Risk Modelling using EVT, GJR GARCH, t Copula for Selected NIFTY Sectoral Indices

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Abstract:- This paper makes an attempt to explain the procedure as well as estimate the VaR of a selected portfolio of the Nifty Sectoral Indices using approaches such as GJR GARCH-EVT-Copula, Filtered Historical Simulation, Generalised Extreme Value Theory and t Copula. The GJR GARCH-EVT-t Copula model extracts the filtered residuals obtained using the GJR GARCH technique and by using the Gaussian Kernel method for interior of the distribution and Extreme Value Theory for upper and lower tails to estimate the cumulative distribution of the residuals. A comparison is made between the estimated VaR simulation by the Monte-Carlo method, the aforementioned method and by using t Copula to get the joint distribution of each sectorial indices. The normalised maxima of the sequence is measured by the GEV distribution. An alternative to the Monte Carlo simulation and the Historical simulation is the FHS technique. The mean equation is modelled using the ARMA model while the volatility is modelled using GARCH with a non-parametric specification of the probability distribution of asset returns. The VaR estimates of the equally weighted portfolio of NIFTY Sectoral indices of 95% and 99% confidence intervals are backtested over a 2478-day estimation window.

Keywords:- *Value at Risk, NIFTY Sectorial Indices.*

I. INTRODUCTION

Traditionally capital markets are considered as barometer of an economy of a country and plays a crucial role in generating capital required for the economy. One of the inherent characteristics of these capital markets is that they are highly volatile in nature. With the implementation of globalization and liberalization policies by the developing countries the unrestricted flow of capital among the markets of the economies has resulted in the financial integration with world markets. Especially the developing markets due to their potential for better returns started attracting large capital inflows. As a result, the volatility of these capital markets also became a major concern for the investors. As the changes in the stock prices are very sensitive to the events happen at economy level there is a need for the economies to maintain the stable economic conditions so that the volatility of stock prices is always kept under control.

Although the VaR has become a very popular assessment of risk, it is not a problem-free solution also. First it is not always possible compare the VaR measured by traditional VaR models. They may often be fairly different, as demonstrated by numerous studies. And most of the studies focus on the univariate case of marginal VaR, component VaR and incremental VaR making it undesirable for portfolio risk management.

An attempt to empirically test and evaluate the VaR estimates of the portfolio consisting of NIFTY Sectoral Indices using GJR GARCH, Copula Theory, Extreme Value Theory and FHS technique is made in this study. The EVT is integrated with a time series model in order to obtain a conditional EVT which can filter the heteroscedasticity and the autocorrelations in the financial data. The multivariate joint probability density function is used to fit the stock market portfolios but it underestimates the VaR of the portfolios. The copula method helps in fitting the multivariate dependence model and is simple and flexible.

The section 2, 3, 4 and 5 deals with the Literature Review, Methodology, Results & Discussions and Summary & conclusions respectively.

II. LITERATURE REVIEW

The Heteroskedastic multivariate financial models have been introduced by Nelson (1981), Kraft and Engel (1982), Bollerslev et al. (1988), Diebold and Nerlove (1989), among others. Different multivariate financial models impose different restrictions on the dynamic behaviour of the variances, co variances and correlations. Since the financial time series are leptokurtic with heavy tails which make VaR being underestimated for i.i.d Gaussian distribution. So we tend to adopt the EVT and Copula in order to understand and model the tails and encapsulate the heavy-tail into the VaR estimation.

Lauridsen (2000) in his paper showed several defects of the VaR models in modelling the distribution of tails of profits and losses and extreme value models based on GARCH can be improvised by integrating changes in the volatility level. Burridge, John, Michael, & Chih (2000) proposed to estimate the market risk based on Extreme Value Theory which attempted to model the rare market events. Mendes & Carvalhal (2003) proposed that the Extreme Value Theory to analyse ten Asian stock market for estimating the VaR is a more conservative way to decide the capital requirements than traditional VaR models. Selcuk, Gencay & Fatuk (2004) demonstrated that the Generalized Pareto Distribution (GDP) and Extreme Value Theory aptly fits the tails of the return distribution in these markets and are more accurate at higher quantiles. Palaro & Hotta (2006) demonstrated that the conditional Copula theory can be an intense tool in estimating the VaR for a portfolio of Nasdaq and S&P 500 stock indices. Gilli & Këllezi (2006) proved that the POT method demonstrated more prevalent in the long term behaviour and it was favourable to compute in the interval estimates as it better endeavours the information about the distribution of the model. Bohdalova (2007) presented few strategies that uses Copula approach for making prudent choices as for the data employed and computational aspects are concerned can be made to decrease the overall cost and computational time. Marimoutou, Raggad, & Trabelsi (2009) results showed that the Conditional Extreme Value Theory and Filtered Historical Simulation procedures are indeed offering a major improvement as suggested for oil markets. Staudt, FCAS & MAAA (2010) highlighted a few of the considerations in modelling joint behaviour with Copulas such as choosing a Copula which appropriately catches the tail dependence and representing the skewness and kurtosis of the fundamental data and have natural interpretation. Huang, Chein & Wang (2011), Gondje-Dacka & Yang (2014) and Zhang, Zhou, Ming, Yang & Zhou (2015) has proved that it has the ability to understand and process the complex structure among the financial market events and even calculated the maximum loss and maximum gain of the distribution. Yi, Y., Feng, X., & Huang (2014) and Xiao & Koeniker (2009) proposed a method to estimate extreme conditional quantiles by combining quantile GARCH model of an Extreme Value theory approach. Zhang, H., Guo, J., & Zhou (2015) observed that the prediction effect of VaR is significantly more beneficial in a relatively stable market and VaR will underestimate market risk if there are large fluctuations in market and suggested to utilize stress testing. Singh, Allen & Powell (2017) applied GARCH (1,1) based by dynamic EVT approach and appeared with backtesting in stable as well as in extreme market conditions for the ASX-AII ordinaries(Australian) index and the S&P-500 (USA) Index.

III. METHODOLOGY

➢ *Extreme value Theory (EVT)*

Let us assume X represents the random variable of loss and is independently identically distributed given by:

$$F(x) = \Pr(X \leq x)$$

EVT heavily uses the Fisher- Trippett theorem and thus giving us practical solutions to n extreme random loss variables to measure the normalised maxima of the sequence. This process is also known as Generalized Extreme value (GEV) distribution given by:

$$H(z;a,b) = \begin{cases} \exp\left[-\left(1+z\dfrac{x-b}{a}\right)^{-\frac{1}{z}}\right] : z \neq 0 \\ \exp\left[-exp\left(\dfrac{x-b}{a}\right)\right] : z = 0 \end{cases}$$

When under the condition of x appears to 1+ z(x-b)/a > 0 and the parameters a and b in the GEV distribution refers to the location and scale parameters of the limiting distribution in H, their meaning is close but they are distinct from mean and standard deviation. The final parameter i.e. z is critical and it corresponds to the tail index as it shows the heaviness of extreme losses in the data sample.

Let's define Extreme value at risk directly relating to the fitted GEV distribution H_n (to n data points) given by: -

$$\Pr[H_n < H'] = p = \Pr[X < H']^n = [\alpha]^n$$

Where α is the VaR confidence level associated with the threshold H'. This can be defined as:

$$EVaR = \begin{cases} b_n[1-(-n\ln(\propto))^{-nz_n}] : (Frechet; z > 0) \\ b_n - a_n\, ln[-nln(\propto)] : (Gumbel; z = 0) \end{cases}$$

➢ *Copula Function*

Copula are primarily used to minimise tail risk. The price dependencies of multivariate distribution which can be split into into k univariate, marginal distribution and a copula theory can be formed. Let $X_1, X_2, X_3, \dots\dots, X_d$ as the random variables. Then, their *cumulative distribution function* is denoted by $H(x_1, x_2, x_3, \dots\dots, x_d) = P[X_1 < x_1, X_2 < x_2, \dots\dots, X_d < x_d]$ and the marginal as per the Sklar's theorem can be seen to be $F_i(x) = P[X_i \leq x]$. Copula can be defined as a multivariate distribution consisting of random variables in which each of its marginal distributions is uniform. It elucidates the dependence amongst two or more variables which possess the characteristic of non-normal distribution.

