On Noteworthy Applications of Laplace Transform in Real Life

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Abstract:- Mathematics is a methodical application of matter. It is so said because the subject makes a man methodical or more systematic. To justify & validate research findings, various mathematical tools are used. Laplace transform plays a vital role in wide field of science & technology which can be considered as a shortcut for complex calculations. This paper provides solid foundation of what Laplace transform is and its properties and its application in various fields which can further be useful in real life as well.

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Keywords:- *Laplace Transform, Mass Spring Damper System, Chemical Pollution, Transfer Function.*

I. INTRODUCTION INTEGRAL TRANSFORM

Let K(s, t) be a function of two variables‘s’ and ‘t’ where‘s’ is a parameter (may be real or complex) independent of t. The function f(s) defined by the integral (assumed to be convergent)
∞

$$f(s) = \int_{-\infty}^{\infty} K(s,t)F(t)dt$$

is called the Integral transform of the function F(t) and is denoted by L{F(t)}

The function $K(s,t)$ is called the Kernel of the transformation (also called Integral Kernel or Nucleus).

A. Laplace Transform:

If the Kernel $K(s,t)$ is defined as

$$\mathrm{K(s,t)} = \begin{cases} 0 & , \quad for\ t < 0 \\ e^{-st} & , \quad for\ t \geq 0 \end{cases}$$

then $f(s) = \int_0^\infty e^{-st} F(t)dt$

The f(s) defined by the above equation is called the **Laplace Transform** of the function F(t) and is also denoted by L{F(t)} or F(s).

B. Existance of Laplace Transforms:

If F(t) is piecewise continuous in every finite interval and is of exponential order ‘a’ as t → ∞, then Laplace Transform of F(t) that is F(s) exist ∀ s > a.The Laplace Transform has several applications in the field of science and technology. In this paper we will discuss about applications of Laplace Transform in real life.

C. Properties of Laplace Transform

➢ *Linearity Property:* -

$$\mathrm{L}\,\{\mathrm{a}F_1(\mathrm{x}) + \mathrm{b}F_2(\mathrm{x})\} = a\ L\{F_1(x)\} + bL\{F_2(x)\}$$

Where a and b are constants

- *Change of scale property: -*

If $L\{F(x)\} = f(s)$ then $L\{F(ax)\} = \frac{1}{a} f\left(\frac{s}{a}\right)$

- *First shifting property: -if L {F(x)} = f(s) then*

$L\{e^{-a}(x)\} = f(s + a)$

- *Laplace transform of derivatives: -*

$\{f^{n}(t)\} = {}^{n}L\{f(t)\} - s^{n-1}f(0) - s^{n-2}f'(0) - \ldots - f^{n-1}(0)$

II. APPLICATIONS

A. *Mass Spring Damper System*

The suspension system of the car is meant for driver's control of the car & comfort of occupants. The spring allows the wheels to move up to absorb bumps in the road & reduce jolting, while the dampers prevent bouncing up & down Consider the mechanical system as shown in figure.

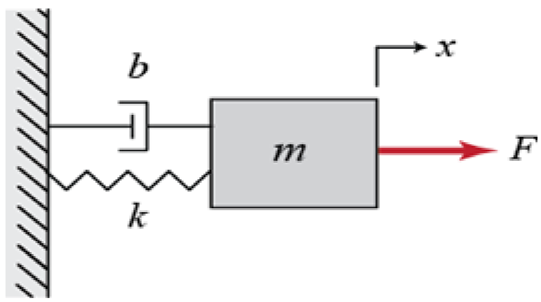

Fig (1.1)

The generalized equation for the system can be formulated as $F = m\ddot{x} + b\dot{x} + kx$ wh m= mass of system
b= damping coefficient k= spring coefficient x= displacement
F= Resultant force

Taking Laplace transform throughout

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From fig (1.1), $\frac{□^2□}{□□^2} + 4\frac{□□}{□□} + 3□ = 10 \sin □□$

Taking Laplace transform throughout

$$L\left[\frac{\square^2\square}{\square\square^2}\right] + 4\,\square\left[\frac{\square\square}{\square\square}\right] + 3\square\,[\square] = 10\,L[\sin \square\square]$$

By (II) & assuming initial conditions ,

$$(0) = \ '(0) = 0$$

$$(\square^2 + 4\square + 3)(\square) = 10\,\frac{\square}{\square^2 + \square^2}$$

Taking $\square = 1$

$$(\square) = \frac{10}{(\square^2 + 1)(\square^2 + 4\square + 3)}$$

Solving it by partial fraction,

$$(\square) = 10\left[\frac{\frac{1}{4}}{\square + 1} + \frac{-\frac{1}{20}}{\square + 3} + \frac{(-)\frac{1}{5}\square + \frac{1}{10}}{\square^2 + 1}\right]$$

Taking inverse Laplace transform,

$$\square\,\square^{()} = 10\left[\frac{1}{4}\square^{-\square} - \frac{1}{20}\square^{-3\square} - \frac{1}{5}\cos\square + \frac{1}{10}\square\square\square\square\right]$$

Hence,

Graph of solution of the system

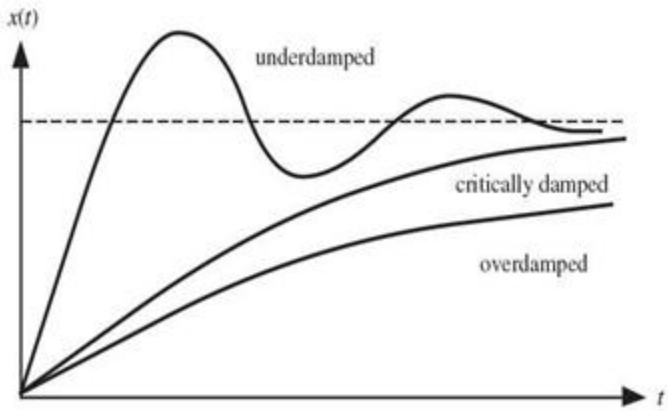

Fig. (1.2)

Depending upon the mass, spring coefficient and damper coefficient, different responses to the system can be recorded. It is necessary to analyze the mass-spring-damper system mathematically to be able to size your spring, damper and the mass of the object you want to stabilize and to be able to describe the reaction for a given system.

