Solution Alternative of Complex Fuzzy Linear Equation System

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Abstract:- In this thesis discuss about alternative to determine solution from fuzzy complex linear system of the form $\tilde{C} \otimes \tilde{z} = \tilde{w}$ where $\tilde{C}$ is complex fuzzy matrix and $\tilde{w}, \tilde{z}$ are arbitrary complex fuzzy vector that will be done by using inverse method. There are some cases that include solution of $\tilde{C} \otimes \tilde{z} = \tilde{w}$ but just 2 cases that will be discussed. Finally, we will give some examples for applying the formula were gotten.

***Keywords**:- Fuzzy Numbers; Fuzzy Complex Numbers; Triangular Fuzzy Numbers; Fuzzy Complex Linear System.*

I. INTRODUCTION

The concept of *fuzzy* number and *fuzzy* arithmetic was introduced in 1965 [21]. *Fuzzy* concepts are applied in various fields such as control theory, decision theory, and some parts of management science. These fields require a *fuzzy*-based equation system as a mathematical model, such as a system of fuzzy linear equations [6-7], the *fully fuzzy* linear equation system[4-5, 11-12, 15-16, 19-20], and a system of *dual fully fuzzy* linear equation [9, 1-14, 18].

Not only system of linear *fuzzy* equations, but also complex *fuzzy* linear equation systems whose coefficients are fuzzy numbers. Fuzzy numbers are extended into complex *fuzzy* numbers which were first introduced by Buckley [3].

Some authors, [1] discuss solutions of *fuzzy* linear equation systems based on *fuzzy* center while [2] used the *fuzzy* center method tosolev *fuzzy* linear equation systems and apply them to circuit analysis. [7] providing a solution of the complex *fuzzy* equation system in the form $C \otimes \tilde{z} = \tilde{w}$ sized $n \times n$ by expanding the matrix to $G \otimes \tilde{x} = \tilde{b}$ where G is the matrix measuring $2n \times 2n$ in the example of calculating C is real matrix and complex number matrices while in [8] it only discusses the real cofficient matrix using the same method. The some method as discussed by [7] and [8] is also discussed by [10]. [17] discussed the application of complex linear systems of equations in circuit analysis. [22] solved a system of complex *fuzzy* linear equation using the *QR* method.

Based on these descriptions, several authors discuss complex *fuzzy* linear equation systems with real number coefficients or complex numbers and complex fuzzy number variables with *fuzzy* numbers in the form of parametric functions so that the authors are interested in discussing the solution of the system of linear *fuzzy* equations in the form of $\tilde{C} \otimes \tilde{z} = \tilde{w}$ with $\tilde{C} = \tilde{A} + i\tilde{B}$, $\tilde{z} = \tilde{x} + i\tilde{y}$, and $\tilde{w} = \tilde{u} + i\tilde{v}$ where $\tilde{C}$ any fuzzy vector complex triangles that will be solved by inverse matrix method. Then the system of linear *fuzzy* equations can also be written as.

$$\left(\tilde{A} + i\tilde{B}\right) \otimes (\tilde{x} + i\tilde{y}) = \tilde{u} + i\tilde{v}$$

Where in determining $\tilde{x}$ and $\tilde{y}$ which is the solution to a system of complex linear *fuzzy* equation depending on the matrix value $\tilde{A}$, $\tilde{B}$, $\tilde{u}$, and $\tilde{v}$, which are related to the multiplication formula of *fuzzy* numbers to be used. So the possible values of $\tilde{A}$, $\tilde{B}$, $\tilde{u}$, and $\tilde{v}$ are positive or negative fuzzy matrices that produce many possible combinations of solutions. In this paper, the author will only discuss two cases of solutions of complex *fuzzy* linear equation systems.

II. PRELIMINARIES

Several supporting theories relating to the problems discussed include triangular *fuzzy* numbers, complex *fuzzy* numbers, and complex linear *fuzzy* equation.

A. *Triangular Fuzzy Number*

Below is given the definition of triangular *fuzzy* number as given by [11-12, 16, and 18-20].

Definitions 2.1. The *fuzzy* set $\tilde{a}$ is defined as $\tilde{a} = \left(x, \mu_{\tilde{a}}(x)\right)$. In pairs $\left(x, \mu_{\tilde{a}}(x)\right)$, x is a member of the set $\tilde{a}$ and $\mu_{\tilde{a}}(x)$ its value in the interval [0, 1] is called the membership function.

