ISSN No:-2456-2165

Fuzzy Join- Semidistributive Semilattice

Kayalvizhi.M, Nandhini.R (Masters in Mathematics)
Thassim Beevi Abdul Kader College for Women, Kilakarai
Ramanathapuram, Tamilnadu, India.

Abstract:- The main idea followed in this paper is Fuzzy join-semidistributive semilattice. In this we worked some theorem related to the concept and give some definition related to the concept.

***Keywords**:- Lattice Semilattice, Sublattice, Modular Lattice, Join.*

I. INTRODUCTION

Some authors published many papers related to lattice concept. Many definitions given in preliminaries are based on the reference paper. Here we worked with the topic of Fuzzy join-semidistributive semilattice.

II. PRELIMINARIES

A. Definition:

Let L be a non-empty set, let ∨ and ∧ is said to be two binary operations defined on L. The (L, ∨, ∧) is said to be a lattice, if the following conditions are satisfied.

- ***Idempotent***
  $x \wedge x = x$ and $x \vee x = x$ for all $x \in L$
- ***Commutative***
  $x \wedge y = y \wedge x$ and $x \vee y = y \vee x$ for all $x, y \in L$
- ***Associative***
  $x \wedge (y \wedge z) = (x \wedge y) \wedge z$
  $x \vee (y \vee z) = (x \vee y) \vee z$ for all $x, y, z \in L$
- ***Absorbtion***
  $x \wedge (x \vee y) = x$ and $x \vee (x \wedge y) = x$ for all $x, y \in L$

B. Definition:

Let S be a non-empty set, let ∨ and ∧ is said to be two binary operations defined on S. The (S, ∨, ∧) is said to be a semilattice, if the following conditions are satisfied.

- ***Idempotent***
  $x \wedge x = x$ and $x \vee x = x$ for all $x \in S$
- ***Commutative***
  $x \wedge y = y \wedge x$ and $x \vee y = y \vee x$ for all $x, y \in S$
- ***Associative***
  $x \wedge (y \wedge z) = (x \wedge y) \wedge z$
  $x \vee (y \vee z) = (x \vee y) \vee z$ for all $x, y, z \in S$

C. Definition:

Let μ be a fuzzy set in L then μ is called a Sublattice of L if

i) $\mu(x+y) \geq \min((\mu(x), \mu(y))$
ii) $\mu(x.y) \geq \min((\mu(x), \mu(y))$

D. Definition:

A lattice (L, ∨, ∧) **is** distributive if the following additional holds for all x, y, z in L.

$$x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z) \text{ or}$$
$$x \vee (y \wedge z) = (x \vee y) \wedge (x \vee z)$$

III. MAIN RESULT

- ***Definition: 3.1***

A Fuzzy lattice L is called a Fuzzy join-semi distributive if

$\mu(x \vee y) = \mu(x \vee z) \Longrightarrow \mu(x \vee y) = \mu(x) \vee \mu(y \vee z)$, for all $\mu(x), \mu(y), \mu(z) \in L$.

- ***Theorem: 3.2***

Every Fuzzy join-semidistributive semilattice is fuzzy semilattice but every fuzzy semilattice is not the fuzzy join-semidistributive semilattice

Proof:

Given S is a Fuzzy join–semi distributive semilattice
$\Longrightarrow \mu(p \vee q) = \mu(p) \vee \mu(q \vee r)$, for all $\mu(p), \mu(q), \mu(r) \in S$.
To prove S is a Fuzzy Semilattice.

That is to prove $\mu(p \vee q) = \mu(p \vee r)$, for all $\mu(p), \mu(q), \mu(r) \in S$.
Let $\mu(p), \mu(q), \mu(r)$ be arbitrary.
Then $\mu(p \vee q) = \mu(p) \vee \mu(q \vee r)$
$\geq \min\{\mu(p), \mu(q \wedge r)\}$
$\geq \min\{\mu(p), \min\{\mu(q), \mu(r)\}\}$
$\geq \min\{\mu(p), \min\{\mu(r), \mu(q)\}\}$ by commutative law
$\geq \min\{\mu(p), \mu(r \wedge q)\}$
$= \mu(p) \vee \mu(r \wedge q)$
$= \mu(p \vee r)$, for all $\mu(p), \mu(q), \mu(r) \in S$.

Hence S is a Fuzzy Semilattice.

The converse need not be true.
(i.e) Every Fuzzy Semilattice need not be Fuzzy join–semidistributive.

ISSN No:-2456-2165

We shall verify it by the following example.
Consider the Fuzzy Semilattice of the following figure.

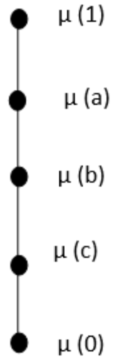

Fig 1

This Fuzzy Semilattice is not Fuzzy join–semidistributive.

