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A Study of Chaotic Behaviour of Multiparticle Production in High Energy Nuclear Interactions

M. Mohisin Khan[1], N. Ahmad[2], A. Kamal[3], W. Bari[4], M.A. Nasr[5]
[1]Department of Applied Physics, ZHCET, Aligarh Muslim University, Aligarh, India.
[2]Department of Physics, Aligarh Muslim University, Aligarh, India.
[3]Department of Physics, Shibli National College, Azamgarh, India.
[4]Physics Department, University of Kashmir, Srinagar, India.
[5]Department of Physics, Misurata University, Misurata, Libya.

Abstract:- Present study is an attempt made to investigate the ocuurrence of erraticity be-haviour as a measure of bin multiplicity fluctuations in multiparticle production in relativistic nuclear interaction. The study has been carried out to see the said erraticity behaviour in ^{16}O-AgBr interactions at 4.5A, 14.5A, 60A and 200A GeV/c. Apart from the experimental results analysis has also been done for the AMPT generated events. Results of this analysis reveal that the observed fluc-tuations of multiplicity in pseudorapidity bins have eratic nature. Furthermore, the results indicating suppression of erratic fluctuations for the events having relatively higher multiplicity and this might be hints that this type of analysis may provide some vital information about the dynamical fluctuations occurring due to some new kind of physical processes in relativistic and ultra-relativistic nuclear collisions.

I. INTRODUCTION

Study of non-linear phenomena in high energy hadronic and nuclear collisions has shown an increasing attentiveness and curiosity[1-4] since the first observation of unexpectedly large local multiplicity fluctuations by JACEE Collaboration[5] and later for the data collected at various colliders(Fixed target and colliding beam accelerator).

A power law-growth of Scaled Factorial Moments (SFMs), F_q, with decreas-ing phase space bin-width, referred to as the intermittency, has been observed in e^+e^-, hadron-hadron(hh),hadron-nucleus(hA) and nucleus-nucleus(AA) colli-sions[1 and ref. therein]. The observed power law behaviour of SFMs indicates the presence of self-similar property in the mechanism of multiparticle produc-tion. This, in turn would suggest that the existence of large particle density in small phase space bin would show a rare occurrence but the phenomenon is not completely impossible. The non-statistical fluctuations in multiplicity and other global observables are envisaged to arise due to the occurrence of a phase transition from the hot and dense baryonic matter produced in the relativistic hadronic and nuclear collisions to the normal hadronic matter[6,7]. Investigations relating to the study of intermittency in F_q can not account for such fluctuations because of the fact that fluctuations determined on event-by-event (E-by-E) basis are likely to be suppressed in estimating the average value of F_q. To account for these fluctuations and the associated scaling behaviour, a new method of analysis, called the erraticity, proposed by R.C. Hwa[7], has been used. It is worthmen-tioning that a few attempts have been made to investigate the erratic nature of particle production[6-13] but most of these are limited to simulated data sam-ples. It was, therefore, considered worthwhile to study the erraticity behaviour in relativistic AA collisions by analysing the experimental and simulated data on

AA collisions over a wide incident energy range. Such event-by-event studies have caught further attention after the avaliability of very high multiplicity events and huge statistics in AA collisions at the relativistic and ultrarelativistic energies(at SPS, RHIC, LHC). Simulation in the framework of perturbative QCD also supports chaotic multi-particle production in the high energy nuclear collisions[10]. The vertically averaged horizontal factorial moments, F^h_q, are estimated from[1,2,5,8];

$$F_q^h = \frac{1}{N_{evt}} \sum_{e=1}^{N_{evt}} F_q^e \qquad (1)$$

where, N_{ev} refers to the total number of events in a sample and $F_q^{(e)}$ is the event factorial moment describing the spatial pattern of an event and is defined as;

$$F_q^e = \frac{<n(n-1)......(n-q+1)>}{<n>_e^q} \qquad (2)$$

where n is the particle multiplicity in a particular pseudorapidity bin. Since $F_q^{(e)}$ is envisaged[5] to result in relatively larger fluctuations, when analysed on event-by-event basis. Hence a $F_q^{(e)}$ distribution for a given q and M is obtained; M is the number of equally spaced bins in a given pseudorapidity space. Such a distribution is visualized to help disentangle useful and interesting information about the fluctuations and chaos if its dependence on q and M are fully explored. Determination of a few moments of $F_q^{(e)}$ distribution, for example, normalized moments $C_q^p = <\varphi_q^p>, where\ \varphi_q^p = \frac{F_q^e}{<F_q^e>}$

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Is likely to serve the purpose. The order q is an integer while p can take on any value > 0 and need not be integers. If C_q^p exhibit a power-law behaviour of the type;

$$C_q^p \propto M^{\Psi_q^p} \qquad (3)$$

For a given q, then such a behaviour is referred to as erraticity[5]; $\psi_q(p)$ represent erraticity exponents[11].

