

The Use of Semi-Markov Process Model to Determine the Optimal Performance of Hydro Power Turbines (A Case Study of Shiroro Hydro Electric Power Plant)

Mohammed A¹, Abdulmumin Z. I² and Mohammed T. G²

¹Department of Mathematics, Federal University of Technology Minna, Nigeria

² Department of Mathematics and Statistics, Niger State Polytechnic, Zungeru

Abstract:- A semi-Markov model in continuous state and time to study the daily running of turbine has been presented in this paper. The model was used to determine the optimal performance of turbines of Shiroro hydro generation station Niger State, Nigeria for a period of 4 years. The result shows that there is no transition from state 1 to state 4, state 2 to state 4 and state 4 to state 3, that is $\phi_{14}(n) = \phi_{24}(n) = \phi_{43}(n) = 0$.

The result also shows that the graphs of interval transition probabilities from one state to another are gradually increasing. From the virtual transition probabilities, the graph is gradually decreasing and after forty-one (41) days $\phi_{11}(n)$, $\phi_{22}(n)$, $\phi_{33}(n)$ and $\phi_{44}(n)$ has the percentages of 99%, 96%, 92% and 99%, which shows that when turbine is in any of the state it stay there for a longer period before transition was made to another state. These results are important information to the engineers and utility staffs to plan against the failure of the turbines.

Keyword:- Semi-Markov Process, Continuous Time, Probability, Interval Transition probability, Waiting Time, Holding Time, Turbine, Virtual Transition probability.

I. INTRODUCTION

Hydro-electric generation is the power potential of falling water have long been identified as very useful in the generation of electricity through the conversion of various form of energies. The water is made to run down a pipe or "penstock", striking the blades of the turbine runner. The runner rotated as connected by a shaft to the generator; provide the necessary power for the production of electricity. Having served its purpose, the water then passed out through the tailrace of the station to join the main stream of the river. In most cases it has been necessary to install massive Dams on waterways in order to produce the required head. This allows several generators to be operated from the same body of water, supplying the required flexibility for complete utilization of the water available (Igboannugo *et al* 2013). Nigerians is experiencing serious challenges in the Energy Sector, particularly Electrical Energy. The power generated dropped from 4,500MW to

1,327MW, while the hydropower stations were able to generate less than 40% of their installation capacities (Udo 2016). Furthermore, Saba *et al* (2016) lamented that about 30 to 40% of electricity generated is usually lost due to poor maintenance, water management, station and sub-stations related problems. To have effective power generation, maintenance of machine/equipment constitutes a major aspect, and contributes significantly to the net cost of production Sule (2010) focused on capacity of electricity generation in Nigeria and the major factors affecting electricity generation, transmission and distribution in the country. Also Bobosat *et al* (1977) applied Markov model to equipment maintenance and pointed out that equipment deterioration could be modeled as a multi-state discrete time controlled Markov process. This related to Markovian deterioration, in which case the degree of deterioration was used to classify the states. Thomas M. W. *et al* (2006), showed that a semi-Markov process with sojourn times given as a general lifetime distribution can be approximated by a conventional Markov process with exponentially distributed sojourn times. This means that the general lifetime distribution is replaced by a sum of exponentially distributed times. One way to estimate the general lifetime distribution in the semi-Markov model is the use of expert opinion. Furthermore Pievatola *et al* (2004) proposed a state space model for electrical power system made by independent semi-Markov components, in which restoration times can have a non-exponential distribution, thus obtaining a more realistic reliability characterization, especially regarding the outage duration distribution. Geunet *et al* (2011) proposed semi-Markov model for power system maintenance. The model was designed to balance costs and benefits because frequent maintenance increase cost while, infrequent maintenance can also be costly due to electricity outage. Also Vulpeet *et al* (2004) developed an approach to the availability evaluation of repairable subsystem and equipment. The unavailability state of a subsystem is split into three to five "smaller" ones. The loss of availability process can be modeled by means of a semi-Markov process and a Markov renewal process which generalize Markov jump processes.

In this paper, Semi- Markov model in continuous state and time was used to study the performance of turbines and prediction of power generated in order to improve the stability of power in the country.

II. MATERIALS AND METHODS

Study Area and Data Source: The data used in this paper work, were collected from the National Control Centre (NCC). In Nigeria, the electricity generating stations are interconnected to form the national grid at 330/132KV with a single National Control Centre (NCC) in Oshogbo and sub control centre at Shiroro (Minna) where power generated is being shared to eleven (11) distribution companies.

Semi- Markov model: Semi-Markov process is a stochastic process in which changes of state occur according to a Markov chain and for which the time interval between two successive transitions is a random variable whose distribution may depend on the present state from which the transition takes place(Bellman, 1957).