IV. RESULTS AND DISCUSSIONS

A. *GJR-GARCH-EVT-Copula Model*

It is necessary that the data needs to independent and identically distributed (i.i.d) before even we use EVT to model the tails of the distribution (i.e. of an individual index). The two important which every financial return exhibit is autocorrelation and heteroskedascity. Now we may look at different figures which depict the relation between ACF of returns as well as ACF of squared returns for a particular nation.

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➢ *ACF plots of Top 5 NIFTY 50 Sectoral Indices*

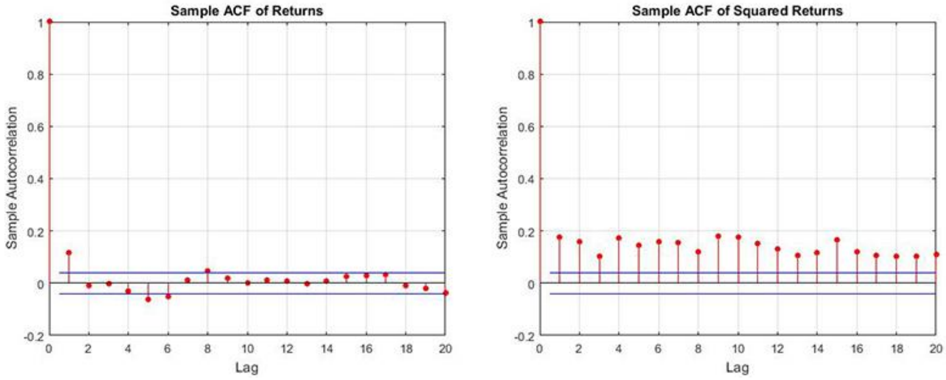

Fig 1A:- Sample ACF of returns and sample ACF of squared returns of NIFTY Bank

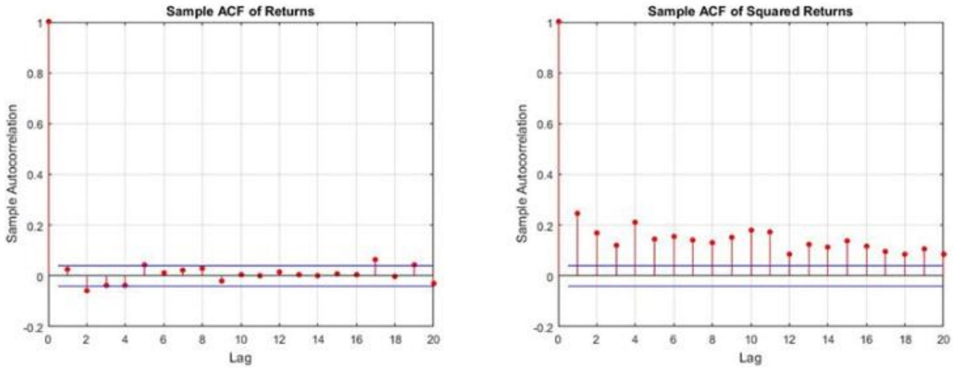

Fig 1B:- Sample ACF of returns and sample ACF of squared returns of NIFTY FMCG

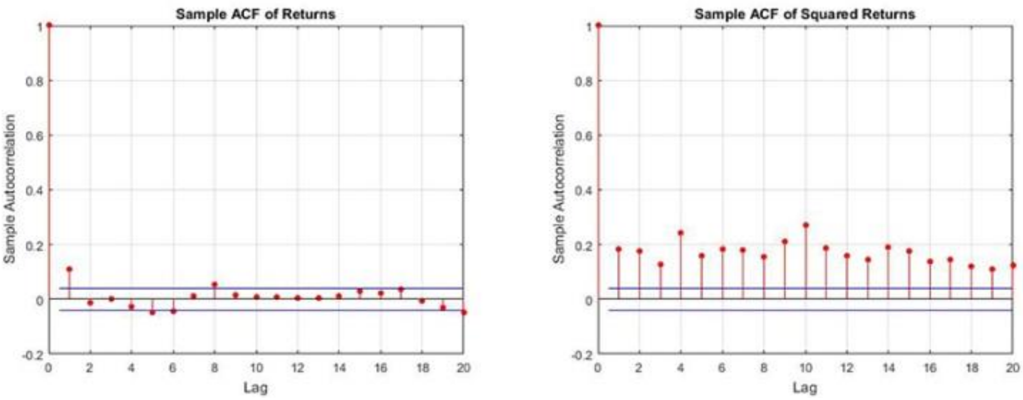

Fig 1C:- Sample ACF of returns and sample ACF of squared returns of NIFTY Private Bank

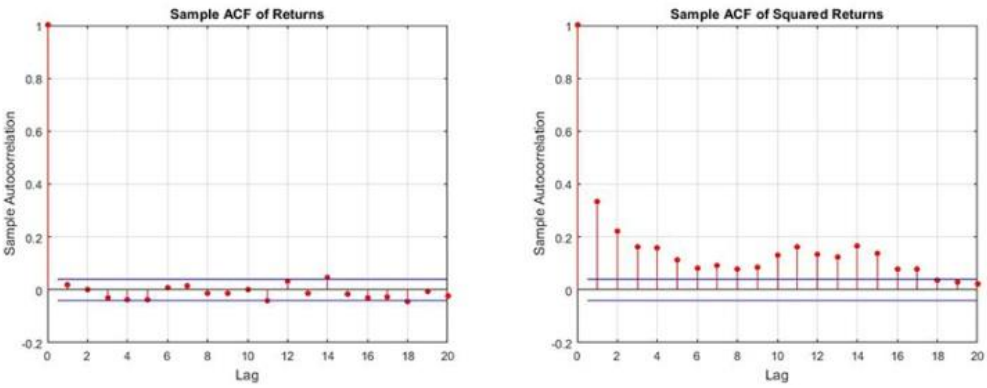

Fig 1D:- Sample ACF of returns and sample ACF of squared returns of NIFTY IT

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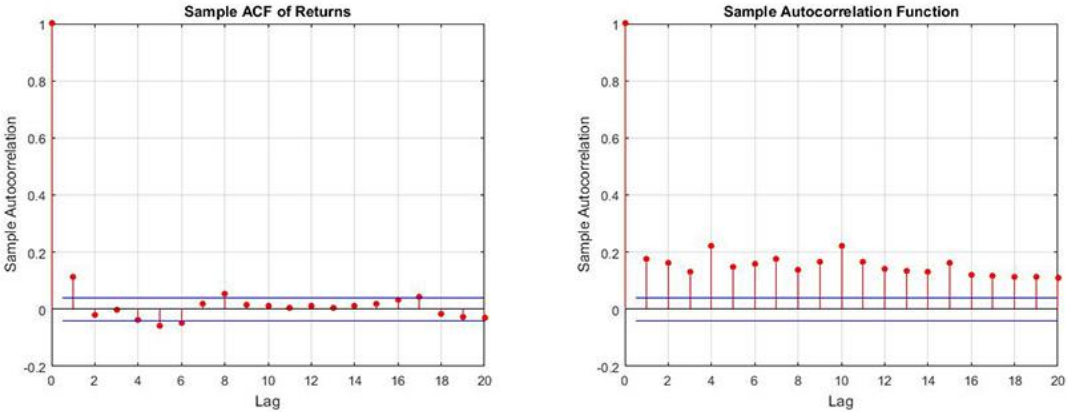


Fig 1E:- Sample ACF of returns and sample ACF of squared returns of NIFTY Financial Services
Fig 1:- ACF plots of Top 5 NIFTY 50 Sectoral Indices