Definitions 2.2. *Fuzzy* number is a fuzzy set $\tilde{a}: \mathbb{R} \rightarrow [0,1]$ which: fulfills the following conditions

1. $\tilde{a}$ is upper semi-continous.
2. $\tilde{a} = 0$ outside some interval $[a, c]$.
3. There exist real number b in the interval $[a, c]$ such that,

3.1 $\tilde{a}$ monotone increases at the interval $[a, b]$.
3.2 $\tilde{a}$ monotone down at the interval $[b, c]$.
3.3 $\tilde{a} = 1$ for the value of $x = b$.

The membership function for triangular *fuzzy* number $\tilde{a} = (a, \alpha, \beta)$ ie

$$\mu_{\tilde{a}}(x) = \begin{cases} 1 - \dfrac{(a-x)}{\alpha}, & a - \alpha \le x \le a \\ 1 - \dfrac{(x-a)}{\beta}, & a \le x \le a + \beta \\ 0 & other \end{cases}$$

With the parametric function $\tilde{a} = [\underline{a}(r), \overline{a}(r)]$ represented as follow $\underline{a}(r) = a - (1-r)\alpha$ and $\overline{a}(r) = a + (1-r)\beta$.

Furthermore, some definitions of the *fuzzy* number parametric functions are given by [4, 6, 8-11, 13-14, 17-19, and 22].

Definitions 2.3. The *fuzzy* number $\tilde{a}$ in $\mathbb{R}$ is defined as a pair of funtions $\tilde{a} = [\underline{a}(r), \overline{a}(r)]$ which satisfies the following properties:

1. $\underline{a}(r)$ monotone ascending, limited, and continuous left at $[0,1]$.
2. $\overline{a}(r)$ monotone descending, limited, and continuous right at $[0,1]$.
3. $\underline{a}(r) \le \overline{a}(r), 0 \le r \le 1$.

Several authors define positive and negative fuzzy numbers based on area [2 and 18] as follows.

Definitions 2.4 Given the triangular *fuzzy* number $\tilde{a} = (a, \alpha, \beta)$. The *fuzzy* number $\tilde{a}$ is said to be positive or negative based on the cases given below which are based on the area concept of the *fuzzy* number triangle.

a) If $a - \alpha \ge 0$ then $\tilde{u}$ positive, conversely if $a + \beta \le 0$ then $\tilde{u}$ negative.
b) If $a - \alpha < 0$ and $a > 0$, *fuzzy* number $\tilde{a}$ is said to be positive if $\frac{\beta-\alpha}{2} + 2a - \frac{a^2}{\beta} > 0$ and $\tilde{a}$ said to be negative if $\frac{\beta-\alpha}{2} + 2a - \frac{a^2}{\beta} < 0$.
c) If $a < 0$ and $a + \beta > 0$, fuzz numbery $\tilde{a}$ is said to be positive if $\frac{\beta-\alpha}{2} + 2a + \frac{a^2}{\beta} > 0$ and $\tilde{a}$ is said to be negative if $\frac{\beta-\alpha}{2} + 2a + \frac{a^2}{\beta} < 0$.
d) If $a = 0$, the *fuzzy* number $\tilde{a}$ is said to be positive if $\beta - \alpha > 0$ and $\tilde{a}$ is said to be negative if $\beta - \alpha < 0$.

General arithmetic of fuzzy numbers as given [4, 7, 14, 19] is as follows.

Definitions 2.5 Given two *fuzzy* number $\tilde{u} = (a, \alpha, \beta)$ and $\tilde{v} = (b, \gamma, \delta)$, the arithmetic of *fuzzy* number is defined as follows:

i. Addition:

$$\tilde{u} \oplus \tilde{v} = (a + b, \alpha + \gamma, \beta + \delta) \tag{1}$$

ii. Substraction:

$$\tilde{u} \ominus \tilde{v} = (a - b, \alpha + \delta, \beta + \gamma) \tag{2}$$

iii. Scalar multiplication:

$$\lambda \otimes \tilde{u} = \lambda \otimes (a, \alpha, \beta)$$

$$= \begin{cases} (\lambda a, \lambda\alpha, \lambda\beta) & \lambda \ge 0, \\ (\lambda a, -\lambda\beta, -\lambda\alpha) & \lambda < 0. \end{cases} \tag{3}$$

iv. Multipllication

a. Case 1, if $\tilde{u}$ positive and $\tilde{v}$ positive then

$$\tilde{u} \otimes \tilde{v} = (ab, a\gamma + b\alpha, a\delta + b\beta) \tag{4}$$

b. Case 2, if $\tilde{u}$ positive and $\tilde{v}$ negative then

$$\tilde{u} \otimes \tilde{v} = (ab, a\gamma - b\beta, a\delta - b\alpha) \tag{5}$$

c. Case 3, if $\tilde{u}$ negative and $\tilde{v}$ positive then

$$\tilde{u} \otimes \tilde{v} = (ab, b\alpha - a\delta, b\beta - a\gamma) \tag{6}$$

d. Case 4, if $\tilde{u}$ negative and $\tilde{v}$ negative then

$$\tilde{u} \otimes \tilde{v} = (ab, -a\delta - b\beta, -a\gamma - b\alpha) \tag{7}$$

B. Complex fuzzy number

The following is the definition of complex *fuzzy* numbers as given by several authors such as [7-8 and 22].