Model Formulation: We model the performance of Turbines and classify the condition of Turbine in to four states using the principle of Markov chain. Let the states be defined as:

- State 1 Operate above the average (1800MW and above per day)
- State 2 Operate below the average (below 1800MW per day)
- State 3 Short time repair (under repair for maximum of seven days)
- State 4 Long time repair (under repair above seven days)

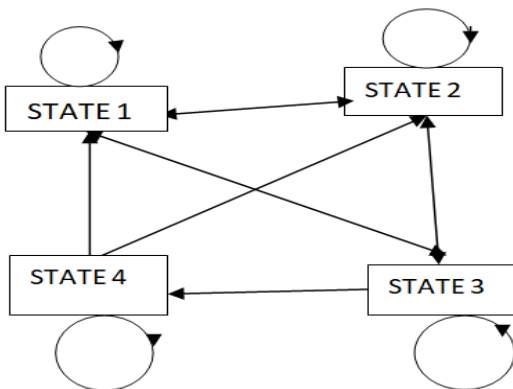


Figure 1: The Transition Diagram for the Turbine Operation

We observed that the states 1, 2 and 3 communicates while 3 and 4 are transit state, and all possible transitions of the process are made between the states 1, 2, 3, 4. We would like a transition to occur at a time the duration of stay in a state is completed, even if the new state is the same as the old. Such transitions are called virtual transition and are represented by loops in the transition diagram.

From the above diagram, we record the transition probability matrix P for the process as shown in equation 1

$$P = \begin{pmatrix} p_{11} & p_{12} & p_{13} & 0 \\ p_{21} & p_{22} & p_{23} & 0 \\ p_{31} & p_{32} & p_{33} & p_{34} \\ p_{41} & p_{42} & 0 & p_{44} \end{pmatrix} \quad (1)$$

We use the semi-Markov process technique to analyses the process with the above set of state. The transitions can be readily identified from the transition probability matrix P . To study this process, we specify the probabilistic nature of the transition. And think of this process as a process whose successive state occupancies are governed by the transition probabilities of a Markov chains, but whose stay in any state is described by a random variable that depends on the state to which the next transition is made.

Holding Time and Waiting Time: Let (P_{ij}) be the probability that the Turbine that is in state i on its last transition will enter state j on its next transition $i, j = 1, 2, 3,4$. The transition probabilities must satisfy the following

$$P_{ij} \geq 0, \quad i, j = 1, 2, 3,4 \text{ and}$$

$$\sum_{j=1}^4 P_{ij} = 1, \quad i = 1, 2, 3,4 .$$

(2)

Whenever the Turbine enters state i it remains there for a time T_{ij} in state i before making a transition to state j . T_{ij} is called the holding time in state i . The holding times are positive integer valued random variables each governed by a probability distribution function $f_{ij}(\cdot)$ called the holding time distribution function for a transition from state i to state j . (Howard 1971)

Thus,

$$P(T_{ij} = m) = f_{ij}(m) \quad i, j = 1, 2, 3,4$$

(3)

We assume that the means μ_{ij} of all holding time distribution are finite and that all holding times are at least one day in length. That is,

$$f_{ij}(0) = 0$$

To completely describe the semi-Markov process, we must specify four holding time distribution functions in addition to the transition probabilities.

For a fixed value of i , T_{ij} is the same for each value of j , ($i, j = 1, 2, 3, 4$).

Let $f_{ij}(\cdot)$ be the probability distribution of continuous random variable T_{ij}

$$F_{ij}(n) = p(T_{ij} \leq n) = \int_{m=0}^n f_{ij}(m) dm \quad (4)$$

and

$\bar{F}_{ij}(\cdot)$ be the complementary cumulative probability distribution of T_{ij}

$$\bar{F}_{ij}(\cdot) = 1 - f_{ij}(n) = p(T_{ij} > n) = \int_{m=n+1}^{\infty} f_{ij}(m) dm \quad (5)$$

Suppose the turbine enters state i . Let Y_i be the time it spent in state i before moving out of the state i . Then Y_i is called the waiting time in state i .

We let $W_i(\cdot)$ be the probability distribution function of Y_i

Then
$$W_i(m) = p(Y_i = n) = \sum_{j=1}^4 p_{ij} f_{ij}(m) \quad (6)$$

The probability distribution $W_i(\cdot)$ and the complementary probability distribution $\bar{W}_i(\cdot)$ for the waiting times are given as follows

$$W_i(n) = p(Y_i \leq m) = \int_{m=1}^n W_i(m) dm \quad (7)$$

$$= \sum_{j=1}^4 p_{ij} F_{ij}(n) \quad (8)$$

And
$$\bar{W}_i(m) = p(Y_i > n) = 1 - W_i(n) \quad (9)$$

$$= \int_{m=n+1}^{\infty} W_i(m) dm \quad (10)$$

$$= \sum_{j=1}^4 p_{ij} \bar{F}_{ij}(n)$$

Interval transition probability in continuous time: We define $\phi_{ij}(n)$ to be the probability that the condition of Turbine will be in state j in day n given that it entered state i in day zero. This is called the interval transition probability from state i to state j in the interval $(0, n]$. Then

$$\phi_{ij}(n) = \delta_{ij} \bar{W}_i(n) + \sum_{k=1}^4 p_{ik} \int_1^n f_{ik}(m) \phi_{kj}(n-m) dm \quad (11)$$

$$\delta_{ij} = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases} \quad i, j = 1, 2, 3, 4 \quad n = 1, 2, 3, 4 \quad (12)$$