As C_q^p are the moments of $F_q^{(e)}$ distribution, C_q^p obviously would be quite sensitive to the E-by-E erratic fluctuations. The derivative of $\psi_q(p)$ at p = 1,

$$\mu_q = \frac{d}{dp}\psi_{q\ at\ p=1}^p \qquad (4)$$

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Describes the anomalous scaling property of the fluctuation-width and is known as entropy index. Another quantity of interest is Σ_q, which is entropy like[6] expressed as:

$$\Sigma_q =< \Phi_q \ln\Phi_q > \qquad (5)$$

Entropy index, μ_q, may also be calculated from the lnM dependence of Σ_q from

$$\mu_q = \frac{\delta\Sigma_q}{\delta \ln M} \qquad (6)$$

It may be mentioned that μ_q is regarded as an appropriate parameter for mea-suring the chaotic behaviour of particle production[6,7,10].

II. THE DATA

Four sets of data on the interactions of ^{16}O−ions with AgBr targets at 4.5, 14.5, 60 and 200AGeV/c, from emulsion experiments performed by EMU01 collabo-ration[11,12,13,14] are analysed. The numbers of events are 530, 519, 422 and 223, respectively. The other details of the data, criteria for selecting events and tracks etc., can be found else-where[15,16]. It is worthmentioning that the con-ventional emulsion technique has two main advantages over the other detectors:

Its 4π coverage and (ii) data are free from biases due to full phase space cov-erage. However, in case of other detectors, only a fraction of charged particles are recorded due to limited acceptance cones. This not only reduces the charged particle multiplicity but also distorts some of the event characteristics, such as particle density fluctuations[16]. For comparing the findings of the present study with the predictions of AMPT[17], event samples matching the real data are sim-ulated using the code ampt-v-1.2.21. The number of events in each simulated data set is equal to that in the real data sample. The events are simulated by taking into account the percentage of interactions which occur in the interactions of projectile with various target nuclei in nuclear emulsion[18] . The values of im-pact parameter for each data is so set that the mean multiplicities of relativistic charged particles become nearly equal to those obtained for the real data sets.

III. RESULTS AND DISCUSSION

All the relativistic charged particles produced in an interaction are arranged in ascending order of their η values. The entire η-space of an event is divided into M bins of equal width and the values of the event factorial moments, $F_q^{(e)}$, are calculated for q=2 and M = 20. Distributions of these moments are shown in Fig. 1 for the experimental and AMPT simulated data at four different energies.

It is clear from the figure that the distributions obtained for the four colli-sion energies have sharp peaks at $F_2 \simeq 1$. The shape of F_2-distributions for the AMPT simulated data is almost similar to that of the experimental data. F_2-distributions for all the data sets reveal that the event-by-event fluctuations are noticebly large. An additional peak at around F_2=0 is clearly observable. This may perhaps be due to the fact that with decreasing bin size lar large value of M(M=20), bin multiplicity would be less than the order of the moment, q for all the bins and hence resulting in a vanishingly event factorial moment. It is also important to mention that the similar trends in $F_q^{(e)}$ distributions are

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reported for the MC simulation of jet fragments of the perturbative QCD[10].