B. *Chemical Pollution in a Reservoir*

Water Pollution due to contaminants has become serious threat to environment as well as to human health. Normally pollution I large reservoirs commonly occurs on a time dependent scale in which system is not in steady state condition for pollutants. The basic idea is,

The formulation that governs time dependent concentration of an aqueous species in a reservoir is,

$$c(t) = \frac{M(t)}{V}$$

- H1: Volume of reservoir is constant
- H2: Flow rate remains constant
- H3 : Reaction rate remains constant
- H4 : Pollutant is uniformly distributed in reservoir
- H5 : Input & output of water is same By H4 & H5 ,

As, $M(t) = VC_0(t)$

dM(t)
$= QC_0 -$
dt
QM(t) V

dC
$V \;\; = QC_0 - QC$
Assuming only fresh water is coming in,

$\therefore C_0 = 0$

dC

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$\Rightarrow$ + —
dt

Q
$C = 0$ —
V

Applying Laplace transform & solving,

Q
$(t) = (0)e^{(-V)t}$ –

Ex.

How much time would it take for pollutant to reach acceptable level if volume of lake is $25 \times 10^6 m^3$,
Flow of fresh water is $15 \times 10^6 m^3$, initial concentration of contaminant is $10^6 parts/m^3$ & acceptable level of pollutant is $5 \times 10^6 m^3$?

Q
$\Rightarrow (t) = (0)e^{(-V)t}$ –

Solving it with given data,required time can be calculated as $t = 11.55$ units approximately

Hence, such a model can be prepared to overcome water pollution which has serious ill effects over human health.

C. *Transfer Function of Control System*

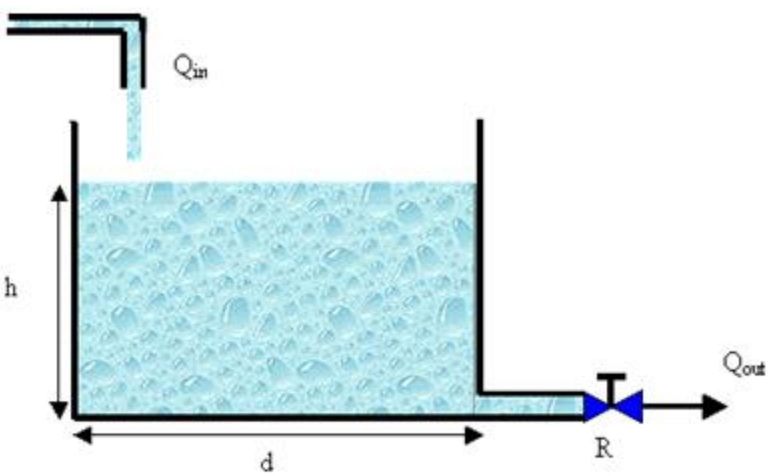

Fig (3.1)

The tank shown in figure is initially empty. A constant flow rate Q_{in} is added for t>0. The rate at which flow leaves the tank (Q_{out}) = CH.
A = cross sectional area M = Mass of fluid
ρ = density of fluid

Since Mass= volume * density M = AH * ρ
Hence mass flow rate

$\frac{dM}{dt} = \frac{(AH * \rho)}{dt}$ —

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$= \rho A * \frac{dH}{dt}$

To construct a differential equation for head H we know, mass flow rate into tank is equal to mass in flow rate – mass out flow rate.

Hence $\rho A * \frac{dH}{dt} = \rho Q_{in} - \rho Q_{out}$

i.e. $A * \frac{dH}{dt} = Q_{in} - CH$……..

(since $Q_{out} = CH$)

Hence $Q_{in} = A * \frac{dH}{dt} + CH$

Taking Laplace Transform on both sides

$L[Q_{in}]_s = A * L[\frac{dH}{dt}]_s + * L[H]_s$

From property (d),

$\frac{H(S)}{Q_{in}(S)} = \frac{1}{SA_S + C_S}$ ………….. (1)

But we know, $_{ut}$ = CH Applying Laplace transform,

$H(S) = \frac{Q_{out}(S)}{C_S}$…………… (2)

From Equation (1) and (2)

$\frac{Q_O(S)}{Q_{in}(S)} = \frac{1}{1 + (\frac{A_S}{C_S})}$

which represents transfer function of control system.

Hence using this transfer function we can control the water level in tank.

III. CONCLUSION

In this paper we have tried focusing on such unusual applications of Laplace Transform which may resolve many practical problems in day to day life in easier way.

Such as Mass Damper System takes care of an individual’s comfort. Chemical Pollution model may help us to reduce hazardous chemicals in water which may in turn be beneficial to human life. Also the Transfer function derived by using Laplace Transform may help us to regulate water which is very important natural resource.

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