This is the interval transition probability from state i to state j in the interval $(0, n]$

Application

Shiroro hydro electricity generation station has four units. These are unit 411G1, 411G2, 411G3 and 411G4, and each of these units generate maximum of 150MW having a total maximum generation capacity of 600MW. We obtained the daily running data of Shiroro hydro electricity generation station of the four (4) turbines for the period of four (4) years i.e. (2012-2015) is summarized in table 1

Table (1): A summary of power generated of Shiroro generation station from 2012-2015

CLASS INTERVAL (MW)	STATES	FREQUENCY
$\geq 1,800$	1	2,635
$1 \leq 1,800$	2	995
Short Time Repair	3	740
Long Time Repair	4	1,478
Total		5,848

Table (2): The Transition Count Matrix for the Turbines

	STATES 1	STATE 2	STATE 3	STATE 4
STATE 1	2475	102	58	0
STATE 2	68	730	197	0
STATE 3	59	157	475	49
STATE 4	20	33	0	1425

The transition probabilities Matrix for Turbines

$$P = \begin{bmatrix} 0.9393 & 0.0387 & 0.0220 & 0 \\ 0.0683 & 0.7337 & 0.1980 & 0 \\ 0.0797 & 0.2122 & 0.6419 & 0.0662 \\ 0.0135 & 0.0223 & 0 & 0.9642 \end{bmatrix} \quad (13)$$

Suppose that the holding times in each state before making a transition to another state follows the exponential distribution with parameter λ . This implies that the mean holding time in each state is $\frac{1}{\lambda}$ (in days). The mean holding time in each state is shown in table (3) below.

Table (3): Mean Holding Time for Turbines

Mean Holding Time			
STATE 1	STATE 2	STATE 3	STATE 4
659	249	185	370

➤ *Results*

Table (3) shows that turbines have highest holding time in state 1 and state 4. From equation (11), we obtain the interval probabilities presented in tables (4), (5) and (6).

Table (4): Interval Transition Probabilities $\phi_{12}(n)$, $\phi_{13}(n)$, $\phi_{21}(n)$ and $\phi_{23}(n)$

n	$\phi_{12}(n)$	$\phi_{13}(n)$	$\phi_{21}(n)$	$\phi_{23}(n)$
1	0.000058		0.000032	0.000272
2	0.000115		0.000065	0.000544
3	0.00017		0.000098	0.000814
4	0.000231		0.000131	0.001084
5	0.000289		0.000164	0.001352
6	0.000346		0.000197	0.0016196
7	0.000404		0.000229	0.0018858
8	0.000461		0.000262	0.002151
9	0.000518		0.000294	0.0024150
10	0.000576		0.000327	0.0026780
11	0.000633		0.000359	0.0029400
12	0.000690		0.0003924	0.0032009
13	0.000747		0.0004248	0.003460
14	0.000804		0.0004571	0.003719
15	0.000860		0.0004892	0.003977
16	0.000917		0.0005216	0.004234
17	0.0009742		0.0005538	0.004490
18	0.001030		0.0005859	0.004744
19	0.0010872		0.0006180	0.0049984
20	0.0011436		0.0006501	0.0052511
21	0.001199		0.0006821	0.0055028
22	0.0012561		0.0007140	0.0057535
23	0.0013122		0.0007459	0.0060032
24	0.0013682		0.0007773	0.0062519

25	0.0014242	0.0008094	0.0064996	0.0188421
26	0.0014800	0.0008419	0.0067463	0.0195573
27	0.0015358	0.0008731	0.006992	0.0202697
28	0.0015915	0.0009047	0.0072367	0.0209792
29	0.0011647	0.0009368	0.0074805	0.0216859
30	0.0017027	0.0009679	0.0077233	0.0223897
31	0.001758	0.0009994	0.0079651	0.0230907
32	0.0018135	0.0010309	0.008206	0.0237890
33	0.0018688	0.0010623	0.0084459	0.0244844
34	0.0019240	0.0010937	0.0086848	0.0251771
35	0.0019791	0.0011250	0.0089228	0.0258670
36	0.0020341	0.0011563	0.0091598	0.0265542
37	0.0020891	0.0011876	0.0093959	0.0272386
38	0.0021439	0.0012188	0.0096311	0.0279203
39	0.0021987	0.0012499	0.0098653	0.0285992
40	0.0022534	0.0012810	0.0098579	0.0292755
41	0.0023081	0.0013121	0.0103309	0.0299490

Table (4) presents the values of interval transition probabilities in continuous time from state 1 to state 2, state 1 to 3, state 2 to 1 and state 2 to 3 respectively using Equation (11), for $n = 1, 2, \dots, 41$. It is illustrated in Figure 2

