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A Belief Based Method for Non-Classical Multi-Criteria Decision Making Problems
(A Method applied in Decision Support Tool for Mainstreaming Sustainability in Social Housing Projects)

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Abstract:- This paper discusses a belief-based method for solving Multi Criteria Decision Making problem while addressing the problem of missing values and rank reversal. The presented method was successfully applied in ranking building materials for social housing projects in India.

Keywords:- *Evidence theory; Belief models; Missing values; Probability Theory; Multi-criteria decision making; Building Materials.*

I. INTRODUCTION

Classical Multi-Criteria Decision-Making (MCDM) consists of choosing an alternative among a known set of alternatives based on their quantitative evaluations or numerical scores obtained with respect to different criteria. The model proposed here can be classified as Multi-Criteria Decision Making (MCDM) tool. Its purpose is to help the user to choose an alternative among a known set of alternatives based on their quantitative evaluations or numerical scores obtained with respect to selection criteria. Although the MCDM problem can be easily formulated, there are difficulties in solving it because of the lack of data or missing score values of an alternative for any criterion. Solving a classical MCDM becomes further complicated as the scores are expressed in different physical units and different scales, due to the diverse nature of the selection criteria. Such differences in the unit and scale of scores necessitate a normalization step that may yield further problems like the rank reversal. A rank reversal is a change in rank order when the structure of the MCDM problem is changed by adding or deleting alternatives. This usually happens when the choice of direct data normalization is made in the MCDM problem. Another barrier in solving such a problem is that no alternative exists which optimizes all the criteria jointly. Hence, MCDM problems are not exactly solved, but a decision is found by means of ranking or other relative compromises.

A. Context

Many methods have been developed to address the classical MCDM problem like Analytic Hierarchy Process (AHP), Elimination and Choice Expressing Reality (ELECTRE), Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS), and Estimator Ranking Vector (ERV). However, very few methods exist in the literature, which deal with non-classical MCDM problems having missing score values [1].

In this paper, a step by step process of such a method developed by the authors is presented to solve a non-classical MCDM problem. This was applied to create a Sustainability Assessment Tool (SAT) for ranking building materials used in Indian social housing projects. The SAT is a component of a Decision Support Toolkit (DST), an interactive and online toolkit comprising of a range of outputs, datasets, tools, and insights. The DST can help prospective users in choosing sustainable building materials and making and monitoring sustainable design interventions and construction practices in social housing projects. This study was carried out as part of the Mainstreaming Sustainable Social Housing in India project (MaS-SHIP) is a research project funded by the United Nations Environment's 10 Year Framework Programme (10YFP) undertaken by the Low Carbon Building Research Group at Oxford Brookes University (UK), The Energy and Resources Institute (TERI), Development Alternatives and the United Nations Human Settlements Programme (UN-Habitat) [2].

TOPSIS would have been an appropriate method to arrive at final individual values for each alternative across all criteria [3]. However, to account for the missing values and to avoid possible rank reversals, there was a need to convert the score matrix using a belief function approach from the Dempster-Shafer Theory [4]. Also known as the theory of belief functions, Dempster Shafer Theory is a generalization of the Bayesian theory of subjective probabilities. Where Bayesian theory requires probabilities for each question, belief functions allow us to assign degrees of belief on probabilities of the questions. Another reason for the authors to choose this method was to avoid possible rank reversals from direct normalization.

B. Structure of the paper

The paper is structured as follows; the section II explains the formulation of the non-classical MCDM problem for ranking of alternatives. The subsequent sections III, IV & V define and explain the derivation of the values of belief, plausibility and uncertainty for all the score values in the MCDM problem formulated in section II. Section VI explains the derivation of global score values to determine the final rankings of the alternatives for the selected criteria.

The stepwise method has been demonstrated by solving a hypothetical MCDM problem where the high school students are required to be ranked on the basis of defined selection criteria. The alternatives in this problem are the different students and the selection criteria are Performance in Physics, Performance in Chemistry, Performance in Mathematics, Discipline Issues Reported and Participation in Extra-curricular Activities.