Here
$\mu(p \vee q) = \mu(1)$
$\mu(p \vee r) = \mu(1)$
$\mu(q \wedge r) = \mu(0)$
$\mu(p) \vee \mu(q \wedge r) \geq \min \{ \mu(p) , \mu(q \wedge r)\}$
$\geq \min \{ \mu(p) , \mu(0)\}$
$= \mu(p)$

Thus $\mu(p \vee q) = \mu(p \vee r)$, but $\mu(p \vee q) \neq \mu(p) \vee \mu(q \wedge r)$ for all $\mu(p), \mu(q), \mu I \in S$.
$\Rightarrow$ Sis not a Fuzzy join–semidistributive semilattice.

➢ ***Theorem: 3.3***

Every Fuzzy distributive semilattice is Fuzzy join–semidistributive but fuzzy join-semidistributive need not to be fuzzy distributive semilattice.

Proof:

Given S is a Fuzzy distributive semilattice.
$\Rightarrow \mu(p) \vee \mu(q \wedge r) = \mu(p \vee q) \wedge \mu(p \vee r)$ for all $\mu(p), \mu(q), \mu I \in S$. $\longrightarrow$ (1)
To prove S is Fuzzy join–semidistributive semilattice.
For let $\mu(p), \mu(q), \mu I \in S$ be arbitrary and $\mu(p \vee q) = \mu(p \vee r)$

$\mu(p) \vee \mu(q \wedge r) = \mu(p \vee q) \wedge \mu(p \vee r)$, by (1)
$\geq \min \{ \mu(p \vee q), \mu(p \vee r)\}$
$\geq \min \{ \mu(p \vee q), \mu(p \vee q)\}$
$= \mu(p \vee q)$.

Thus $\mu(p \vee q) = \mu(p \vee r) \Longrightarrow \mu(p \vee q) = \mu(p) \vee \mu(q \wedge r)$ for all $\mu(p), \mu(q), \mu I \in S$.
$\Longrightarrow$ S is a Fuzzy join–semi distributive semilattice.

The converse need not be true.
(i.e) Every Fuzzy join–semidistributive semilattice need not be Fuzzy distributive semilattice.
We shall verify it by the following example.

Consider the lattice K_7 of the following figure

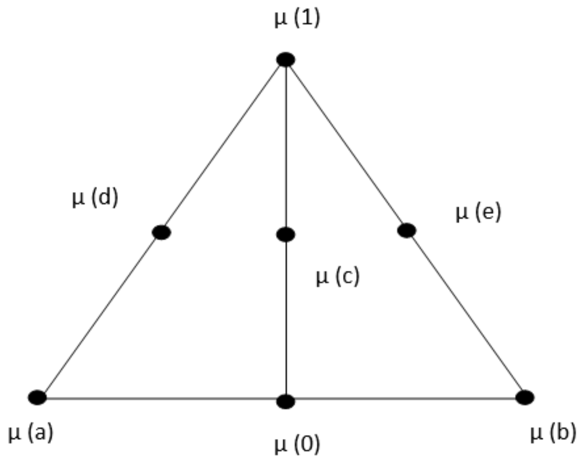

Fig 2

This Fuzzy semilattice is Fuzzy join–semidistributive but not Fuzzy distributive.
$\mu(p) \vee \mu(s \wedge q) \geq \min \{ \mu(p), \mu(s \wedge q)\}$
$\geq \min \{ \mu(p), \mu(p)\}$
$= \mu(p) \vee \mu(p)$
$= \mu(p)$
$\mu(p \vee s) \wedge \mu(p \vee q) \geq \min \{ \mu(p \vee s), \mu(p \vee q)\}$
$\geq \min \{ \mu(s), \mu(1)\}$
$= \mu(s) \wedge \mu(1)$
$= \mu(s)$
Therefore $\mu(p) \vee \mu(s \wedge q) \neq \mu(p \vee s) \wedge \mu(p \vee q)$
$\Rightarrow K_7$ is not Fuzzy distributive.

➢ ***Theorem: 3.4***

Every Fuzzy meet–semidistributive semilattice need not be Fuzzy
join–semidistributive semilattice.

Proof:

By an example,
Consider the Fuzzy semilattice K_7 of the following figure.

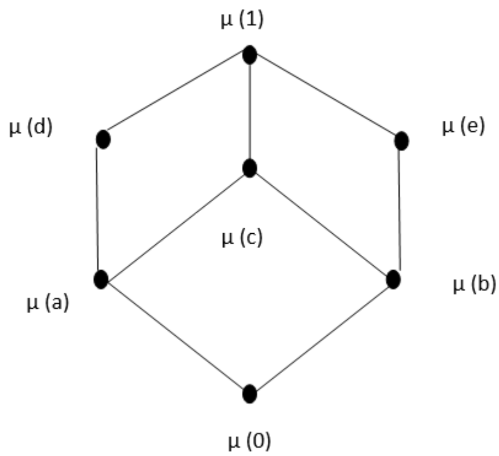

Fig 3

ISSN No:-2456-2165

This Fuzzy semilattice is Fuzzy meet–semidistributive but not Fuzzy
Join–semidistributive.