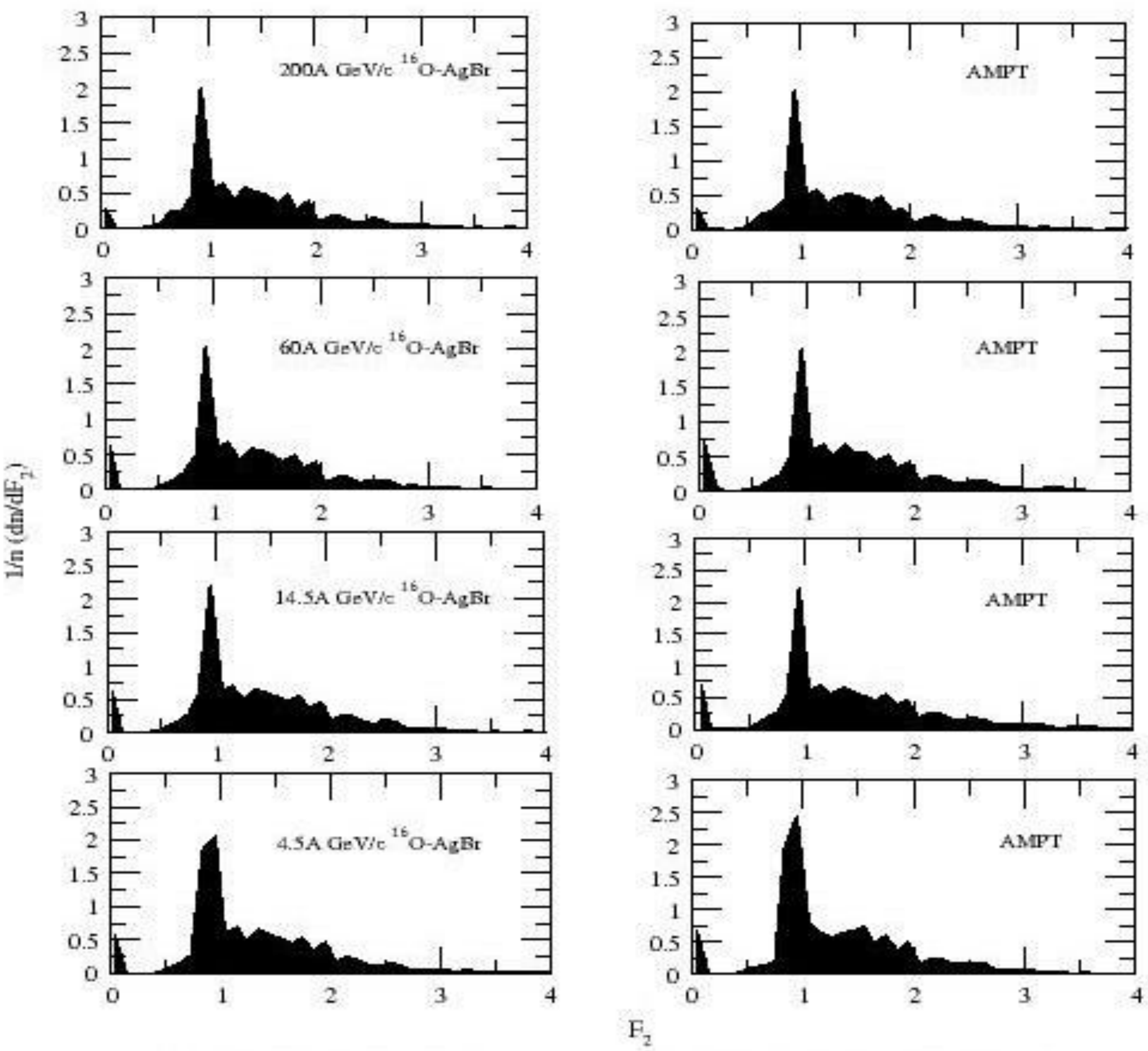

Fig. 1 F_2-distributions for the experimental and AMPT simulated for M=20 data.

It has been argued that the large event-by-event fluctuations disappear when $F_q^{(e)}$ is averaged over the entire data sample and one should measure these fluctuations in terms of C_q^p moments.

For this purpose values of C_p^q moments are calculated for p = 0.5, 1.0, 1.5, 2.0, q=2,3,4 and M = 2^ν where ν = 0, 1, 2, 3....... The variations of $\ln C_q^p$ with lnM for the experimental data sets at 4.5A, 14.5A, 60A and 200A GeV/c ^{16}O-AgBr interactions are displayed in Fig. 2.