II. FORMULATING THE MCDM

Shafer [4] defined the set of alternatives as the representative of the Frame of Discernment (FoD) of the problem. FoD is defined as the set of mutually exclusive "elementary" propositions. In this case, the FoD of our problem is represented by a set of 7 alternatives denoted by

$$A \triangleq \{A_1, A_2, A_3 \ldots\ldots A_7\} \quad (1)$$

The alternatives consist of students of a high school. The MCDM problem also consists of a set of 5 criteria for selection, denoted by

$$C \triangleq \{C_1, C_2, C_3 \ldots\ldots C_5\} \quad (2)$$

The selection criteria consist of both quantitative (Performance in Physics, Performance in Chemistry, Performance in Mathematics) as well as qualitative (Discipline Issues Reported, Participation in Extra-curricular Activities) score values. Each criterion has a weighting factor which characterizes its relative importance. The set of normalized weights for the criteria denoted by

$$w \triangleq \{w_1, w_2, w_3 \ldots\ldots w_5\} \quad (3)$$

The weights could be derived from AHP pairwise comparison survey done with experts from the field. This method was also adopted as part of the MaS-SHIP project to assign relative weights to the 18 attributes. The relative importance (weight) of each criterion is between 0 and 1,

i.e. $w_i = [0,1] \; \forall \, w_i \in w$ (4)

and the sum of all weights in equal to 1,

i.e. $\sum_{X=1}^{5} w_{\mathrm{x}} = 1$ (5)

The score value is related to the evaluation of an alternative A_i for a criterion C_j, hence denoted by

$$S_{ij} = S_j(A_i) \quad (6)$$

The scores for criteria - Performance in Physics, Performance in Chemistry, and Performance in Mathematics are quantitative and would fall between the maximum (100) and the minimum (0) score values only. But, the scores for criteria – Discipline Issues Reported and Participation in Extra-curricular Activities are qualitative and represented as 'High', 'High-Medium', 'Medium', 'Medium-Low', and 'Low'. If the score value for a particular alternative-criterion pair is missing, it is denoted by the "theta" symbol (θ).

So the MCDM problem in this paper could be formulated as follows: given the (7×5) score matrix $S_{ij} = S_j(A_i)$, elements of which will either have a numerical value or θ value (when the score value is missing) and knowing the relative importance of criteria denoted by w , how to rank the elements of $A \triangleq \{A_1, A_2, A_3 \ldots\ldots A_{17}\}$ to make the final decision?

Students	Performance			Discipline Issues Reported	Participation in Extra-curricular Activities
	Physics	Chemistry	Mathematics		
	Score	Score	Score	High-Medium-Low	High-Medium-Low
1	42	93	82	L	VH
2	56	45	θ	L	M
3	48	70	95	L	VH
4	79	100	67	M-L	L
5	92	64	70	M-L	L
6	θ	58	86	L	M
7	83	36	99	L	M-L

Table 1:- Sample Score Matrix

III. BELIEF VALUES FROM 'POSITIVE SUPPORT'

The *belief* of a score for an alternative in a particular criterion is the minimum chance of that alternative being better than another alternative in the same criterion. The belief value is a representation of the minimum evidence we have regarding the performance of an alternative in a criterion. It is the minimum probability of the alternative being better than another alternative in the same criterion. Hence, it is the summation of how much the score of the alternative is better than others in the criterion divided by the best score in the criterion. In this case, the best score is the one with the least numerical value. How much an alternative is better than other alternatives in a criterion is called as its positive support. For an alternative A_i in a criterion C_j, we denote its *belief* as $Bel_j(A_i)$ and its positive support as $Sup_j(A_i)$.

$$Bel_j(A_i) \triangleq \frac{Sup_j(A_i)}{A^j_{min}} \quad (7)$$

$$Sup_j(A_i) \triangleq \sum_{Y \in A\,:\,S_j(Y) \geq S_j(A_i)} |S_j(Y) - S_j(A_i)| \quad (8)$$

As the score values in all criteria were made uniform by making lesser to be better, the positive support for any alternative in any criterion would be calculated on how numerically lesser it is than other alternatives in the same criterion. For the same reason the alternative with the best score for any criterion is the one with the lowest numerical score value, denoted here by A^j_{min}.