Definition 2.6 Any complex *fuzzy* number represented as $\tilde{x} = \tilde{p} + i\tilde{q}$ with $\tilde{p}$ and $\tilde{q}$ *fuzzy* numbers.

Definition 2.7 Given any two complex *fuzzy* numbers $\tilde{x} = \tilde{p} + i\tilde{q}$ and $\tilde{y} = \tilde{u} + i\tilde{v}$ where $\tilde{p}$, $\tilde{q}$, $\tilde{u}$, and $\tilde{v}$ are triangular fuzzy numbers, the arithmetic of complex *fuzzy* numbers is given as follows:

i. Addition:

$$\tilde{x} + \tilde{y} = (\tilde{p} + \tilde{u}) + i(\tilde{q} + \tilde{v})$$

ii. Scalar Multiplication:

$$k\tilde{x} = k\tilde{p} + ik\tilde{q}$$

with $k \in R$.

iii. Multiplication:

$$\tilde{x} \times \tilde{y} = (\tilde{p} \otimes \tilde{u} - \tilde{q} \otimes \tilde{v}) + i(\tilde{p} \otimes \tilde{v} + \tilde{q} \otimes \tilde{u}) \tag{8}$$

III. SOLUTION ALTERNATIVE OF FUZZY COMPLEX LINEAR SYSTEM

The system of complex *fuzzy* linear equations is a system consisting of several complex *fuzzy* linear equations with fuzzy number coefficient [7-8, 17, and 22]. The following is given a system of complex *fuzzy* linear equations.

$$\left.\begin{array}{c} \tilde{c}_{11}\tilde{z}_1 \oplus \tilde{c}_{12}\tilde{z}_2 \oplus \cdots \oplus \tilde{c}_{1n}\tilde{z}_n = \tilde{w}_1 \\ \tilde{c}_{21}\tilde{z}_1 \oplus \tilde{c}_{22}\tilde{z}_2 \oplus \cdots \oplus \tilde{c}_{2n}\tilde{z}_n = \tilde{w}_2 \\ \vdots \\ \tilde{c}_{n1}\tilde{z}_1 \oplus \tilde{c}_{n2}\tilde{z}_2 \oplus \cdots \oplus \tilde{c}_{nn}\tilde{z}_n = \tilde{w}_n \end{array}\right\} \tag{9}$$

where $\tilde{c}_{ij}$, $\tilde{w}_i$, $\tilde{z}_i$, $1 \le i, j \le n$ is a complex *fuzzy* number. Ifthe *fuzzy* matrix $\tilde{C} = (\tilde{c}_{ij})$, $\tilde{w} = (\tilde{w}_i)$, $\tilde{z} = (\tilde{z}_i)$, then equation (9) can be written as

$$\tilde{C} \otimes \tilde{z} = \tilde{w} \tag{10}$$

Since $\tilde{C}$, $\tilde{w}$, and $\tilde{z}$ are complex fuzzy matrices, for example, $\tilde{C} = \tilde{A} + i\tilde{B}$, $\tilde{z} = \tilde{x} + i\tilde{y}$, and $\tilde{w} = \tilde{u} + i\tilde{v}$ so that equation (10) becomes

$$(\tilde{A} + i\tilde{B}) \otimes (\tilde{x} + i\tilde{y}) = \tilde{u} + i\tilde{v} \tag{11}$$

By multiplying using algebra in equation (8), equation (11) becomes

$$\left(\tilde{A} \otimes \tilde{x} - \tilde{B} \otimes \tilde{y}\right) + i\left(\tilde{A} \otimes \tilde{y} + \tilde{B} \otimes \tilde{x}\right) = \tilde{u} + i\tilde{v} \quad (12)$$

Based on similarity of the left and right sides, equation (12) can be divided into

$$\tilde{A} \otimes \tilde{x} - \tilde{B} \otimes \tilde{y} = \tilde{u} \quad (13)$$
$$\tilde{A} \otimes \tilde{y} + \tilde{B} \otimes \tilde{x} = \tilde{v} \quad (14)$$