Table (5): Interval Transition Probabilities $\phi_{31}(n)$, $\phi_{32}(n)$, $\phi_{34}(n)$, $\phi_{41}(n)$ and ϕ_{42}

n	$\phi_{31}(n)$	$\phi_{32}(n)$	$\phi_{34}(n)$	$\phi_{41}(n)$	$\phi_{42}(n)$
1	0.0004292	0.001142	0.000356	0.0000364	0.0000601
2	0.0008561	0.002274	0.000711	0.0000727	0.0001200
3	0.0012807	0.003409	0.001063	0.0001089	0.0001798
4	0.0017030	0.004534	0.001414	0.0001450	0.0002395
5	0.0021231	0.005652	0.001763	0.0001810	0.0002989
6	0.0025408	0.006765	0.002110	0.0002169	0.0003583
7	0.0029564	0.007814	0.002456	0.0002527	0.0004174
8	0.0033697	0.008971	0.002798	0.0002884	0.0004746
9	0.0037808	0.010066	0.003140	0.0003240	0.0005353
10	0.0041896	0.011154	0.003479	0.0003596	0.0005939
11	0.0045963	0.012237	0.003817	0.0003950	0.0006525
12	0.0050007	0.013314	0.0041537	0.0004303	0.0007108
13	0.0054030	0.143855	0.0044878	0.0004656	0.0007690
14	0.0058031	0.0154509	0.0048202	0.0005007	0.0008271
15	0.0062011	0.0165104	0.0051507	0.0005358	0.0008850
16	0.0065969	0.0175643	0.0054795	0.0005707	0.00094275
17	0.0069906	0.0186125	0.0058065	0.0006056	0.001000
18	0.0073822	0.0196551	0.0061318	0.0006404	0.0010577
19	0.0077717	0.0206920	0.0064553	0.0006750	0.0011150
20	0.0081590	0.0217234	0.0067770	0.0007096	0.0011721
21	0.0085446	0.0227492	0.0070970	0.0007441	0.0012291
22	0.0089275	0.0237695	0.0074153	0.0007785	0.0012859
23	0.0093087	0.0247842	0.0077319	0.0008128	0.0013425

24	0.0096877	0.0257936	0.0080468	0.0008470	0.0013990
25	0.0100648	0.0267974	0.008360	0.0008811	0.0014525
26	0.0104398	0.0277959	0.0086715	0.0009152	0.0015116
27	0.0108128	0.028789	0.0089813	0.0009491	0.0015676
28	0.0111838	0.0297768	0.0092894	0.0009829	0.0016235
29	0.0115528	0.0307592	0.0095959	0.0010167	0.0016793
30	0.0119198	0.0317363	0.0099007	0.0010503	0.0017439
31	0.0122848	0.0327082	0.0102039	0.0010839	0.0017903
32	0.0126479	0.0336748	0.0105055	0.0011174	0.0018456
33	0.0130090	0.0346333	0.0108054	0.0011508	0.0019007
34	0.0133681	0.0355959	0.0111038	0.0011841	0.0019557
35	0.0137254	0.0365437	0.0114005	0.0012173	0.0020642
36	0.0140807	0.0374896	0.0116956	0.0012504	0.0020653
37	0.0144341	0.0384305	0.0119891	0.0012834	0.0021198
38	0.0147855	0.0393664	0.0122811	0.0013164	0.0021743
39	0.0151351	0.0402971	0.0125715	0.0013492	0.0022228
40	0.0158289	0.4122296	0.0128603	0.0013820	0.0022826
41	0.0152873	0.0421437	0.0131475	0.0014147	0.0023366

Table(5) presents the values of interval transition probabilities in continuous time from state 3 to state 1, state 3 to 2, state 3 to 4, state 4 to 1 and state 4 to 2 respectively using Equation (11), for $n = 1, 2, \dots, 41$. This is graphically shown in Figure 3

Table (6): Virtual Transition Probabilities $\phi_{11}(n)$, $\phi_{22}(n)$, $\phi_{33}(n)$ and $\phi_{44}(n)$

n	$\phi_{11}(n)$	$\phi_{22}(n)$	$\phi_{33}(n)$	$\phi_{44}(n)$
1	0.9999027	0.998936	0.0080714	0.9914431
3	0.9998181	0.997878	0.9978751	0.9882341
3	0.9997274	0.9968234	0.996828	0.9870073
4	0.9996892	0.995773	0.9957917	0.9865151
5	0.9995464	0.994726	0.9947655	0.9862957
6	0.9994562	0.993684	0.9939498	0.9861766
7	0.9993660	0.992647	0.9927439	0.9860954
8	0.999276	0.991633	0.9912488	0.9860283
9	0.9991861	0.990583	0.9907632	0.9859664
10	0.9990963	0.989558	0.989788	0.9859066
11	0.9990067	0.988336	0.988233	0.9858477
12	0.9989172	0.9872555	0.9878685	0.9857892
13	0.9988279	0.9861358	0.9869237	0.9857309
14	0.9987387	0.9850149	0.985989	0.9856728
15	0.9986496	0.9838918	0.985064	0.9856149
16	0.9985606	0.9827697	0.984149	0.9855572
17	0.9984718	0.9816435	0.983245	0.9854996
18	0.9983838	0.9805103	0.98235	0.9854422
19	0.9982946	0.9793711	0.9814657	0.9853849
20	0.9982062	0.9782258	0.9805909	0.9853277
21	0.9981179	0.9770844	0.979725	0.9852708
22	0.9980297	0.9759457	0.9788707	0.9852140