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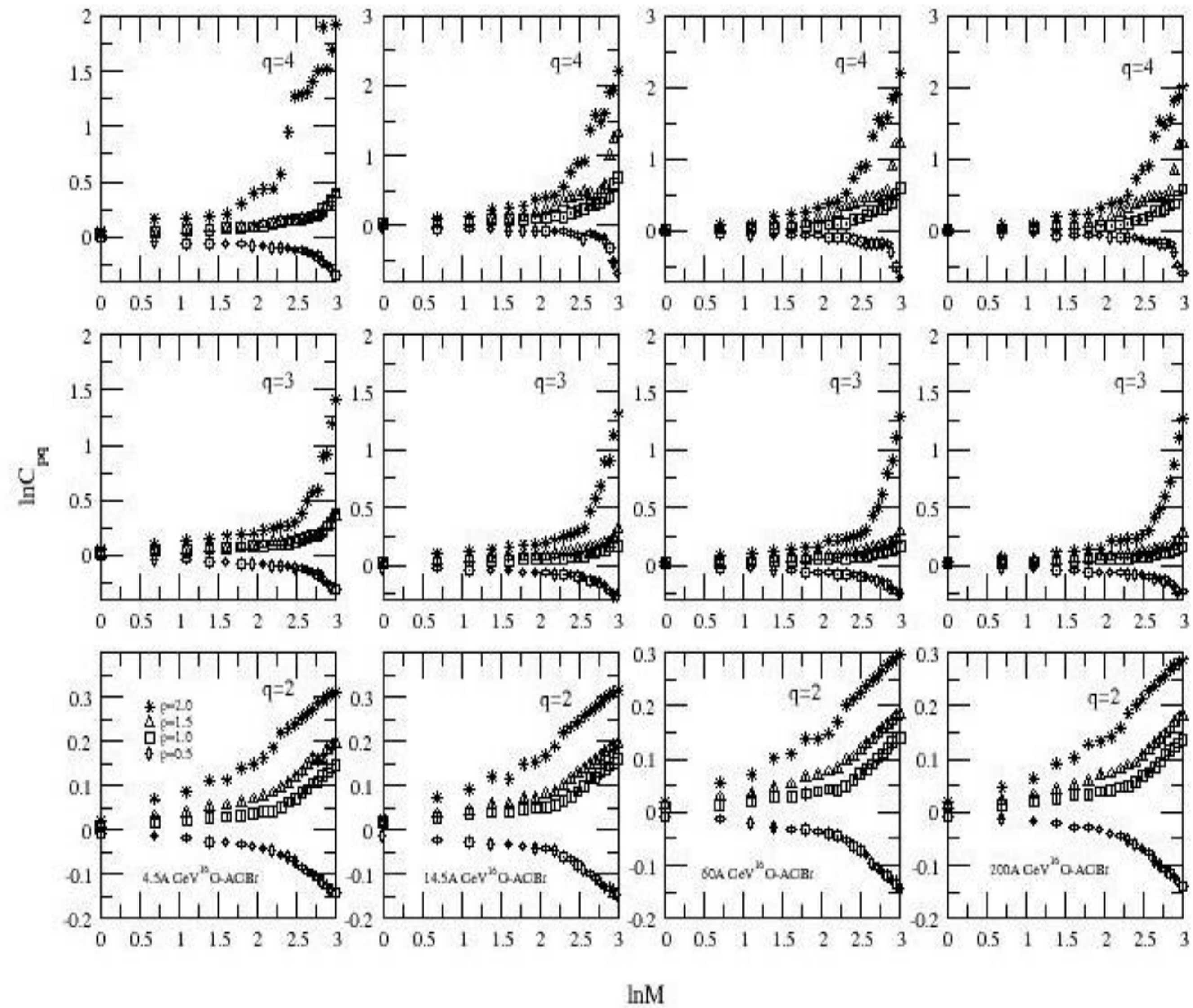

Fig.2 Variations of C_{pq}-moments with M on logarithmic scale.

It is clear from the figure 2 that the shapes of the plots at four different energies are similar. However there are slight differences in the values of C^q_p for the data at these energies.

The comparisons of C_p^4 vs lnM plots for the experimental and AMPT simulated data at 60A and 200A GeV are shown in Fig.3. It is clear from Fig.3 that AMPT produced almost the same variation as for the experimental data.