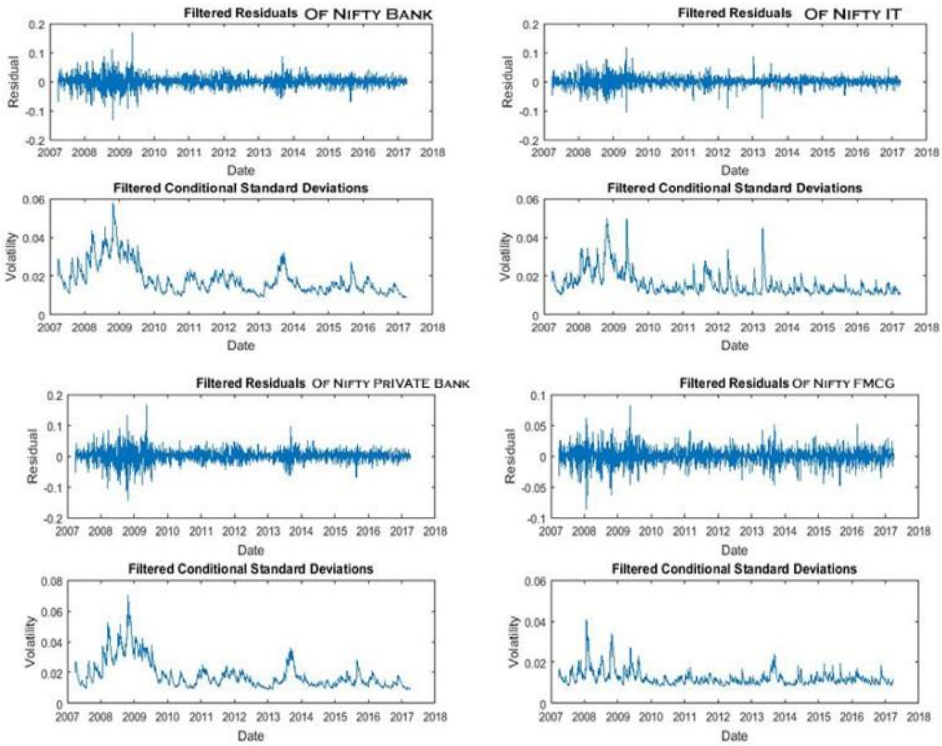
We require the GARCH model essentially to condition the data for the tail estimation process. The reason being that the squared returns illustrates a high degree of persistence w.r.t to variance. GARCH will also be crucially helpful in filtering out the serial dependence which is exhibited by the data. One which is quite noticeable is the fact that the returns are not independent from one day to the next. But AR (1)-GJR GARCH (1, 1) model helps in producing i.i.d observation which sorts out the requirement necessary for EVT. Now we try to fit AR (1)-GJR GARCH (1, 1) models to each index: -

$$r_t = c + \theta r_{t-1} + \varepsilon_t,\ \varepsilon_t \sim N(0, \sigma_t)$$

$$\sigma^2 = k + \alpha\sigma_{t-1}^2 + \phi\varepsilon_{t-1}^2 + \psi\varepsilon_{t-1}^2 I_{t-1}$$

Where: $I_{t-1} = \begin{cases} 0 & if \quad \varepsilon_{t-1} \geq 0 \\ 1 & if \quad \varepsilon_{t-1} < 0 \end{cases}$ and r_t is the index return, and σ_t the volatility.

After we fit AR (1)-GJR GARCH (1, 1) models to each index, we can then compare model residuals as well as the equivalent conditional standard deviation which is separated out from the raw returns.



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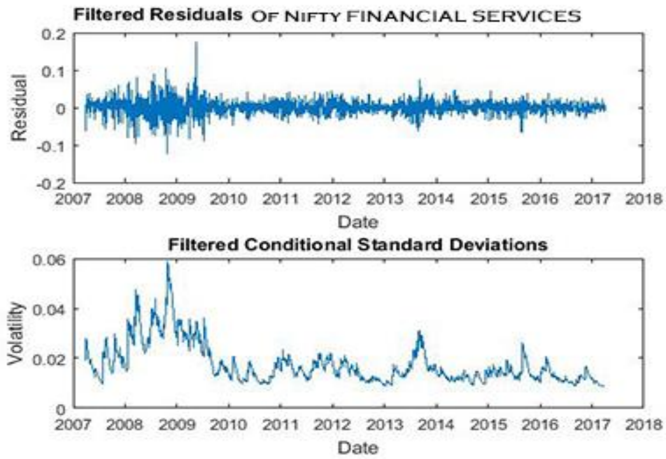

Fig 2:- Filtered residuals and filtered conditional standard deviations for NIFTY Sectorial Indices

When we closely perceive the lower graphs we observe a persistent variation in volatility present in the filtered residuals. Later on we can standardize the residuals. These standardized residuals follow zero – mean and unit- variance (i.i.d series). Therefore, it shows the EVT estimation of the sample CDF tail.

➢ *The ACF of the standardized residuals and squared standardized residuals.*

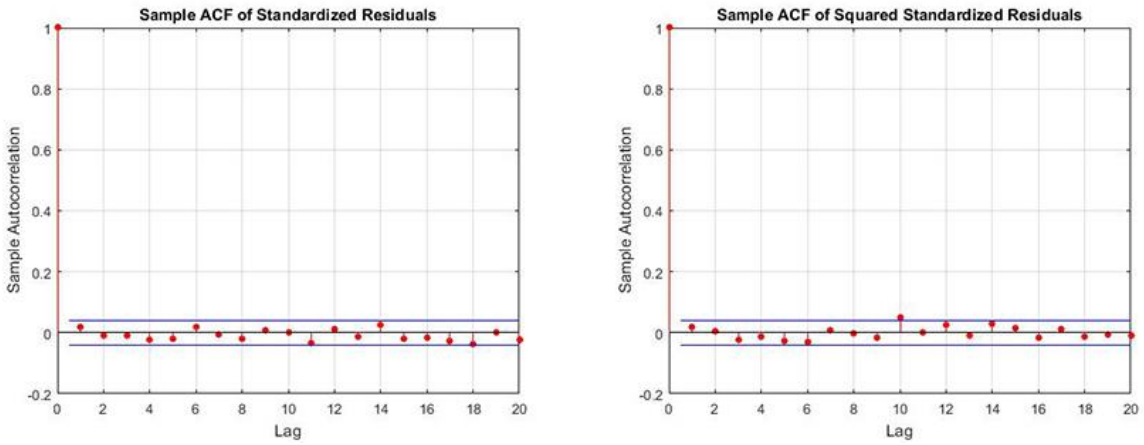

Fig 3A:- Sample ACF of the standardized returns and squared standardized residuals of NIFTY Bank

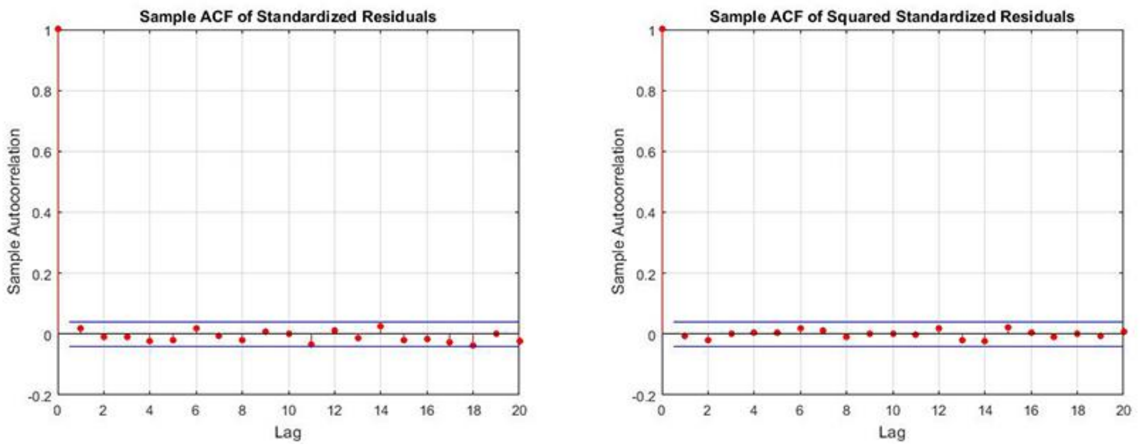

Fig 3B:- Sample ACF of the standardized returns and squared standardized residuals of NIFTY FMCG