Since $\tilde{A}, \tilde{B}, \tilde{x}, \tilde{y}, \tilde{u}$, and $\tilde{v}$ are triangular *fuzzy* number matrices, let $\tilde{A} = (A, M, N)$, $\tilde{B} = (B, P, Q)$, $\tilde{x} = (x, a, b)$, $\tilde{y} = (y, c, d)$, $\tilde{u} = (u, \alpha, \beta)$ and $\tilde{v} = (v, \gamma, \delta)$ so that equations (13) and (14) becomes

$$(A, M, N) \otimes (x, a, b) \ominus (B, P, Q) \otimes (y, c, d) = (u, \alpha, \beta) \quad (15)$$
$$(A, M, N) \otimes (y, c, d) \ominus (B, P, Q) \otimes (x, a, b) = (v, \gamma, \delta) \quad (16)$$

To determine the multiplication operation used in equations (15) and (16) depends on the value of the matrix $\tilde{A}$, $\tilde{B}$, $\tilde{x}$, and $\tilde{y}$ which depend on the values $\tilde{u}$ and $\tilde{v}$. First, if $\tilde{u}$ is positive or negative then there are many possibilities for the matrix values $\tilde{A}$, $\tilde{B}$, $\tilde{x}$, and $\tilde{y}$ that correspond to equation (15). From some of these possibilities, it will be adjusted to equation (16). So that there are several possible solutions for equations (15) and (16). This paper will discuss 2 possible solutions for the system of complex linear *fuzzy* equations in equations (9) which are as follows.

A. Solution for fuzzy linear equation systems where $\tilde{A}$, $\tilde{B}$, $\tilde{u}$, and $\tilde{v}$ are positive

For the case with matrices $\tilde{A}$, $\tilde{B}$, $\tilde{u}$, and $\tilde{v}$ are positive, it is possible that the values for $\tilde{x}$ and $\tilde{y}$ are positive. So that using the positive multiplication formula in equation (4) to equations (15) and (16), is obtained

$$(Ax, Aa + Mx, Ab + Nx) \ominus (By, Bc + Py, Bd + Qy) = (u, \alpha, \beta) \quad (17)$$
$$(Ay, Ac + My, Ad + Ny) \oplus (Bx, Ba + Px, Bb + Qx) = (v, \gamma, \delta) \quad (18)$$

By using the *fuzzy* number reduction formula in equation (2) to equation (17) and the fuzzy number addition formula in equation (1) to equation (18), it is obtained

$$(Ax - By, Aa + Mx + Bd + Qy, Ab + Nx + Bc + Py) = (u, \alpha, \beta) \quad (19)$$
$$(Ay + Bx, Ac + My + Ba + Px, Ad + Ny + Bb + Qx) = (v, \gamma, \delta) \quad (20)$$

Next, equating the left and right sides in equation (19) and equation (20) is generated

$$Ax - By = u \quad (21)$$
$$Ay + Bx = v \quad (22)$$
$$Aa + Mx + Bd + Qy = \alpha \quad (23)$$
$$Ad + Ny + Bb + Qx = \delta \quad (24)$$
$$Ab + Nx + Bc + Py = \beta \quad (25)$$
$$Ac + My + Ba + Px = \gamma \quad (26)$$

By using the substitution method in equations (21)-(26), the system solution of complex *fuzzy* linear equations is obtained , $\tilde{z} = (x, a, b) + i(y, c, d)$ with

$$x = (A + BA^{-1}B)^{-1}(u + BA^{-1}v) \quad (27)$$
$$y = (A + BA^{-1}B)^{-1}(v - BA^{-1}u) \quad (28)$$
$$a = (B - C^3A)^{-1}(\gamma - My - Px + C(-\beta + Nx + Py) + C^2(\delta - Ny - Qx) + C^3(-\alpha + Mx + Qy)) \quad (29)$$
$$b = (B - C^3A)^{-1}(\delta - Ny - Qx + C(-\alpha + Mx + Qy) + C^2(\gamma - My - Px) + C^3(-\beta + Nx + Py) \quad (30)$$
$$c = (B - C^3A)^{-1}(-(-\beta + Nx + Py) - C(\delta - Ny - Qx) - C^2(-\alpha + Mx + Qy) - C^3(\gamma - My - Px)) \quad (31)$$
$$d = (B - C^3A)^{-1}(-(-\alpha + Mx + Qy) - C(\gamma - My - Px) - C^2(-\beta + Nx + Py) - C^3(\delta - Ny - Qx)) \quad (32)$$

Where $C = BA^{-1}$.