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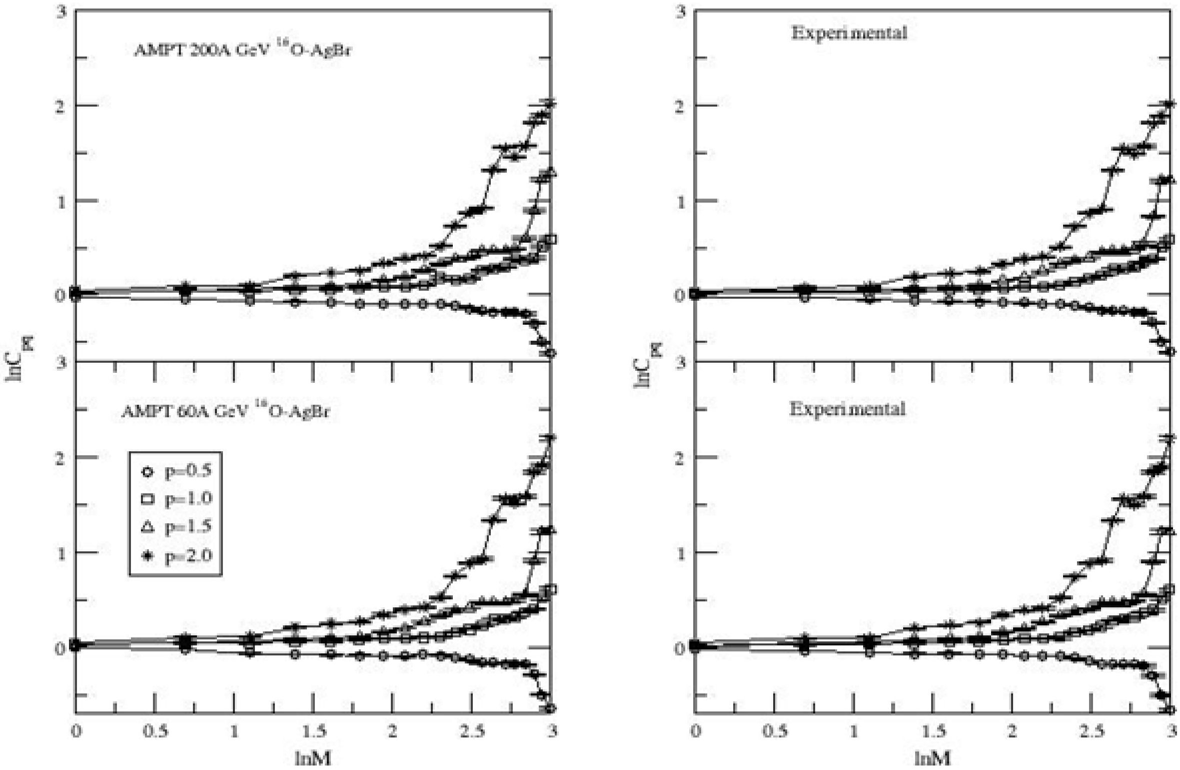

Fig.3. Comparison of $\ln C_{pq}$-moments vs. lnM plots for the experimental and AMPT simulated data sets at two different energies.

As suggested[1,2] that the information about the spatial and e-by-e fluctuations can be obtained by studying the nature of erraticity exponents, $\Psi_q(p)$, vs p plots. Fig.4 shows the dependence of Ψ_4 on the parameter p for the experimental and simulated data sets at two different energies. Despite the slight differences in the values of Ψ_4 for the experimental and simulated data the dependence on the p is of similar nature.