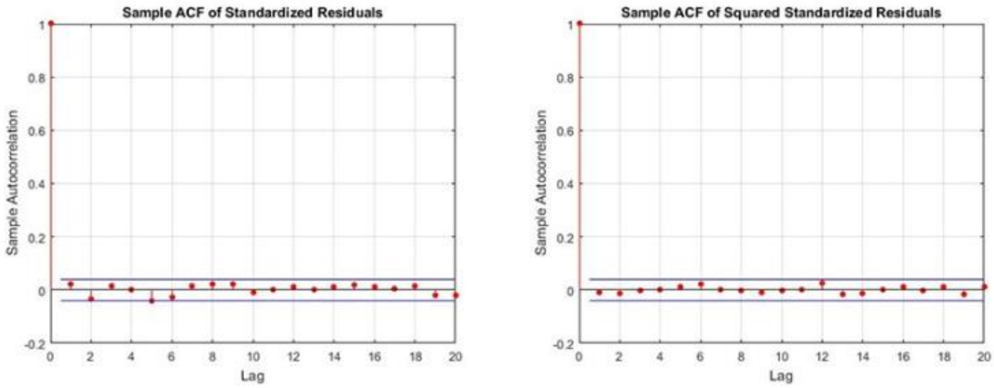

Fig 3C:- Sample ACF of the standardized returns and squared standardized residuals of NIFTY Private Bank

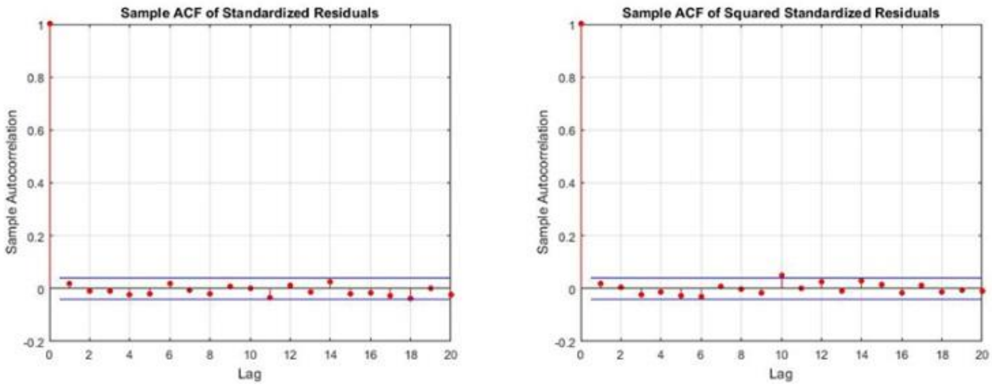

Fig 3D:- Sample ACF of the standardized returns and squared standardized residuals of NIFTY IT

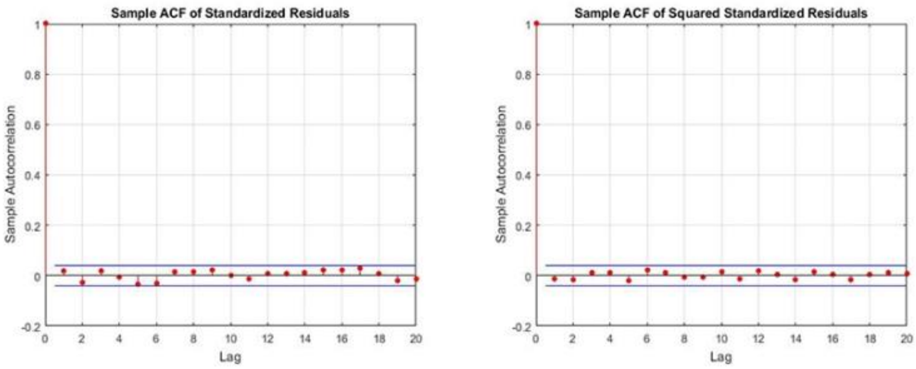

Fig 3E:- Sample ACF of the standardized returns and squared standardized residuals of NIFTY Financial Services
Fig 3:- Sample ACF of the standardized returns & squared standardized residuals of NIFTY Sectorial Indices

The next task involves fitting a probability distribution for each index so that their daily movements can be traced (this can be done after we filter out the data). While doing we are not concerned whether the data that is being analyzed is from normal distribution or any other form of simple parametric distribution.

Interior of the distribution is where we find the majority of the data, so we can use kernel density estimate for it. One of the biggest drawback of it is that it executes poorly when it is applied it to upper and lower tails. In the practice of risk management, we notice that it is of utmost importance that we accurately portray the tails of the distribution, even when the observed data in the tails is scarce. This gap is bridged with the help of GPD (generalized Pareto distribution).

After approximating three distinct regions of composite semi-parametric empirical CDF, we graphically join them and we will be able to see the results.

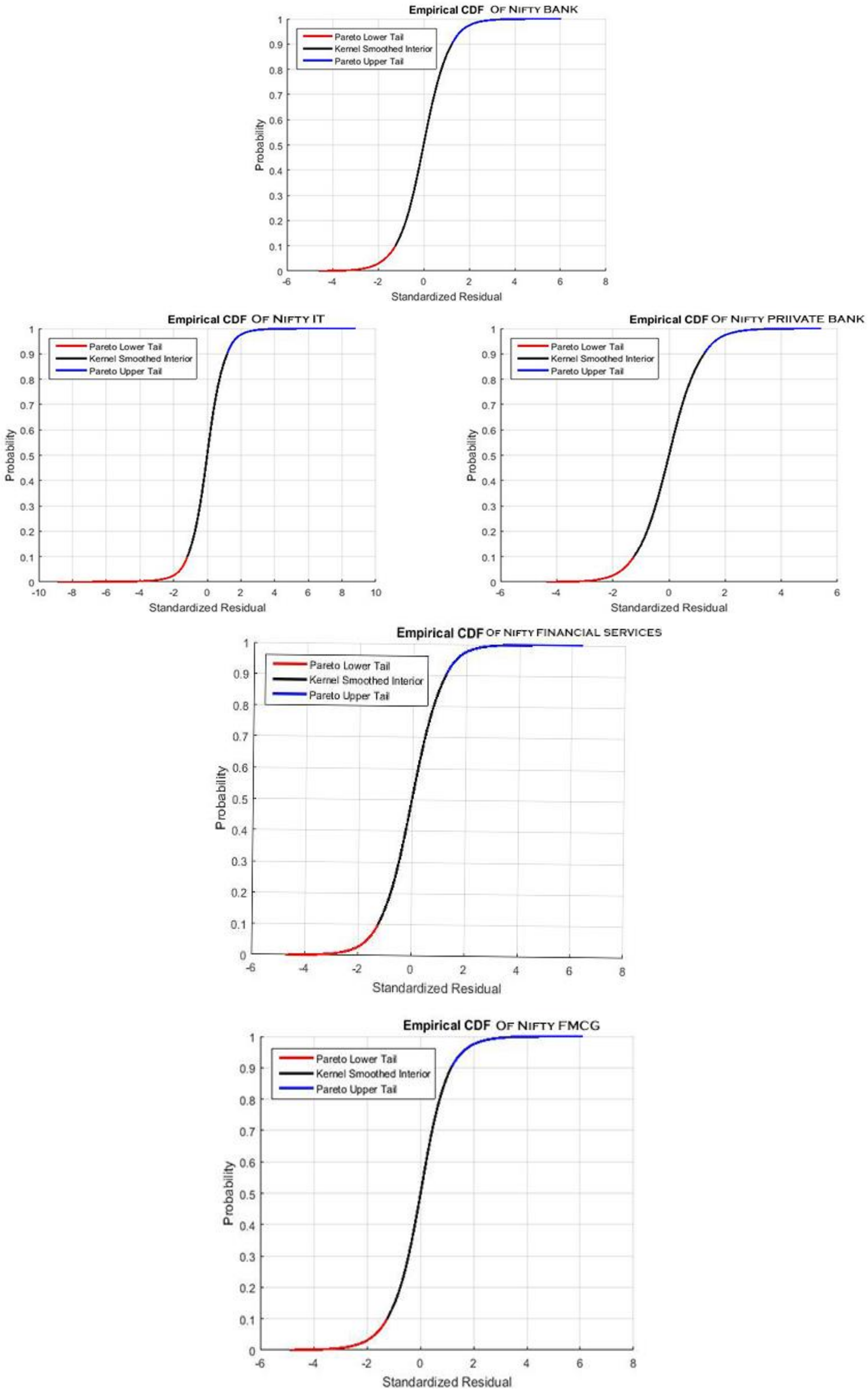

Fig 4:- Empirical CDF of a Top 5 NIFTY Sectoral Indices

We then suture together three distinct regions of the composite semi parametric empirical CDF which were estimated before and the results are displayed above. By observation we get to know that both lower and upper tail regions are appropriate for extrapolation. While the kernel smooth interior denoted in black can be used for interpolation.