B. Solution for fuzzy linear equation systems where $\tilde{A}$, $\tilde{B}$, $\tilde{u}$, and $\tilde{v}$ are negative

For cases where the matrices $\tilde{A}$, $\tilde{B}$, $\tilde{u}$, and $\tilde{v}$ are negative, it is possible that the $\tilde{x}$ and $\tilde{y}$ values are negative. So that the negative multiplication formula is used in equation (7) to equation (15) and (16) so that

$$(Ax, -Ab - Nx, -Aa - Mx) \ominus (By, -Bd - Qy - Bc - Py) = (u, \alpha, \beta) \quad (33)$$

$$(Ay, -Ad - Ny, -Ac - My) \oplus (Bx, -Bb - Qx, -Ba - Px) = (v, \gamma, \delta) \quad (34)$$

By using the fuzzy number reduction formula in equation (2) to equation (33) and the addition formula for equation (1) to equation (34), it is obtained

$$(Ax - By, -Ab - Nx - Bc - Py, -Aa - Mx - Bd - Qy) = (u, \alpha, \beta) \quad (35)$$
$$(Ay + Bx - Ad - Ny - Bb - Qx, -Ac - My - Ba - Px) = (v, \gamma, \delta) \quad (36)$$

Next, equalize the two sides of equation (35) and (36) such that

$$Ax - By = u \quad (37)$$
$$Ay + Bx = v \quad (38)$$
$$-Ab - Nx - Bc - Py = \alpha \quad (39)$$
$$-Aa - Mx - Bd - Qy = \beta \quad (40)$$
$$-Ad - Ny - Bb - Qx = \gamma \quad (41)$$
$$-Ac - My - Ba - Px = \delta \quad (42)$$

Similar to the previous case, the solution of equations (37) - (42) will be determined using the substitution method so that the system solution of complex *fuzzy* linear equations is obtained, $\tilde{z} = (x, a, b) + i(y, c, d)$ with

$$x = (A + BA^{-1}B)^{-1}(u + BA^{-1}v),$$
$$y = (A + BA^{-1}B)^{-1}(v - BA^{-1}u),$$
$$a = (B - C^3A)^{-1}(-(\delta + My + Px) + C(\alpha + Nx + Py) - C^2(\gamma + Ny + Qx) + C^3(\beta + Mx + Qy)),$$
$$b = (B - C^3A)^{-1}(-(\gamma + Ny + Qx) + C(\beta + Mx + Qy) - C^2(\delta + My + Px) + C^3(\alpha + Nx + Py)),$$
$$c = (B - C^3A)^{-1}(-(\alpha + Nx + Py) + C(\gamma + Ny + Qx) - C^2(\beta + Mx + Qy) + C^3(\delta + My + Px)),$$
$$d = (B - C^3A)^{-1}(-(\beta + Mx + Qy) + C(\delta + My + Px) - C^2(\alpha + Nx + Py) + C^3(\gamma + Ny + Qx)),$$

where $C = BA^{-1}$.

IV. EXAMPLE OF COMPLEX LINEAR FUZZY EQUATION SYSTEM CALCULATION

The following is an example of a calculation for solving a complex *fuzzy* linear equation system with positive triangular fuzzy numbers. Given a complex *fuzzy* linear equation system as follows:

$$\big((2,1,4)+i(7,3,3)\big)\otimes(\tilde{x}_1+i\tilde{y}_1)\oplus\big((5,2,3)+i(2,1,1)\big)\otimes(\tilde{x}_2+i\tilde{y}_2)=\big((-1,68,74)+i(38,51,78)\big)$$

$$\big((1,1,1)+i(3,2,1)\big)\otimes(\tilde{x}_1+i\tilde{y}_1)\oplus\big((3,1,5)+i(1,1,2)\big)\otimes(\tilde{x}_2+i\tilde{y}_2)=\big((3,37,52)+i(19,31,47)\big)$$

Based on equation (11), the *fuzzy* matrices $\tilde{A}$ and $\tilde{B}$ and *fuzzy* vectors $\tilde{u}$ and $\tilde{v}$ are obtained

$$\tilde{A}=\begin{bmatrix}(2,1,4)&(5,2,3)\\(1,1,1)&(3,1,5)\end{bmatrix},\quad \tilde{u}=\begin{bmatrix}(-1,68,74)\\(3,37,52)\end{bmatrix},$$
$$\tilde{B}=\begin{bmatrix}(7,3,3)&(2,1,1)\\(3,2,1)&(1,1,2)\end{bmatrix},\quad \tilde{v}=\begin{bmatrix}(38,51,78)\\(19,31,47)\end{bmatrix}.$$