Here $\mu(r \vee s) \geq \min \{ \mu®, \mu(s) \}$
$\geq \min \{ \mu®, \mu(t) \}$
$= \mu® \vee \mu(t)$
$= \mu(1)$

$\mu® \vee \mu(s \wedge t) \geq \min \{ \mu®, \mu(s \wedge t) \}$
$\geq \min \{ \mu®, \mu(0) \}$
$= \mu® \vee \mu(0)$
$= \mu®$

Therefore $\mu® \vee \mu(s \wedge t) \neq \mu(r \vee s)$
$\Longrightarrow K_7$ is not Fuzzy join–semidistributive semilattice.

- ***Definition: 3.5***

A Fuzzy semilattice satisfying the above theorem is called upper locally Fuzzy distributive.

- ***Theorem: 3.6***

Every Fuzzy join–semi distributive semilattice need not be Fuzzy
meet–semidistributive semilattice.
By an example,
Consider the Fuzzy semilattice K_7 of the following figure.

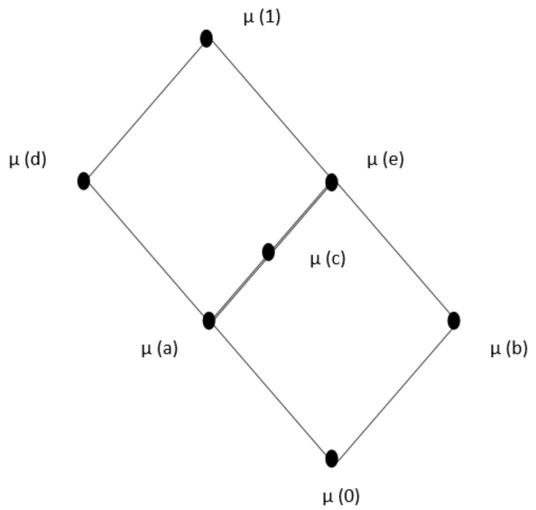

Fig 4

This Fuzzy semilattice is Fuzzy join–semidistributive but not Fuzzy
meet–semidistributive.

Here $\mu(r \vee p) \geq \min \{ \mu(r), \mu(p) \}$
$\geq \min \{ \mu(r), \mu(q) \}$
$= \mu(r \wedge q)$
$= \mu(0)$

$\mu(r) \vee \mu(p \vee q) \geq \min \{ \mu(r), \mu(p \vee q) \}$
$\geq \min \{ \mu(r), \mu(t) \}$
$= \mu(r) \wedge \mu(t)$
$= \mu(r)$

Therefore $\mu(r \vee p) \neq \mu(r) \vee \mu(p \wedge q)$
$\Longrightarrow K_7$ is not Fuzzy meet–semidistributive.

- ***Definition: 3.7***

A Fuzzy semilattice satisfying the above theorem is called lower locally Fuzzy distributive.

- ***Theorem: 3.8***

Fuzzy Dual of Fuzzy meet – semi distributive semi lattice is a Fuzzy
Join–semidistributive semilattice.

Proof:

Given S is a Fuzzy meet–semidistributive semilattice.
$\Longrightarrow \mu(p \wedge q) = \mu(p \wedge r)$ implies $\mu(p \wedge q) = \mu(p) \wedge \mu(q \vee r)$, for all $\mu(p), \mu(q), \mu(r) \in S$.
$\Longrightarrow$ Fuzzy dual of above is
$\mu(p \vee q) = \mu(p \vee r)$ implies $\mu(p \vee q) = \mu(p) \vee \mu(q \wedge r)$, for all $\mu(p), \mu(q), \mu(r) \in \bar{S}$, the Fuzzy dual of S.
$\Longrightarrow \bar{S}$ is a Fuzzy join – semidistributive.

- ***Theorem: 3.9***

Fuzzy Dual of Fuzzy join – semidistributive semilattice is a Fuzzy
meet–semidistributive semilattice.

Proof:

Given S is a Fuzzy join – semi distributive semi lattice.
$\Longrightarrow \mu(p \vee q) = \mu(p \vee r)$ implies $\mu(p \vee q) = \mu(p) \vee \mu(q \wedge r)$, for all $\mu(p), \mu(q), \mu(r) \in S$.
$\Longrightarrow$ Fuzzy dual of above is
$\mu(p \wedge q) = \mu(p \wedge r)$ implies $\mu(p \wedge q) = \mu(p) \wedge \mu(q \vee r)$, for all $\mu(p), \mu(q), \mu(r) \in \bar{S}$, the Fuzzy dual of S.
$\Longrightarrow \bar{S}$ is a Fuzzy meet–semidistributive.

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