We already know that the older graph illustrated CDF so it is indispensable to check whether the GPD would fit in detail. The parameterized Cumulative density function of GPD is given as:

$$F(y) = 1 - (1 + \xi y/\beta)^{-1/\xi}, y \geq 0, \beta > 0, \xi > -0.5 \quad (5)$$

for exceedances (y), tail index parameter ξ and scale parameter β.

Let us plot the empirical CDF of upper tail in excess of the residuals. This has to be supplemented with the CDF being fitted with the GPD.

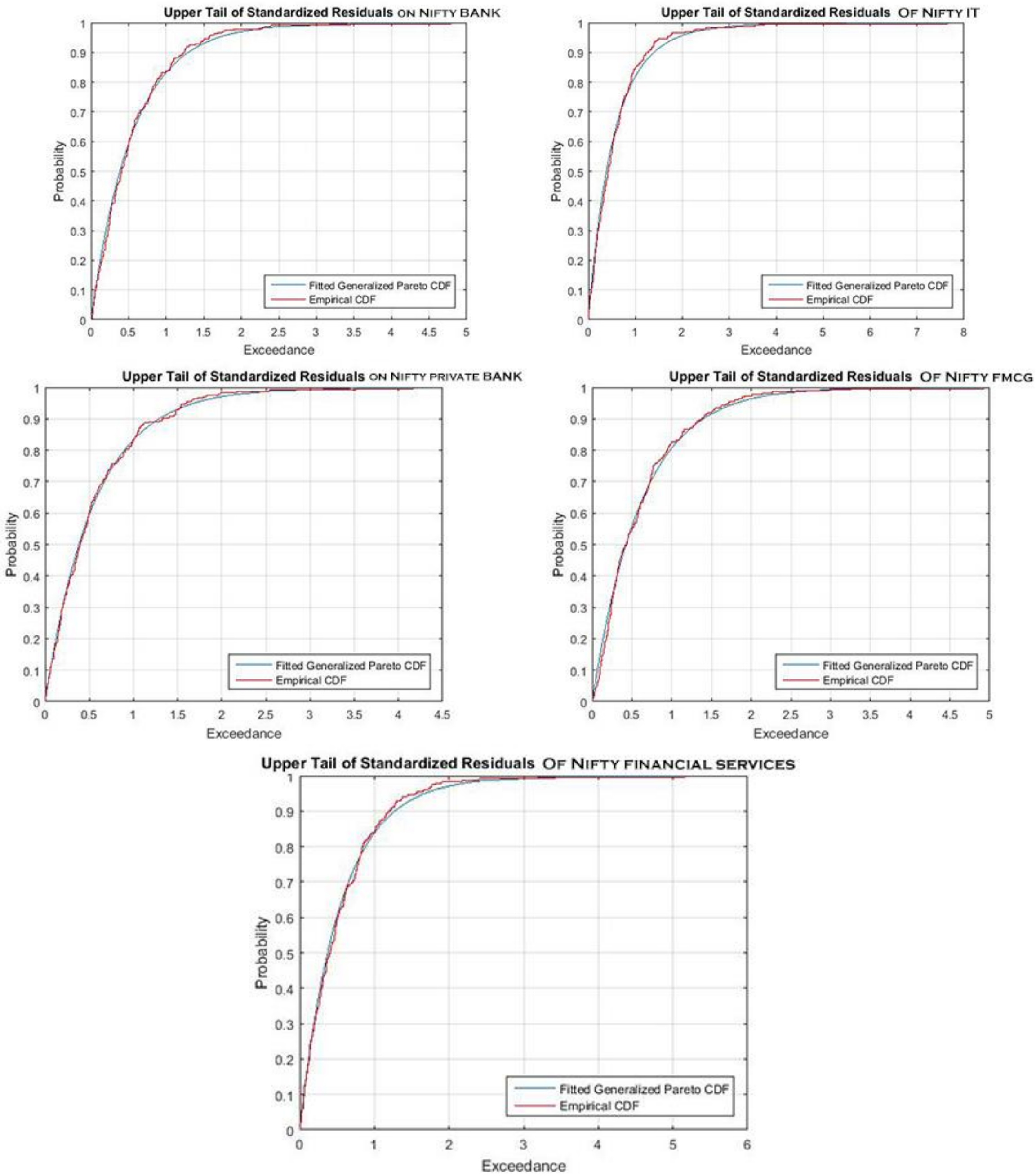

Fig 5:- Upper Tail of Standardized Residuals of NIFTY Sectoral Indices

From the last figure we can perceive that the empirically generated CDF curve for a specific nation matches well with the fitted GPD results. Thus far we have only used 10% of the standardized residuals and we notice that the fitted distribution quiet closely resembles the exceedances data. Hence we can conclude that the GPD model is a good choice. Consequently, for each of the five NIFTY Sectoral Indices we have five separate univariate models which describes the distribution of daily gains and losses. But the problem arises in tying these models together and this is done by Copula model. As per the definition of the Copula we know that it is a multivariate probability distribution whose individual variables are uniformly distributed. Thus we take these resultant univariate distributions to transform the individual data of each index to uniform scale. This form is crucial for fitting a Copula.

B. Filtered Historical Simulation

FHS combines a relatively sophisticated model-based treatment of volatility (GARCH) with a nonparametric specification of the probability distribution of asset returns. FHS retains the non-parametric nature of historical simulation by bootstrapping (sampling with replacement) from standardised residuals.

This method requires the observations to be i.i.d. But as we have already seen that the vast majority of the financial return series display various degrees of autocorrelation and heteroskedascity.

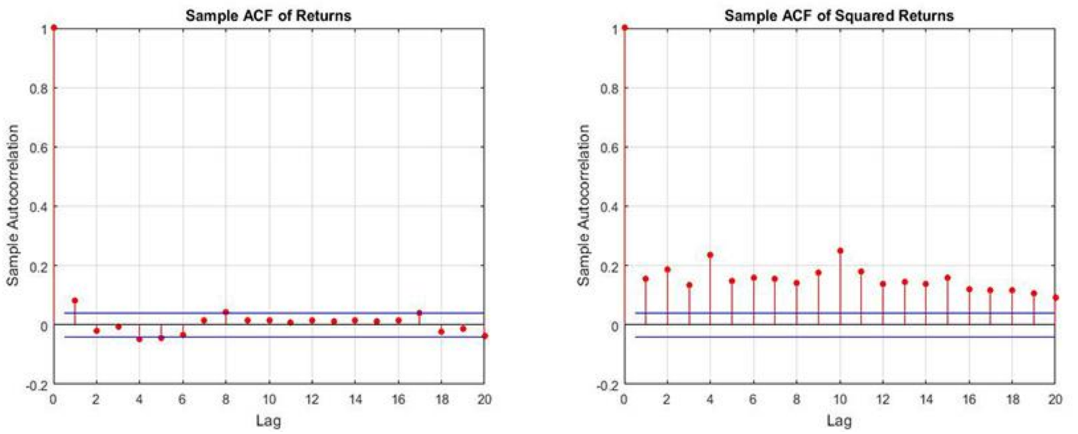

Fig 6:- Sample ACF and Sample ACF of Squared of the Portfolio Returns

The sample ACF of the portfolio returns exhibit a mild serial correlation. However, when the Sample ACF is squared it illuminates the degree of persistence in variance. Thus makes it necessary for the GARCH model to condition the data used in the bootstrapping method.

Thus for generating a series of i.i.d observations, we can fit AR (1) +EGARCH (1, 1) model given below:

$$r_t = c + \theta r_{t-1} + \epsilon_t, \epsilon_t \, N(0, \sigma_t)$$

And a symmetric EGARCH for conditional variance looks like

$$\log\left[\sigma_t^2\right] = k + \alpha \log\left[\sigma_{t-1}^2\right] + \phi\left(\left|z_{t-1}\right| - E\left[\left|z_{t-1}\right|\right]\right) + \psi z_{t-1}$$

Thus this shows that where AR model could only compensate for auto correlation, EGARCH model is able to compensate for heteroskedascity.