23	0.9979417	0.9765936	0.9780254	0.9851572
24	0.9978538	0.9756239	0.9771899	0.985100
25	0.9977661	0.974658	0.976364	0.9850445
26	0.997685	0.973696	0.975548	0.9849883
27	0.997591	0.972738	0.9747419	0.9848322
28	0.9975036	0.9717839	0.973934	0.9848764
29	0.9974164	0.9708335	0.973158	0.9848206
30	0.9973293	0.9698869	0.972381	0.9847650
31	0.9972438	0.968944	0.9716137	0.9847096
32	0.997155	0.9680049	0.9708557	0.9845992
33	0.9970687	0.9670696	0.9701073	0.9845442
34	0.9969822	0.9663769	0.969368	0.9844593
35	0.9968957	0.96521	0.968639	0.9844365
36	0.9968094	0.9642859	0.967919	0.9843465
37	0.9967232	0.963365	0.967209	0.9843010
38	0.9966371	0.9624485	0.966508	0.9842964
39	0.9965512	0.9615354	0.965817	0.9842714
40	0.9964654	0.9608665	0.965135	0.9842173
41	0.9963777	0.9597200	0.9628909	0.996287

Table (6) presents the values of interval transition probabilities in continuous time from state 1 to state 1, state 2 to 2, state 3 to 3 and state 4 to 4 respectively using Equation (11), for $n = 1, 2, 3 \dots 41$. This is graphically shown in Figure 4