As there is no evidence regarding the missing scores, the belief for the same will be zero. As we do not know the missing score, it could take any value, including the worst in the criterion which would make the probability of the alternative being better than another to be zero. Hence the minimum chance that a missing score is better than another score in the same criterion is zero

i.e. $$Bel(\theta) = 0 \quad (9)$$

Students	Performance			Discipline Issues Reported	Participation in Extra-curricular Activities
	Physics	Chemistry	Mathematics		
	Belief Assignments from Positive Support				
1	0.41	0.03	1.00	0.03	0.09
2	0.26	0.38	0.00	0.03	0.00
3	0.34	0.15	1.00	0.03	0.02
4	0.08	0.00	0.04	0.18	0.22
5	0.02	0.19	0.04	0.18	0.19
6	0.00	0.24	0.31	0.03	0.06
7	0.05	0.49	0.15	0.03	0.00

Table 2:- Belief Values

IV. PLAUSIBILITY FROM 'NEGATIVE SUPPORT'

The *plausibility* of a score for an alternative in a particular criterion is the maximum chance of that alternative being better than another alternative in the same criterion. The plausibility value is a representation of the maximum evidence we have regarding the performance of an alternative in a criterion. It is the maximum probability of the alternative being better than another alternative in the same criterion. The Plausibility is derived from the *disbelief* of that alternative in a particular criterion. *Disbelief* is representative of the minimum evidence against an alternative.

The *Disbelief* of an alternative A_i is the negative support of that alternative for that category divided by the score value of the worst alternative for that criterion, which is the maximum of all the score values here. By negative support, we mean how much A_i is worse than all other alternatives for criterion C_j. The Disbelief value represents the minimum value that could be taken by the probability that A_i is worse than another alternative in criterion C_j. For an alternative A_i in a criterion C_j, we denote its Disbelief as $Dis_j(A_i)$ and its negative support as $Inf_j(A_i)$.

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$$Dis_j(A_i) \triangleq \frac{Inf_j(A_i)}{A^j_{max}} \quad (10)$$

$$Inf_j(A_i) \triangleq \sum_{Y \in A\,:\,S_j(Y) \leq S_j(A_i)} |S_j(Y) - S_j(A_i)| \quad (11)$$

As the score values in all criteria were made uniform by making lesser to be better, the negative support for any alternative in any criterion would be calculated on how numerically greater it is than other alternatives in the same criterion. For the same reason the alternative with the worst score for any criterion is the one with the highest numerical score value, denoted here by A^j_{max}.

The plausibility of an alternative for a particular criterion is defined as the maximum value that could be taken by the probability that the alternative is better another alternative for that criterion. Hence, it is one minus the doubt (Disbelief) about the alternative. We denote plausibility of an alternative A_i in a criterion C_j as $Pls_j(A_i)$.

$$Pls_j(A_i) = 1 - Dis_j(A_i) \quad (12)$$

The Plausibility of an alternative represents the maximum possible evidence we have for it directly and hence the probability of the alternative being better than another alternative cannot be higher than this. As there is no evidence on the score value, it might even be the best alternative in that criterion. So the maximum value that the probability of it being better than others is 1, which is when it is actually the best alternative. Hence, the plausibility of a missing score value is 'one', i.e. $Pls\ (\theta) = 1$

$$Dis_j(A_i), Bel_j(A_i), Pls_j(A_i) \in [0,1], \forall\ i, j \epsilon N \quad (13)$$

$$Bel_j(A_i) \leq Pls_j(A_i)\ , \forall\ i, j \in N \quad (14)$$

The belief and plausibility measures represent the lower and upper bounds which surround the belief value. Hence, the interval between the Belief and Plausibility of a score value S_{ij} represents the exact range in which the belief of the score resides. The smaller the interval, the higher is the certainty of the score value. For instance, if $Bel_j(A_i) = Pls_j(A_i)$, there is absolute certainty regarding the probability of A_i is being better than another alternative in criterion C_j. We must reinforce that the certainty is not about the score being better than another but about the probability of the same. In the literature, these intervals are often expressed as the *Belief Intervals* (Khatibi, 2010). This leads us to the third crucial element to be derived using this theory for each of our score values, which is Uncertainty.