Furthermore, because the values of the *fuzzy* matrix $\tilde{A},\tilde{B},\tilde{u}$, and $\tilde{v}$ are posistive *fuzzy* matrices, the solution formula is used in equations (27)-(32). Furthermore, since $\tilde{A}=(A,M,N)$, $\tilde{B}=(B,P,Q)$, $\tilde{x}=(x,a,b)$, $\tilde{y}=(y,c,d)$, $\tilde{u}=(u,\alpha,\beta)$, and $\tilde{v}=(v,\gamma,\delta)$, then

$$A=\begin{bmatrix}2&5\\1&3\end{bmatrix},\quad B=\begin{bmatrix}7&2\\3&1\end{bmatrix},\quad u=\begin{bmatrix}-1\\3\end{bmatrix},\quad v=\begin{bmatrix}38\\19\end{bmatrix},$$
$$M=\begin{bmatrix}1&2\\1&1\end{bmatrix},\quad P=\begin{bmatrix}3&1\\2&1\end{bmatrix},\quad \alpha=\begin{bmatrix}68\\37\end{bmatrix},\quad \gamma=\begin{bmatrix}51\\31\end{bmatrix},$$
$$N=\begin{bmatrix}4&3\\1&5\end{bmatrix},\quad Q=\begin{bmatrix}3&1\\1&2\end{bmatrix},\quad \beta=\begin{bmatrix}74\\52\end{bmatrix},\quad v=\begin{bmatrix}78\\47\end{bmatrix}.$$

Next determine the value of A^{-1} is

$$A^{-1}=\begin{bmatrix}3&-5\\-1&2\end{bmatrix}$$

Then it can be calculated the value of the matrix $A+BA^{-1}B$ is

$$A+BA^{-1}B=\begin{bmatrix}42&12\\18&6\end{bmatrix}$$

Then the value of $(A+BA^{-1}B)^{-1}$ is

$$(A+BA^{-1}B)^{-1}=\begin{bmatrix}\frac{1}{6}&-\frac{1}{3}\\-\frac{1}{2}&\frac{7}{6}\end{bmatrix}.$$

First calculate the value of *x* and *y*. Based on the formula in equation (27), the value of *x* is obtained

$$x=\begin{bmatrix}\frac{1}{6}&-\frac{1}{3}\\-\frac{1}{2}&\frac{7}{6}\end{bmatrix}\left(\begin{bmatrix}-1\\3\end{bmatrix}+\begin{bmatrix}7&2\\3&1\end{bmatrix}\begin{bmatrix}3&-5\\-1&2\end{bmatrix}\begin{bmatrix}38\\19\end{bmatrix}\right)=\begin{bmatrix}2\\4\end{bmatrix},$$

And the value of *y* based on equation (28) is

$$y=\begin{bmatrix}\frac{1}{6}&-\frac{1}{3}\\-\frac{1}{2}&\frac{7}{6}\end{bmatrix}\left(\begin{bmatrix}38\\19\end{bmatrix}+\begin{bmatrix}7&2\\3&1\end{bmatrix}\begin{bmatrix}3&-5\\-1&2\end{bmatrix}\begin{bmatrix}-1\\3\end{bmatrix}\right)=\begin{bmatrix}3\\2\end{bmatrix},$$

After obtaining the values of *x* and *y* the values of *a*, *b*, *c*, and *d* can be determined. First, calculate the value of $C=BA^{-1}$. Therefore, we will determine the inverse of matrix *B* first.

$$B^{-1}=\begin{bmatrix}1&-2\\-3&7\end{bmatrix}.$$

The values of the matrix C,C^2,C^3 is

$$C=\begin{bmatrix}2&5\\1&3\end{bmatrix}\begin{bmatrix}1&-2\\-3&7\end{bmatrix}=\begin{bmatrix}-13&31\\-8&19\end{bmatrix},$$
$$C^2=\begin{bmatrix}-13&31\\-8&19\end{bmatrix}\begin{bmatrix}-13&31\\-8&19\end{bmatrix}=\begin{bmatrix}-79&186\\-48&113\end{bmatrix},$$
$$C^3=\begin{bmatrix}-79&186\\-48&113\end{bmatrix}\begin{bmatrix}-13&31\\-8&19\end{bmatrix}=\begin{bmatrix}-461&1085\\-280&659\end{bmatrix},$$

Next, we will determine the values of $B-C^3A$ is

$$B-C^3A=\begin{bmatrix}-156&-948\\-96&576\end{bmatrix}$$

The inverse of the matrix $B-C^3A$ is

$$(B-C^3A)^{-1}=\begin{bmatrix}\frac{1}{2}&-\frac{79}{96}\\-\frac{1}{22}&\frac{13}{96}\end{bmatrix}.$$