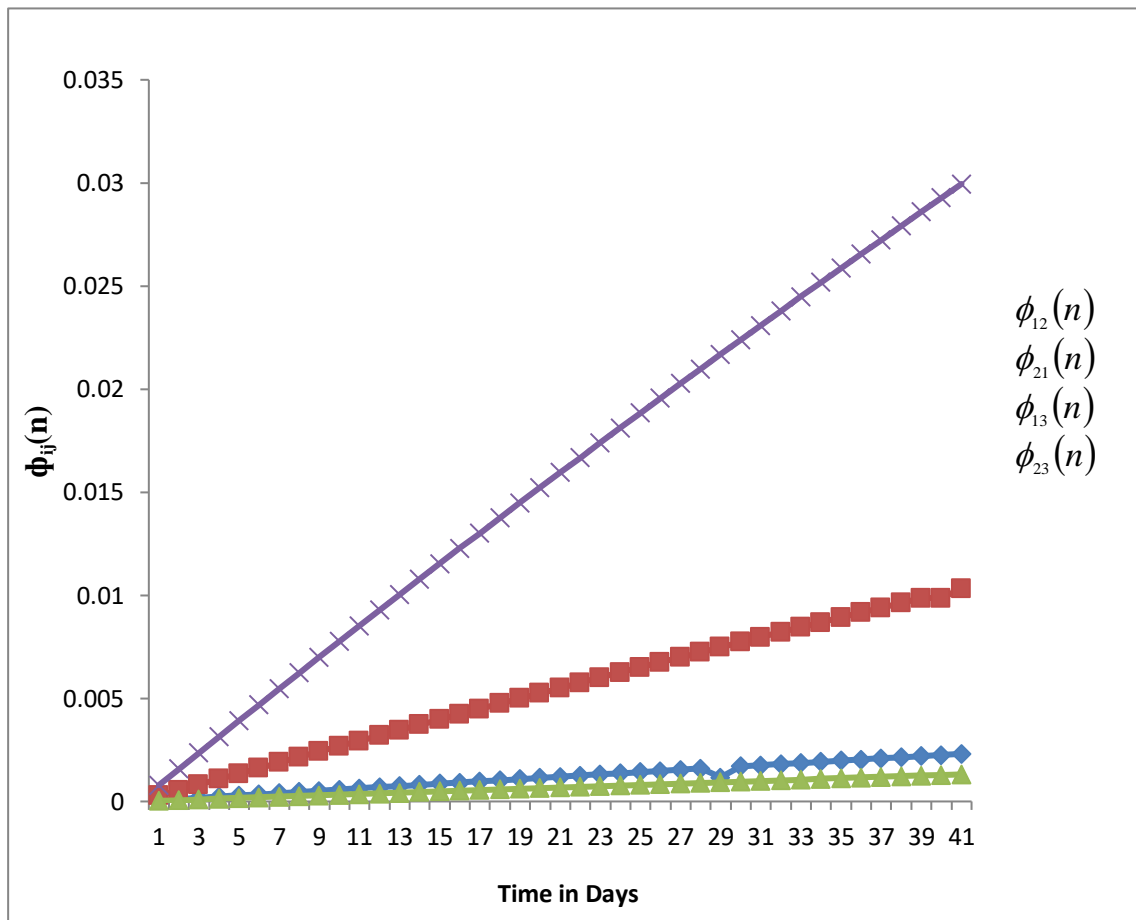


Figure 2: The Graph of Interval Transition Probabilities for $\phi_{12}(n)$, $\phi_{13}(n)$, $\phi_{21}(n)$ and $\phi_{23}(n)$

Figure 2 presents the graph of interval transition probabilities in continuous time from state 1 to state 2, state 1 to state 3, state 2 to state 1 and state 2 to state 3 for $n = 1, 2, \dots, 41$.

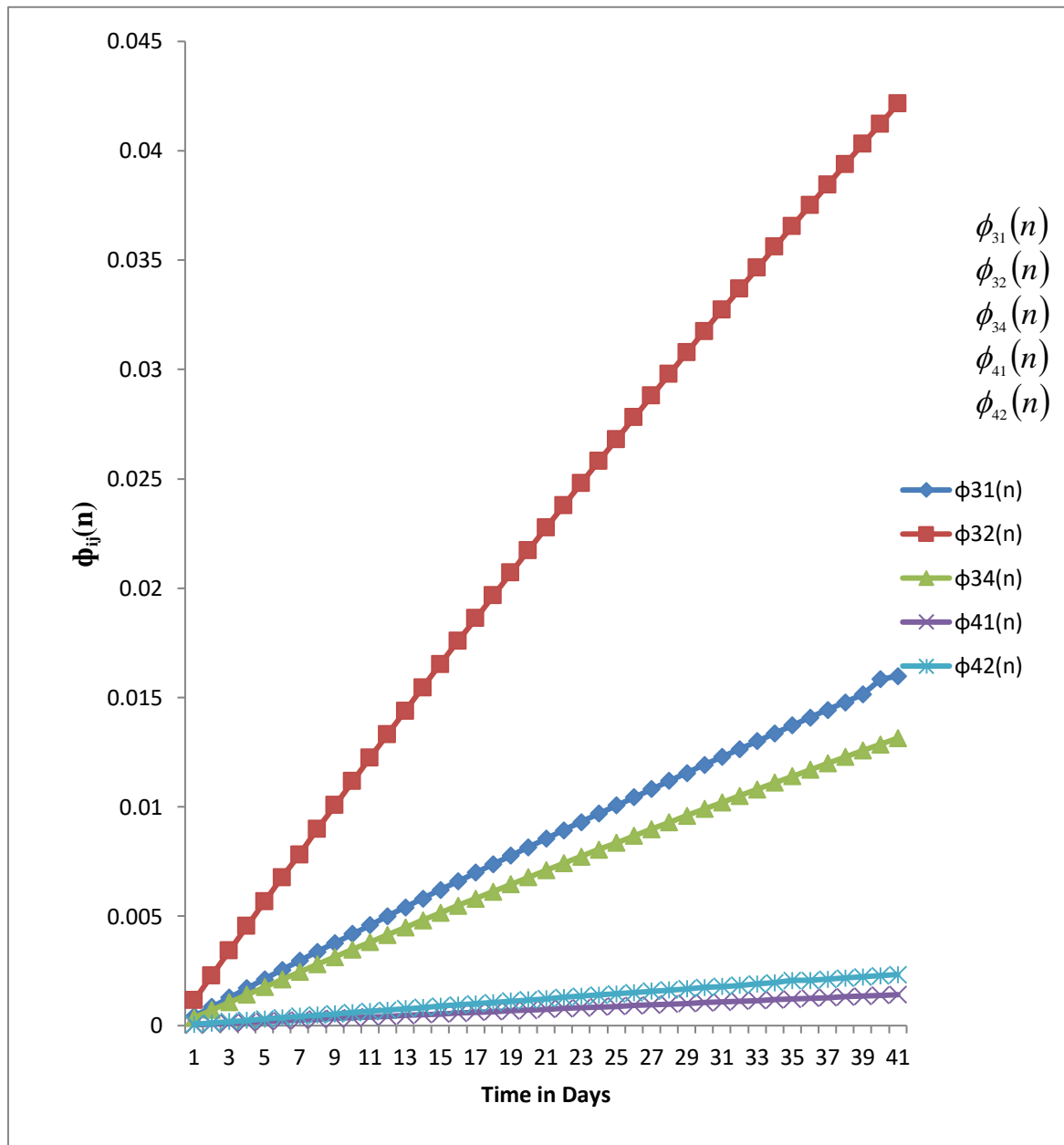


Figure 3: The Graph of Interval Transition Probabilities $\phi_{31}(n)$, $\phi_{32}(n)$, $\phi_{34}(n)$, $\phi_{41}(n)$, and $\phi_{42}(n)$

Figure 3 presents the graph of interval transition probabilities in continuous time from state 3 to state 1, state 3 to state 2, state 3 to state 4, state 4 to state 1 and state 4 to state 2 for $n = 1, 2, \dots, 41$.