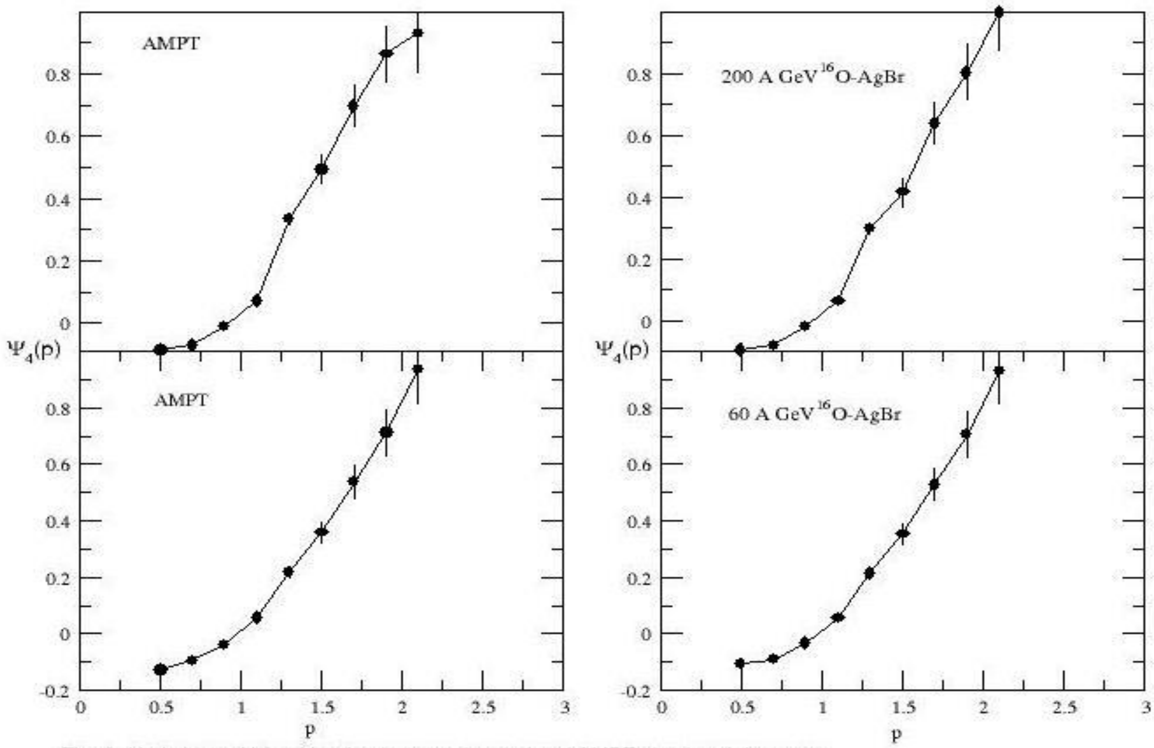

Fig. 4. Variations of Ψ_4 with p for the experimental and AMPT simulated data sets.

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In the figure 5 the variation of $\sum_4$ with lnM for the experimental and simulated data. It is observed from this figure that $\sum_4$ exhibits similar kind of dependence on lnM for both the data sets. The variations of Ψ_4 and $\sum_4$ as obtained in the present study are similar as reported by other workers[8,9].

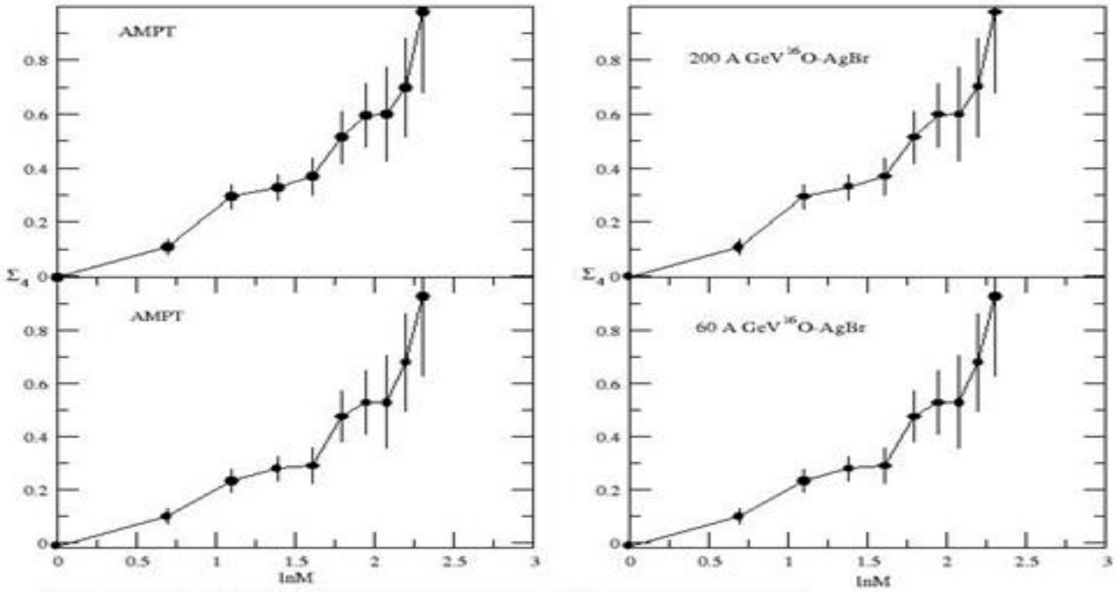

Fig. 5 Variations of Σ_4 with lnM for the experimental and AMPT simulated data sets.