ARIMA(1,0,0) Model:

Conditional Probability Distribution: t

Parameter	Value	Standard Error	t Statistic
Constant	0.000651232	0.000211451	3.07983
AR{1}	0.0645931	0.0208937	3.09151
DoF	7.60779	1.02597	7.41525

Table 1:- Result of ARMA (1,0,0) Model

EGARCH(1,1) Conditional Variance Model:

Conditional Probability Distribution: t

Parameter	Value	Standard Error	t Statistic
Constant	-0.0936049	0.025349	-3.69264
GARCH{1}	0.989409	0.00291115	339.868
ARCH{1}	0.125349	0.0175651	7.13627
Leverage{1}	-0.0810271	0.0118851	-6.81756
DoF	7.60779	1.02597	7.41525

Table 2:- Result of EGARCH (1,1) Conditional Variance Model:

We see that the estimation depicts that there are six estimated parameters accompanied by their corresponding standard errors. (I.e. AR conditional mean model has two parameters while EGARCH conditional variance model has four parameters.

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Thus the fitted model can be written as:

$$r_t = -0.00065 * 10^{-7} + 0.0645\, r_{t-1} + \epsilon_{t,}\ \epsilon_t = N(0, \sigma_t)$$

$$\log[\sigma^2{}_t] = -0.0936049 + 0.99\log[\sigma^2{}_t] + 0.13\ (|z_{t-1}| - E[|z_{t-1}|) - 0.08 z_{t-1}$$

The t-statistic of AR (1) in ARCH (1, 0,0) model is greater than two which means that the parameter should be statistically significant, while for GARCH and ARCH in EGARCH (1,1) model).

Now next major step involves in modelling the residuals and the resultant standard deviation which are filtered out from the raw returns. Below graph depicts the variation in heteroskedascity present in the filtered residual. The i.i.d property is of significant for it allows bootstrapping that uses sampling procedures to safely avoid the downsides of choosing the sample from a population. The reason being that the successive observation is critically dependent upon each other.

Now let us take a look at the ACF of the standardized residual as well as the squared standardized residual.

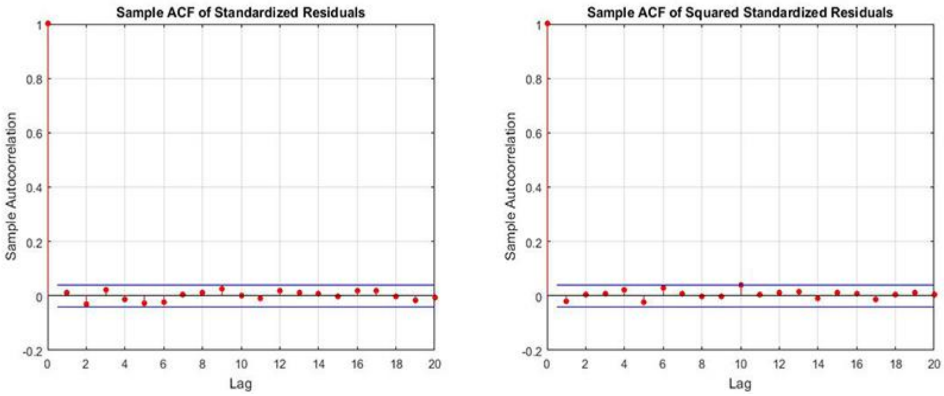

Fig 7:- Sample ACF of Standardized Residuals and Sample ACF of Squared Standardized Residuals

As we try to match both the ACF of the standardized residuals as well as the raw returns, it is revealed that the standardized residuals are exhibiting properties of being approximately i.i.d. therefore it is more amenable for subsequent bootstrapping.
We then sample for 10000 times on the filtered standard residual based on the bootstrapping method. This may be taken to input of i.i.d noise process of the holding period.

The below figure shows the cumulative distribution function and probability density function of simulation of one-month return.

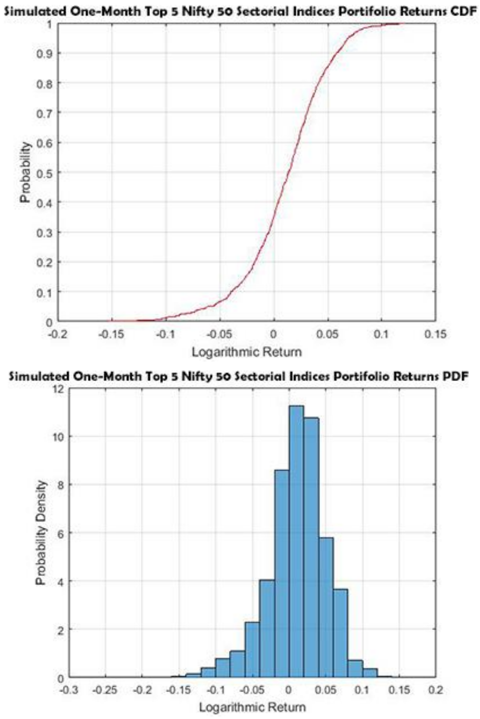

Fig 8:- Simulated One-month of Top 5 NIFTY 50 Sectoral Indices Portfolio Returns CDF

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C. Copula Simulation

Computing value at risk is shown in the following example. These are used for portfolio using multivariable Copula simulation with fat tailed marginal distribution. To calculate optimal risk-return portfolios, these simulations are used.

➢ *Returns & Marginal Distributions:*

The distributions of returns of each index are characterized individually to make Copula modelling. Although each return series distribution can be featured parametrically, it is needed to fit a semi-parametric model by utilizing a piecewise distribution with generalized pareto tails. To improve the behaviour in each tail, extreme value theory is used.

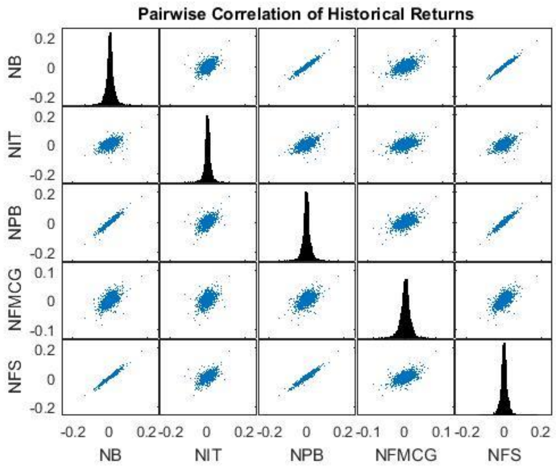

Fig 9:- Pairwise Correlation of Historical Returns

For top 5 NIFTY 50 Sectoral Index return series the code segment makes an object of type pareto tails. To create a composite semi-parametric CDF for every index, pareto tail objects encloses the estimates of parametric pareto lower and upper tail and the nonparametric kernel – smoothed interior.

```
Marginal distribution for NB:

Piecewise distribution with 3 segments
        -Inf < x < -0.0221941     (0 < p < 0.1): lower tail, GPD(0.0775833,0.0125568)
   -0.0221941 < x < 0.0230072   (0.1 < p < 0.9): interpolated kernel smooth cdf
         0.0230072 < x < Inf      (0.9 < p < 1): upper tail, GPD(0.13564,0.0117229)

Marginal distribution for NIT:

Piecewise distribution with 3 segments
        -Inf < x < -0.0176313     (0 < p < 0.1): lower tail, GPD(0.116593,0.011823)
   -0.0176313 < x < 0.0183578   (0.1 < p < 0.9): interpolated kernel smooth cdf
         0.0183578 < x < Inf      (0.9 < p < 1): upper tail, GPD(0.111013,0.0112286)

Marginal distribution for NPB:

Piecewise distribution with 3 segments
        -Inf < x < -0.0218927     (0 < p < 0.1): lower tail, GPD(0.120851,0.0136036)
   -0.0218927 < x < 0.0233097   (0.1 < p < 0.9): interpolated kernel smooth cdf
         0.0233097 < x < Inf      (0.9 < p < 1): upper tail, GPD(0.163863,0.0119621)

Marginal distribution for NFMCG:

Piecewise distribution with 3 segments
        -Inf < x < -0.0141652     (0 < p < 0.1): lower tail, GPD(0.0820131,0.00863497)
   -0.0141652 < x < 0.0152537   (0.1 < p < 0.9): interpolated kernel smooth cdf
         0.0152537 < x < Inf      (0.9 < p < 1): upper tail, GPD(0.122919,0.0069325)

Marginal distribution for NFS:

Piecewise distribution with 3 segments
        -Inf < x < -0.0208654     (0 < p < 0.1): lower tail, GPD(0.0713946,0.013367)
   -0.0208654 < x < 0.0219442   (0.1 < p < 0.9): interpolated kernel smooth cdf
         0.0219442 < x < Inf      (0.9 < p < 1): upper tail, GPD(0.165795,0.0113528)
```

The outcome which is a piecewise distribution object permits interpolation index in interior of CDF whereas extrapolation (function evaluation) in each tail. To estimate quantities out of historical record, extrapolation can be used though it has not valued for operations of risk management. The fit coming from pareto tail distribution is compared with normal distribution here.