Students	**Performance**			**Discipline Issues Reported**	**Participation in Extra-curricular Activities**
	Physics	**Chemistry**	**Mathematics**		
	Plausibility Assignments from Negative Support				
1	**0.86**	**0.15**	**1.00**	**0.53**	**0.46**
2	**0.74**	**0.84**	**0.68**	**0.53**	**1.00**
3	**0.82**	**0.56**	**1.00**	**0.53**	**0.15**
4	**0.43**	**0.00**	**0.29**	**0.73**	**0.67**
5	**0.19**	**0.65**	**0.29**	**0.73**	**0.64**
6	**1.00**	**0.72**	**0.68**	**0.53**	**0.38**
7	**0.37**	**0.89**	**0.50**	**0.53**	**0.03**

Table 3:- Plausibility Values

V. UNCERTAINTY FROM PLAUSIBILITY AND BELIEF VALUES

Uncertainty of a score value is defined as its belief interval or the difference between its plausibility and belief values. We denote the uncertainty of score value S_{ij} as $Unc_j(A_i)$. As we gather more evidence, the uncertainty value diminishes. The score value of the best alternative for any criterion will have a 'zero' uncertainty.

$$Unc_j(A_i) = Pls_j(A_i) - Bel_j(A_i) \quad (15)$$

By both mathematical construction and intuitive sense, the uncertainty value of missing score value is 1. It means there is no certainty or absolute uncertainty regarding the probability of the score value being better than another in the same criterion, i.e. $Unc(\theta) = 1$ (16)

Students	**Performance**			**Discipline Issues Reported**	**Participation in Extra-curricular Activities**
	Physics	**Chemistry**	**Mathematics**		
	Uncertainty Assignments from Belief and Plausibility Values				
1	**0.45**	**0.13**	**0.00**	**0.51**	**0.37**
2	**0.48**	**0.46**	**0.37**	**0.51**	**1.00**
3	**0.48**	**0.42**	**0.00**	**0.51**	**0.14**
4	**0.36**	**0.00**	**0.25**	**0.55**	**0.45**
5	**0.17**	**0.46**	**0.25**	**0.55**	**0.45**
6	**1.00**	**0.48**	**0.37**	**0.51**	**0.32**
7	**0.31**	**0.41**	**0.35**	**0.51**	**0.03**

Table 4:- Uncertainty Values

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VI. DETERMINING A GLOBAL SCORE

For each score value S_{ij} (numerical and 'θ') we have derived 3 corresponding values from the Dempster Shafer Theory of evidence; Belief, Plausibility and Uncertainty. Hence for each pair of alternatives A_i in a criterion C_j we have a triplet of Belief, Plausibility and Uncertainty. We denote the set of all such triplets as BBA (Basic Belief Assignment).

$$S \rightarrow BBA \triangleq BBA_{ij} \,\forall i, j \,\epsilon\, N \qquad (17)$$

$$S_{ij} \rightarrow BBA_{ij} = \{Bel_{ij}, Pls_{ij}, Unc_{ij}\} \qquad (18)$$

Unlike the score matrix, the BBA matrix has no missing value; hence TOPSIS could be applied at this stage. For each criterion, TOPSIS principally measures the distance of each alternative from the best and worst alternatives [3]. For distances between belief intervals we refer to the measure provided by Khatibi (2010). This distance measure has the properties of non-negativity, non-degeneracy, symmetry and triangular inequality, which makes this a true distance metric (Khatibi, 2010). Khatibi denoted $BID(A, B)$ as the Belief Interval Distance between two belief intervals A and B.