Then, we will determine the values of *Mx*, *My*, *Nx*, *Ny*, *Px*, *Py*, *Qx*, and *Qy* because the formula *a*, *b*, *c*, and *d* contain these values.

$$Mx=\begin{bmatrix}1&2\\1&1\end{bmatrix}\begin{bmatrix}2\\4\end{bmatrix}=\begin{bmatrix}10\\6\end{bmatrix},$$
$$My=\begin{bmatrix}1&2\\1&1\end{bmatrix}\begin{bmatrix}3\\2\end{bmatrix}=\begin{bmatrix}7\\5\end{bmatrix},$$
$$Nx=\begin{bmatrix}4&3\\1&5\end{bmatrix}\begin{bmatrix}2\\4\end{bmatrix}=\begin{bmatrix}20\\22\end{bmatrix},$$
$$Ny=\begin{bmatrix}4&3\\1&5\end{bmatrix}\begin{bmatrix}3\\2\end{bmatrix}=\begin{bmatrix}18\\13\end{bmatrix},$$
$$Px=\begin{bmatrix}3&1\\2&1\end{bmatrix}\begin{bmatrix}2\\4\end{bmatrix}=\begin{bmatrix}10\\8\end{bmatrix},$$
$$Py=\begin{bmatrix}3&1\\2&1\end{bmatrix}\begin{bmatrix}3\\2\end{bmatrix}=\begin{bmatrix}11\\8\end{bmatrix},$$
$$Qx=\begin{bmatrix}3&1\\1&2\end{bmatrix}\begin{bmatrix}2\\4\end{bmatrix}=\begin{bmatrix}10\\10\end{bmatrix},$$
$$Qy=\begin{bmatrix}3&1\\1&2\end{bmatrix}\begin{bmatrix}3\\2\end{bmatrix}=\begin{bmatrix}11\\7\end{bmatrix}.$$

Furthermore, to simplify and shorten the calculation, the values of $\gamma-My-Px$, $-\beta+Nx+Py$, $\delta-Ny-Qx$, and $-\alpha+Mx+Qy$ will be determined as follows.

$$\gamma-My-Px=\begin{bmatrix}51\\31\end{bmatrix}-\begin{bmatrix}7\\5\end{bmatrix}-\begin{bmatrix}10\\8\end{bmatrix}=\begin{bmatrix}34\\18\end{bmatrix},$$
$$-\beta+Nx+Py=-\begin{bmatrix}74\\52\end{bmatrix}+\begin{bmatrix}20\\22\end{bmatrix}+\begin{bmatrix}11\\8\end{bmatrix}=\begin{bmatrix}-43\\-22\end{bmatrix},$$
$$\delta-Ny-Qx=\begin{bmatrix}78\\47\end{bmatrix}-\begin{bmatrix}18\\13\end{bmatrix}-\begin{bmatrix}10\\10\end{bmatrix}=\begin{bmatrix}50\\24\end{bmatrix},$$
$$-\alpha+Mx+Qy=-\begin{bmatrix}68\\37\end{bmatrix}+\begin{bmatrix}10\\6\end{bmatrix}+\begin{bmatrix}11\\7\end{bmatrix}=\begin{bmatrix}-47\\-24\end{bmatrix},$$

By using the formula in equations (29)-(32) it is obtained

$$a=\begin{bmatrix}\frac{1}{2}&-\frac{79}{96}\\-\frac{1}{22}&\frac{13}{96}\end{bmatrix}\left(\begin{bmatrix}34\\18\end{bmatrix}+\begin{bmatrix}-13&31\\-8&19\end{bmatrix}\begin{bmatrix}-43\\-22\end{bmatrix}+\begin{bmatrix}-79&186\\-48&113\end{bmatrix}\begin{bmatrix}50\\24\end{bmatrix}+\begin{bmatrix}-461&1085\\-280&659\end{bmatrix}\begin{bmatrix}-47\\-24\end{bmatrix}\right)=\begin{bmatrix}1\\4\end{bmatrix}$$

$$b = \begin{bmatrix} \frac{1}{2} & -\frac{79}{96} \\ -\frac{1}{22} & \frac{13}{96} \end{bmatrix} \left(\begin{bmatrix} 50 \\ 24 \end{bmatrix} + \begin{bmatrix} -13 & 31 \\ -8 & 19 \end{bmatrix} \begin{bmatrix} -47 \\ -24 \end{bmatrix} + \begin{bmatrix} -79 & 186 \\ -48 & 113 \end{bmatrix} \begin{bmatrix} 34 \\ 18 \end{bmatrix} + \begin{bmatrix} -461 & 1085 \\ -280 & 659 \end{bmatrix} \begin{bmatrix} -43 \\ -22 \end{bmatrix} \right) = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$$

