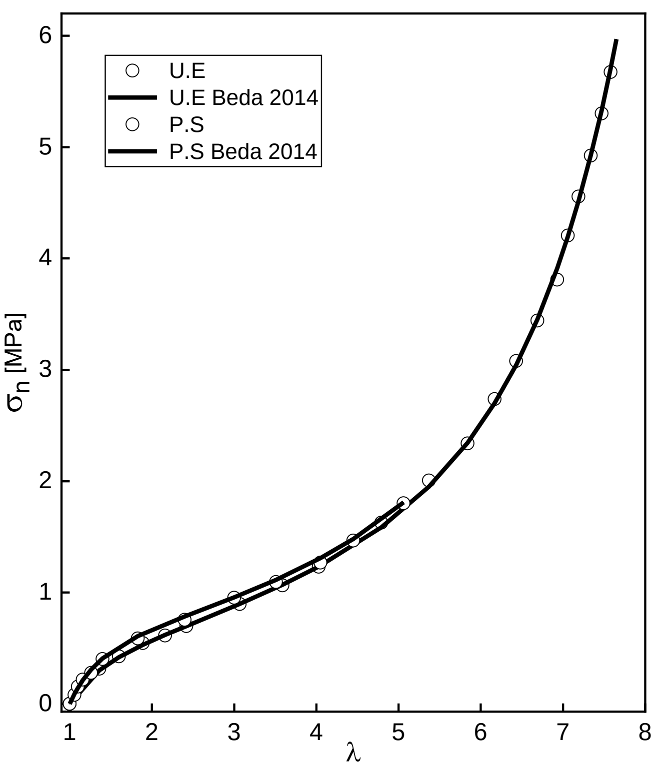

Fig. 7 :- Comparison of the Beda model with the Treloar data according to uniaxial extension, and pure shear experience versus equations (24) and (28).

VII. THE PHENOMENOLOGICAL MODEL PROPOSED

Based on the model of Hart-Smith, model of the Gornet-Marcmann and Beda, there is a proposal for a powerful isotropic and incompressible hyperelastic model that generalizes these models. The model proposed is given by the following expression:

$$W = C_1 \int \exp\left(\beta\left(I_1 - 3\right)^{\alpha}\right) dI_1 + C_2 \int \frac{1}{\sqrt{I_2}} dI_2 \qquad (37)$$

This model has distinguished itself by its ability to describe the three common modes of deformation in the domain of the large strain. This strain energy has good accuracy to reproduce the Treloar experimental data in equibiaxial, uniaxial extension and pure shear test. The proposed model responses are depicted in figure **8** for the three modes of deformation.

The Treloar experimental has permitted to identify the hyperelastic parameters of the constitutive models. Those are performed using the three experiences synchronously. The computational results of this present word are shown in figures **4, 7** and **8**. The different values of these previous phenomenological constitutive hyperelastic parameters for rubber-like materials are given in the following table **1**.

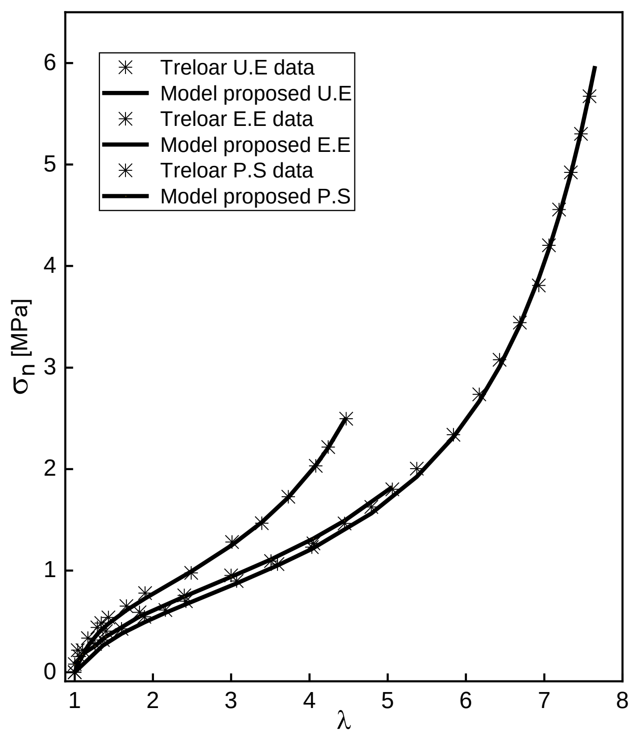

Fig. 8 :- Comparison of model proposed with Treloar data according to uniaxial extension, equibiaxial extension and pure shear.

Models	$C[MPa]$	β	α	$K[MPa]$
Gornet-Marckmann	0.145	$3.2.10^{-4}$	/	$1.36.10^{-3}$
Beda 2014	0.140	$7.5.10^{-4}$	1.800	0.210
Model proposed	0.137	$5.891.10^{-3}$	1.850	58.10^{-3}

Table 1:- the estimated value of the large strain parameters

VIII. CONCLUSION

In this present paper, many constitutive hyperelastic models: Gornet-Marckmann, Beda and proposal model were compared. These phenomenological models had a common link due to the exponential form in the term of I_1 identic to the Hart-Smith model. The Hart-Smith hyperelastic model offers an advantage that will facilitate the numerical implementation in the finite element method. The practical method that called the approach-in-stages had been used for identifying all the different optimal and accurate hyperelastic parameters. At the end of this work, the simple isotropic and incompressible hyperelastic model that generalizes both constitutive equations for rubber-like materials models based has been proposed.

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