$$BID(A, B) \triangleq \frac{1}{2}\left(|Bel(A) - Bel(B)| + |Pls(A) - Pls(B)| + |Unc(A) - Unc(B)|\right) (20)$$

Using Khatibi's distance measure, for each triplet in the BBA matrix, we estimate its distance from the best and worst alternatives in the same criterion. We denote the new matrix as the BID matrix consisting of d_{ij}^{best} and d_{ij}^{worst} for each BBA_{ij}.

$$d_{ij}^{best} = BID(BBA_{ij}, BBA_j^{best}) \qquad (21)$$

Here, BBA_j^{best} is the BBA triplet of the alternative with best score in the criterion C_j. Similarly, for each BBA_{ij} we estimate the distance from the BBA triplet with the worst score value in that criterion.

$$d_{ij}^{worst} = BID(BBA_{ij}, BBA_j^{worst}) \qquad (22)$$

Here, BBA_j^{worst} is the BBA triplet of the alternative with worst score in the criterion C_j. Following the principles of TOPSIS, we have derived 2 corresponding values for each BBA triplet.

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$$BBA_{ij} \rightarrow BID_{ij} \triangleq \left(d_{ij}^{best}, d_{ij}^{worst}\right) \qquad (22)$$

From the BID matrix, for each alternative A_i we compute weighted average of d_{ij}^{best} values with relative importance weightages w_j . Similarly, for each alternative A_i we compute the weighted average of d_{ij}^{worst} values across all criteria. We denote the best and worst weighted averages for each alternative A_i as $D^{best}(A_i)$ and $D^{worst}(A_i)$ respectively.

$$D^{best}(A_i) \triangleq \sum_{j=1}^{18} w_j \times d_{ij}^{best} \qquad (23)$$
$$D^{worst}(A_i) \triangleq \sum_{j=1}^{18} w_j \times d_{ij}^{worst} \qquad (24)$$

After this step, we have arrived at 2 global scores, $D^{best}(A_i)$ and $D^{worst}(A_i)$ for each alternative A_i. From here, we calculate the relative closeness (R) of the alternative A_i with respect to the ideal best solution A^{best} defined by

$$R(A_i, A^{best}) \triangleq \frac{D^{worst}(A_i)}{D^{worst}(A_i) + D^{best}(A_i)} \quad (25)$$

A higher $R(A_i, A^{best})$ means a better alternative A_i. Hence, the preference ordering of the alternatives is made according to the descending order of the $R(A_i, A^{best}) \in [0,1]$.

Student	**Weighted Average**		**Relative Closeness**
	Best	**Worst**	
1	**137.99**	**162.01**	**0.54**
3	**138.88**	**161.12**	**0.54**
2	**160.98**	**139.02**	**0.46**
7	**171.25**	**128.75**	**0.43**
6	**174.63**	**125.37**	**0.42**
5	**175.52**	**124.48**	**0.41**
4	**179.56**	**120.44**	**0.40**

Table 5

To solve the above MCDM problem, the authors have anchored the maximum and minimum values for each criterion. The maximum and minimum score values for the quantitative criteria lies between 0 and 100. Therefore, the minimum and maximum possible score would be 0 and 100 respectively. Similarly, in the case of the qualitative criterion, the minimum possible score would be 'Low' and the maximum would be 'High'. This would help in eliminating the issues of possible rank reversal in case of the addition of more alternatives in the problem.

VII. CONCLUSION

In this paper, we attempted to detail out the process involved in aiding the decision making in ranking of 7 students based on qualitative, quantitative and missing scores across 5 criteria. The method demonstrated is relatively easy to use and does not require normalization of the data at the initial stage. It also deals with the missing score values in the initial MCDM problem. The non-uniformity of the weights has no effect on the model. This provides a potential decision maker with an option to choose the criteria that must be evaluated for estimation of the final R matrix. The weights of the deselected criteria in the model will be redistributed to the selected criteria in their relative proportions.

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