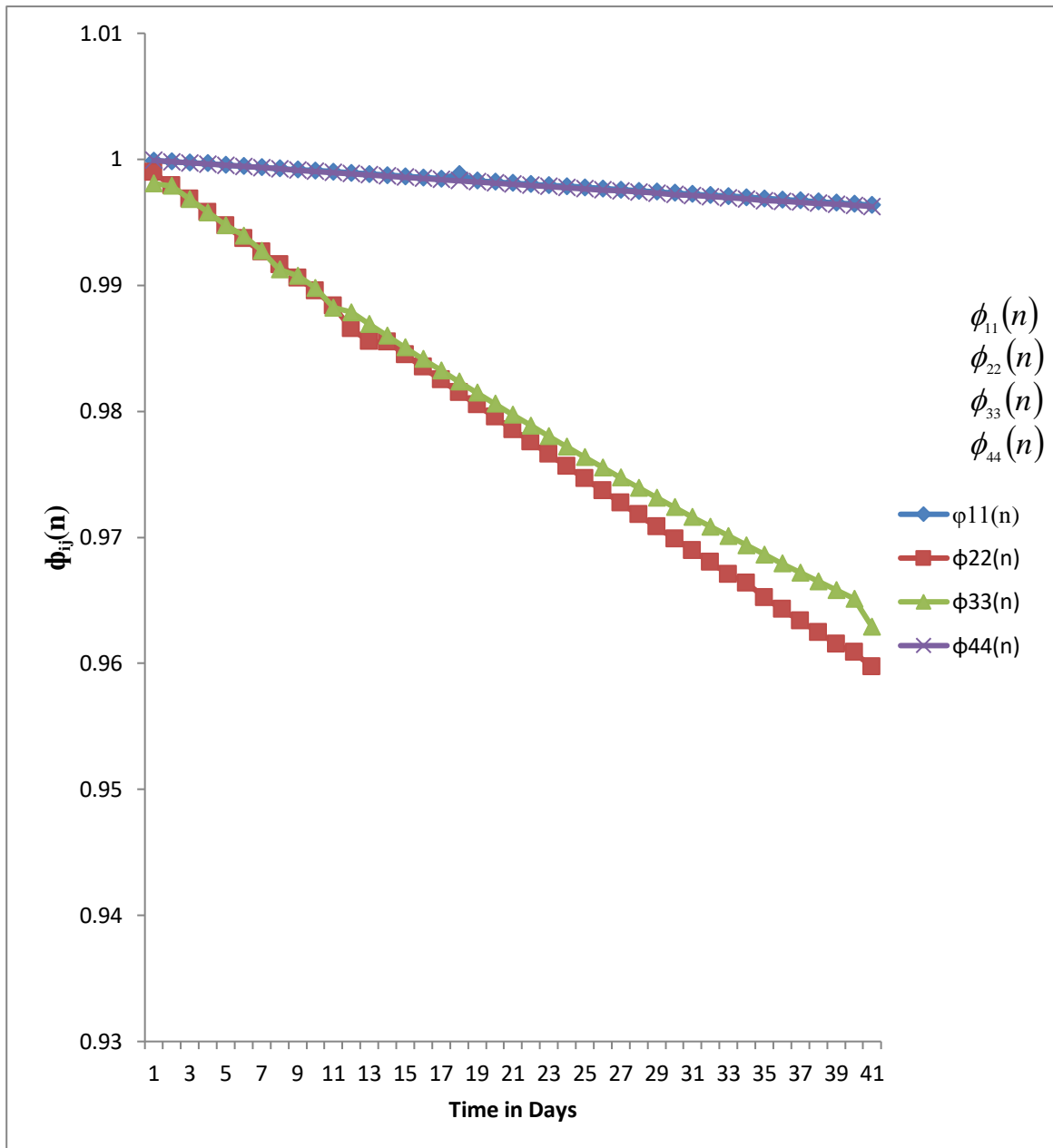


Figure 4: The Graph of Virtual Transition Probabilities $\phi_{11}(n)$, $\phi_{22}(n)$, $\phi_{33}(n)$ and $\phi_{44}(n)$

Figure 4 presents the graph of Virtual transition probabilities in continuous time from state 1 to state 1, state 2 to state 2, state 3 to state 3 and state 4 to state 4 for $n = 1, 2, \dots, 41$

III. DISCUSSION OF RESULTS

The paper presented Semi- Markov model to study the performance of the turbines in Shiroro generation station in continuous time. From the empirical analysis of the data collected the result shows that there is no transition from state 1 to state 4, state 2 to state 4 and state 4 to state 3, that is $\phi_{14}(n) = \phi_{24}(n) = \phi_{43}(n) = 0$ for all n. when turbine is in state 1 it make transition to state 2 most time than

states 3 and 4. Also when turbine is in state 3 (short time repair) or state 4 (long time repair) it normally make transition to state 2 most of time than any other states.