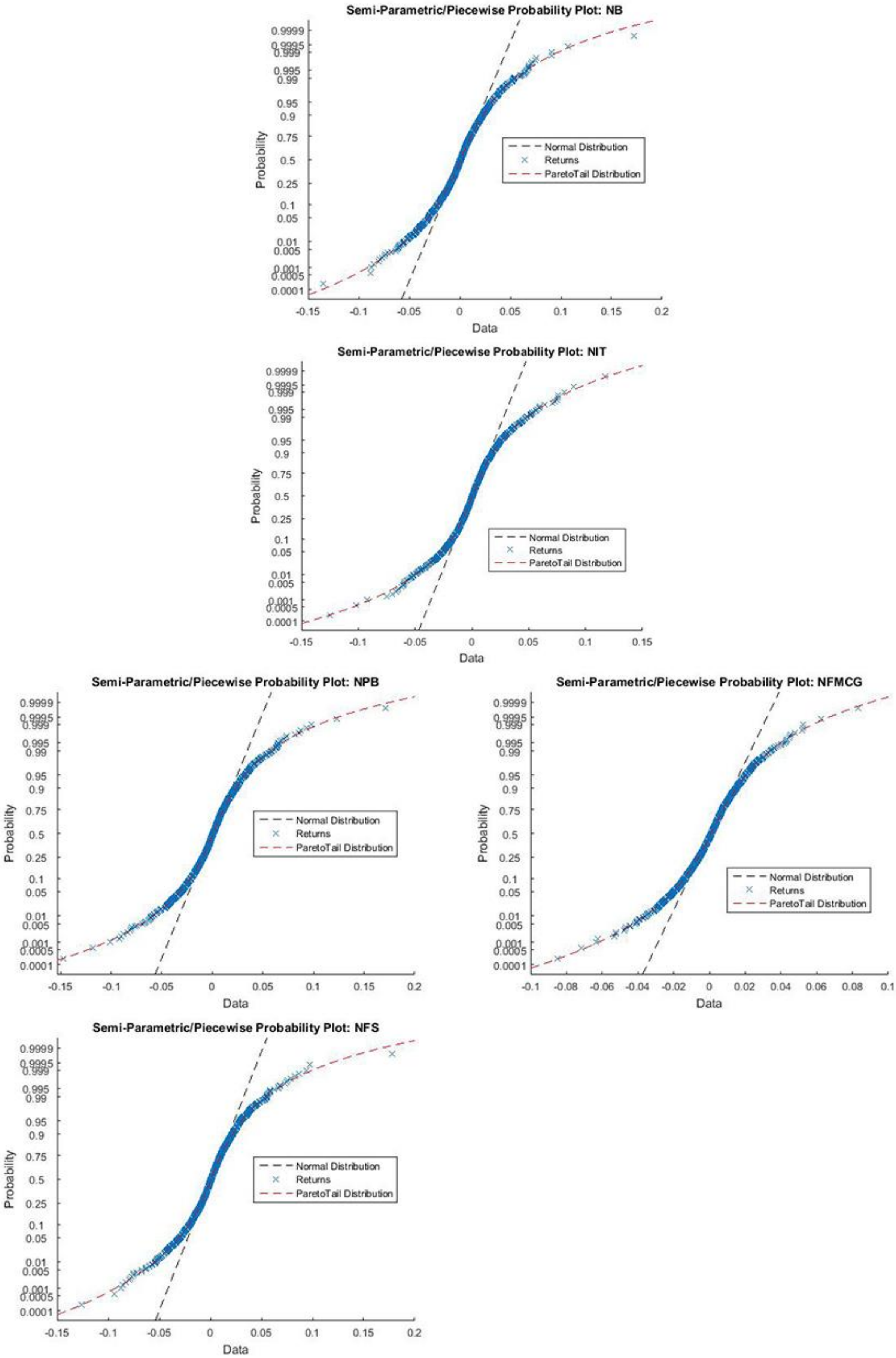

Fig 10:- Semi-Parametric Piecewise Probability Plot for NIFTY Sectoral Indices

➢ *Copula Calibration*

The Statistics toolbox function can be used to calibrate and simulate a t-Copula to data. Daily index returns are used to estimate the parameters of Gaussian and t-Copula are used to estimate the function Copula fit. When the scalar degrees of freedom become infinitely large then the t-Copula becomes a Gaussian Copula. These two Copula shares linear correlation as basic parameter and become same family.

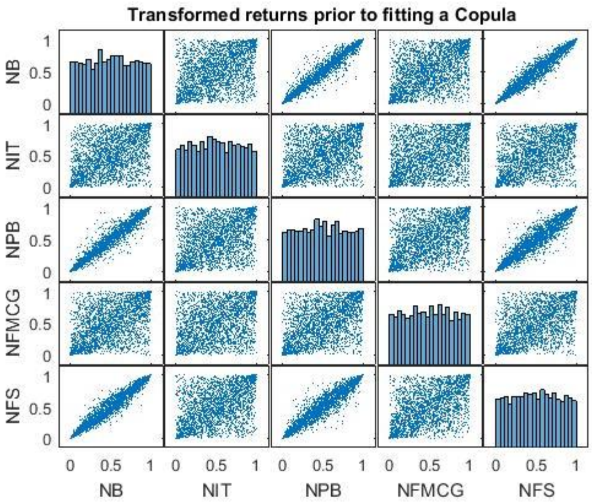

Fig 11:- Transformed returns prior to fitting a Copula

The Calibration of a linear correlation matrix of a Gaussian Copula is straightforward whereas it is not the same case for t-Copula. So in order to calibrate a t-Copula, Statistics tool box software give two techniques. The following code segment first transform the daily centred returns into uniform variates by using the piecewise, semi-parametric Cumulative Distribution Functions derived from above and then Gaussian and t-Copulas are fitted into the transformed data.

```
rhoT =

    1.0000    0.4699    0.9733    0.5043    0.9742
    0.4699    1.0000    0.4750    0.3737    0.4961
    0.9733    0.4750    1.0000    0.5029    0.9572
    0.5043    0.3737    0.5029    1.0000    0.5347
    0.9742    0.4961    0.9572    0.5347    1.0000


DoF =

    4.4232
```

The estimated correlation matrix is quite similar to linear correlation matrix though they are not identical.

```
ans =

    1.0000    0.4924    0.9672    0.5366    0.9778
    0.4924    1.0000    0.5103    0.4175    0.5268
    0.9672    0.5103    1.0000    0.5381    0.9590
    0.5366    0.4175    0.5381    1.0000    0.5704
    0.9778    0.5268    0.9590    0.5704    1.0000
```

t-Copula parameters have to be by the parameters obtained from t-Copula calibration which are of lower degree of freedoms have to be noted and a significant exodus from the Gaussian situation has to be indicated.

```
rhoT =

    1.0000    0.4693    0.9736    0.5045    0.9739
    0.4693    1.0000    0.4745    0.3745    0.4955
    0.9736    0.4745    1.0000    0.5030    0.9573
    0.5045    0.3745    0.5030    1.0000    0.5347
    0.9739    0.4955    0.9573    0.5347    1.0000


DoF =

    4.2935
```

The expected correlation matrix is related but not identical to the linear correlation matrix

```
ans =

    1.0000    0.4924    0.9672    0.5366    0.9778
    0.4924    1.0000    0.5103    0.4175    0.5268
    0.9672    0.5103    1.0000    0.5381    0.9590
    0.5366    0.4175    0.5381    1.0000    0.5704
    0.9778    0.5268    0.9590    0.5704    1.0000
```

➢ *Copula Simulation*

As the parameter of the Copula have been estimated. The combined dependent uniform variates have to be simulated by utilizing the function Copularnd. The uniform variates from Copularnd has to be transformed into daily central returns by extrapolating pareto tails and interpolating smoothed interior. The historical data set is tallied with simulated centred returns and the returns obtained are consistent. The returns obtained are assumed to independent of time but at any instant may possess dependence and rank correlation induced by Copula.