$$c = \begin{bmatrix} \frac{1}{2} & -\frac{79}{96} \\ -\frac{1}{22} & \frac{13}{96} \end{bmatrix} \left(-\begin{bmatrix} -43 \\ -22 \end{bmatrix} - \begin{bmatrix} -13 & 31 \\ -8 & 19 \end{bmatrix} \begin{bmatrix} 50 \\ 24 \end{bmatrix} - \begin{bmatrix} -79 & 186 \\ -48 & 113 \end{bmatrix} \begin{bmatrix} -47 \\ -24 \end{bmatrix} - \begin{bmatrix} -461 & 1085 \\ -280 & 659 \end{bmatrix} \begin{bmatrix} 34 \\ 18 \end{bmatrix} \right) = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

$$d = \begin{bmatrix} \frac{1}{2} & -\frac{79}{96} \\ -\frac{1}{22} & \frac{13}{96} \end{bmatrix} \left(-\begin{bmatrix} -47 \\ -24 \end{bmatrix} - \begin{bmatrix} -13 & 31 \\ -8 & 19 \end{bmatrix} \begin{bmatrix} 34 \\ 18 \end{bmatrix} - \begin{bmatrix} -79 & 186 \\ -48 & 113 \end{bmatrix} \begin{bmatrix} -43 \\ -22 \end{bmatrix} - \begin{bmatrix} -461 & 1085 \\ -280 & 659 \end{bmatrix} \begin{bmatrix} 50 \\ 24 \end{bmatrix} \right) = \begin{bmatrix} 3 \\ 2 \end{bmatrix}.$$

So that the solution to the system of complex linear *fuzzy* equation is $\tilde{z}_1 = (2,1,4) + i(3,2,3)$ dan $\tilde{z}_2 = (4,4,3) + i(2,3,2)$. To find out whether the $\tilde{z}_1$ and $\tilde{z}_2$ are compatible solutions, the values $\tilde{z}_1$ and $\tilde{z}_1$ will be substituted into complex *fuzzy* linear equations in order to obtain

$$\big((2,1,4) + i(7,3,3)\big) \otimes (2,1,4) + i(3,2,3) \oplus \big((5,2,3) + i(2,1,1)\big) \otimes (4,4,3) + i(2,3,2) = \big((-1,68,74) + i(38,51,78)\big)$$

$$\big((1,1,1) + i(3,2,1)\big) \otimes (2,1,4) + i(3,2,3) \oplus \big((3,1,5) + i(1,1,2)\big) \otimes (4,4,3) + i(2,3,2) = \big((3,37,52) + i(19,31,47)\big)$$

By multiplying using algebra in equation (8), the multiplication formula, and subtraction of *fuzzy* numbers, are generated

$$\big((-17,34,39) + i(20,20,52)\big) \oplus \big((16,34,35) + i(18,31,26)\big) = \big((-1,68,74) + i(38,51,78)\big)$$

$$\big((-7,15,18) + i(9,12,20)\big) \oplus \big((10,22,34) + i(10,19,27)\big) = \big((3,37,52) + i(19,31,47)\big)$$

Furtermore, using the formula for adding *fuzzy* numbers on the left side, it is obtained

$$\big((-1,68,74) + i(38,51,78)\big) = \big((-1,68,74) + i(38,51,78)\big)$$

$$\big((3,37,52) + i(19,31,47)\big) = \big((3,37,52) + i(19,31,47)\big)$$

Because the right and left sides are the same, the the solution $\tilde{z}_1 = (2,1,4) + i(3,2,3)$ and $\tilde{z}_2 = (4,4,3) + i(2,3,2)$ is a compatible solution.

V. CONCLUSION

The conclusion that can be drawn in this paper is in determining alternative solutions of complex fuzzy linear equation systems

$\tilde{C} \otimes \tilde{z} = \tilde{w}$ with the example that $\tilde{C} = \tilde{A} + i\tilde{B}$, $\tilde{z} = \tilde{x} + i\tilde{y}$, and $\tilde{w} = \tilde{u} + i\tilde{v}$ there are many solutions that depend on the value of the *fuzzy* triangle matrix $\tilde{A}$, $\tilde{B}$, $\tilde{u}$, and $\tilde{v}$ but in this paper the author only discusses 2 possible solutions. In the same way other possible solutions can be determined. Furthermore, the resulting formulas are applied in the form of examples and the solution obtained when substituted back into a complex fuzzy linear equation system produces a value of $\tilde{w}$ which means the given solution is compatible.

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