In the continuous time, from Tables 4 and 5 and Figures 2 and 3 the result shows that there are some increments in the transition probabilities from states 2, 3 and 4 to state 1 from about 0.000272, 0.0004292 and 0.0000364 in the first day ($n = 1$) to about 0.0023081, 0.0152873 and 0.0014147 in the forty one days ($n = 41$). The percentage increase is about 1.033%, 1.5287% and 0.141147% for states 2, 3 and 4 respectively in forty-one days. These represent the percentage of generating 1,800MW and above in day when the system is presently producing below

1,800MW per day or undergoing a short time repair or a long time repair.

Table 6 and Figure 4 show that $\phi_{11}(n)$, $\phi_{22}(n)$, $\phi_{33}(n)$ and $\phi_{44}(n)$ attained the values of about 0.9963777, 0.9597200, 0.9628909 and 0.996287 respectively for the first few days. They however dropped slowly and diminish to zero at infinity except $\phi_{11}(n)$, and $\phi_{44}(n)$. These show that $\phi_{11}(n)$ (operating above the average) and $\phi_{44}(n)$ (long time repair) are higher than the rest. The result suggests that if the process is in any of these states, it remains/persists in that state for some days before a change of state could occur. Thus, change of state occurs less frequently over the time. The behavior of $\phi_{11}(n)$ and $\phi_{44}(n)$, $n = 1, 2, 3, \dots$ are very interesting. This is because they produced almost the same values of probabilities. This is very clear in the graphs as they almost form a straight line along $y = 1$.

$\phi_{11}(n)$, $n = 1, 2, 3, \dots, 41$, shows that there shall be optimal production (1,800MW and above) consistently for several days by about 99% before a gradual decline due to some repairs either in a short/long time. $\phi_{44}(n)$, $n = 1, 2, 3, \dots, 41$, represents a long time of no production largely due to lack of water in the dam during dry season or breakdown of many turbines and component(s) that needed importation from outside Nigeria.

Therefore, the continuous time Semi-Markov models could be used to predict the optimal megawatt to be generated. The prediction is information that could be useful in the management of hydro electric plant.

IV. CONCLUSION

A semi-Markov model in continuous state and time to determine the performance of turbines of Shiroro generation station has been presented. The model has been able to ascertain long-run performance of the turbines and the prediction of power generated in the organization studied. The results from the model are important information that could assist the engineers and utility staffs to plan against the failure of turbine, in order to improve the stability of power generation in view to accelerate the economic growth of the nation.

REFERENCES

- [1]. Bhat U. N (1984). "Elements of Applied Stochastic Processes" John Wiley New York
- [2]. Bobos, A. G. & Protonotarios, E. N (1977). Optimal system for Equipment Maintenance and Replacement under Markovian Deterioration. *European Journal of Operational Research* 3, 257-264.
- [3]. Bellman, R. (1957) Dynamic Programming Princeton. Princeton University Press
- [4]. Geun-P. P., Jae H. H., Sang S. L. & Yong T. Y. (2011). Generalized Reliability Centered Maintenance Modeling Through Modified Semi-Markov Chain in Power System. *Journal of Electrical Engineering and Technology* Vol. 6 No 1 PP 25-31
- [5]. Howard R. A (1971). Dynamic Probabilistic systems, John Wiley, New York Vol 1 and 2,
- [6]. Igboannugo, A. C. and Aigbe, S. O (2013). Markovian study of Maintenance Practice in a Production Firm-A Case Study. *Nigeria Journal of Technological Research* pg 24-28
- [7]. Pievatola A., Tironi E. & Valade I. (2004). Semi-Markov Processes for Power System Reliability Assessment with Application to Uninterruptible Power. *Journal and Magazines IEE Transactions on Power System Volume 19 Issue 3*
- [8]. Saba, T. M, Tsado, J, Bukar, B. & Raymond, E. (2016). Influence of socio-demographic Variables on Electrical Energy Management Practices among Residential of Niger
- [9]. Thomas, M. W. Jörn, V. and Jörn, H. (2006). Markov State Model for Optimization of Maintenance and Renewal of Hydro Power Components. In Bertling, L. (ed.), *PMAPS2006, Stockholm*, 11-15, Stockholm: KTH.
- [10]. Udo, B. (2016) Nigeria's energy crises worsens: only 5 of 23 power plants functional NERC Premium Times, Tues. June 14
- [11]. Vulpe A., Corausu A (2004). Stochastic Evaluation of Availability for Subsystem by Markov and Semi-Markov Model 13th World Conference on Earth Quake Engineering