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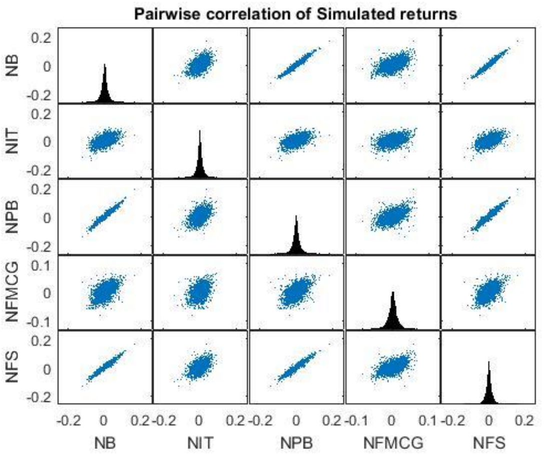

Fig 12:- Pairwise correlation of simulated returns

D. Generalised extreme value theory and extreme VaR

GEV distribution alone can be used to measure the normalised maxima of the sequence. This distribution is also called Fisher Tippet Distribution because it measures the chance of deviation of an event from the probability distribution's central tendency i.e. median. The family of GEV have converged to three types of Extreme value distributions i.e. Gamma, Gumbel and the Frechet distributions.

So let's look at the sample of N=5 largest losses on per NIFTY sectoral indices over last 2478 days, we can effortlessly fit it with GEV distribution and get the best estimates for the parameter z, an and b parameters. But if you see is a very small parameter. Instead of that why can't extract 5 worst losses that took place in the last 2478 days. thus it increases our n substantially to n=25.

As mentioned earlier the MATLAB's cell array is holding 2478 return series (each 2478 day long). We Increase the sample size to n=30 points by taking the top 5 maximal daily losses for each stock. Now we fit the GEV distribution

$$H(z;a,b) = \exp\left[-\left(1 + z\frac{x-b}{a}\right)^{-\frac{1}{z}}\right]$$

While engaging the ready to use function gevfit from Matlab statistics toolbox we get,

```
parmhat =

 -0.5073   0.0245  -0.0974

parmci =

 -0.7457   0.0178  -0.1078

 -0.2688   0.0336  -0.0870
```

The best estimates of the model's parameters are Z_{25}, a_{25}, b_{25}. The negative value of z actually comes from the Fretchet distribution since we fitted data with negative signs.

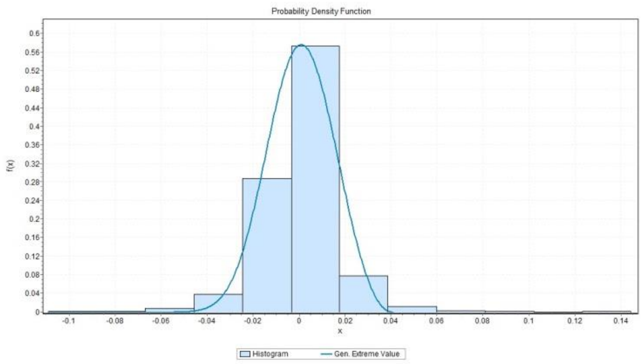

Fig 13:- Probability Density Function of Generalised Extreme Value Distribution

The best estimation of the 1 month EVAR is given as: -

$EVaR = b_{25} - \frac{a_{25}}{-z_{25}}[1 - (-n \ln(0.95))^{nz_{25}}]$ = -0.0729

$EVaR = b_{25} - \frac{a_{25}}{-z_{25}}[1 - (-n \ln(0.99))^{nz_{25}}]$ = -0.1241

The EVaR value is indicative of the fact that among the 5 NIFTY Sectoral Indices in our portfolio we are definitely expecting an extreme loss of 12.41% & 7.29% on the following month (taken from the last 2478 trading days). The cumulative distribution function for NIFTY 50 Sectoral Indices are given as:

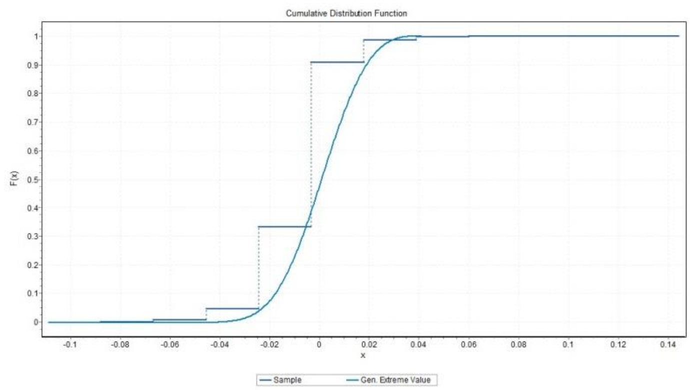

Fig 14:- Cumulative Distribution Function of Generalised Extreme Value Distribution

- *Computing VaR using different models*

In this section, we go through the methods of calculation and the approach adopted to establish our findings. Initially, we embark upon the task of transforming the individual standardized residuals pertaining to the AR (1)-GJR GARCH (1, 1) models into uniform variates. We attain this by utilizing the semi-parametric empirical CDF after which we fit the t Copula to the transformed data. It is important to note that the estimated optimal degrees of freedom for the t Copula is 8.4108. In this research, we also adopt the vital techniques of Filtered Historical Simulation, t Copula and Generalized Extreme Value method for comparison. Using the same, we simulate 1, 00,000 independent random trails of dependent standardized index residuals spanning a month-long horizon of 22 trading days. Lastly, we form a 1/5 equally weighted index portfolio composed of the individual indices assuming that we are given the simulated returns of each index. Next, we work on calculating the VaR at 99% and 95% confidence levels, again spanning the month-long risk horizon. In the table constructed below, we list out the estimated figures of 95% and 99% VaRs for t(8.4108) and other models.

	Value at Risk for different models			
Models	CEVT+t(8.4108) copula	FHS	t Copula	GEV appraoch
Max Loss	27.33%	30.86%	29.86%	35.81%
max Gain	17.69%	15.18%	16.20%	21.69%
95% VaR	-6.73%	-5.55%	-4.32%	-7.29%
99% VaR	-11.73%	-9.94%	-4.22%	-12.41%

Table 3:- Value-at-Risk Calculations for the different models

We can see that the table 3, that the CEVT-Copula based approach given the estimated optimal degree of freedom as 8.4108 performs best to be only followed by t Copula. Finally, it is to be noted that The Generalized Extreme Value approach and Filtered Historical Simulation overestimate the portfolio VaR.

V. CONCLUSIONS

The paper is an attempt to find an appropriate VaR model among the set of models namely GJR GARCH, EVT-Copula, t copula, Filtered Historical Simulation and General Extreme Value Distribution to estimate the VaR of returns of the NIFTY Sectoral Indices.

First, the market risk of the NIFTY Sectoral Indices portfolio is modelled by the Monte Carlo simulation using the t copula and EVT. Second, the distribution of the residuals is modelled using the POT based EVT. Lastly, the data and the simulated residuals are checked for their strong or weak correlation by fitting a seven-dimensional t copula.

Hence there is a perennial conflict as the method chosen by a financial institution decides the risk capital it holds. It is a non-trivial issue because choosing a method for portfolio VaR problems inaccurately measures market risk can have adverse impact on the functioning of the financial institutions. So the results of this study can be used to perform a good risk management on Global investors.

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