The effect of event multiplicity on the erraticity behaviour of the produced particles in high energy nuclear collisions can be study by calculating entropy indices, μ_q for the group of events with fixed relativistic charged particle multiplicity,n_s . In the present study we have calculated fourth order entropy index, μ_4 and its variations with n_s for the experimenat and AMPT data at two different energies is plotted in Fig.6.It may be noted from the figure 6 that the experimental and AMPT data exhibit essentially the similar dependence of μ_4 on n_s. This, in turn, would indicate that the observed chaoticity/erraticity is arising due to some kind of fluctuations as majority of the events are those in which projectile interacts with AgBr targets and eventualy have high values of multiplicity, n_s.

It is interesting to note that the values of the erraticity moments and entropy indices for the experimental and AMPT simulated data are comparable. These results, also, tend to suggest that the erratic fluctuations, observed in the present study, are arising mostly due to the statistical reason as these fluctuations seem to vanish for $n_s > 150$. Therefore, for the events having very high multiplicities , $F_q^{(e)}$ moments may be used to describe the event spatial pattern associated with it and the erraticity behaviour of the multiparticle system may be satisfactorily represented by the erraticity moments.

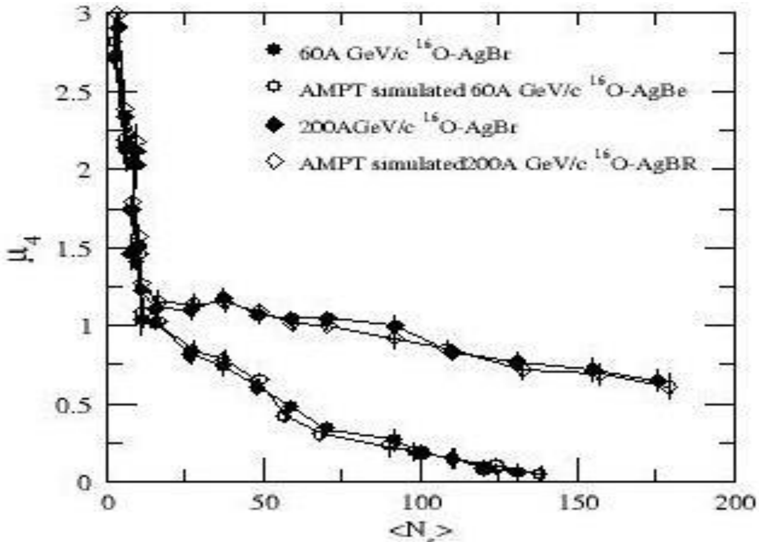

Fig.6. Variation of m_4 with averaged multiplicity of relativistic charged particle.

IV. CONCLUSIONS

In the present study erraticity behaviour of the multiparticle production in 4.5A, 14.5A, 60A and 200A GeV/c ^{16}-AgBr interactions has been studied for the ex-perimental and simulated data. Event factorial moments, $F_q^{(e)}$, has been used as a primary tool to describe the spatial pattern of produced particles in an event. All other parameters studied with the help of Figs 2,3,4,5 and 6 which are used to categorize the erraticity behaviour specifically are drived from the event fac-torial moments. Findings of present study suggest that the fluctuations due to small multiplicity of an event can control the erraticity behaviour. The chaotic behaviour may be thought to show insensitivity to the physical conditions such as the incident energy. Our results also suggest that the erraticity observed in the present study seem largely of the statistical nature and there is no clear evi-dence of the presence of dynamical fluctuations. The observed fluctuations may be thought due to some new kind of physics if the study is extended to multi-particle production at RHIC and LHC energies, where one can study very high multiplicity events. The results of present study are in good agreemnet with the results reported by other workers[8,9,11]. We are trying to carry out the similar erraticity analysis for the ALICE data for Pb-Pb collisions at $\sqrt{s_{NN}}$= 2.76 and 5.02 TeV.

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