ISSN No:-2456-2165

A Mapping Scheme for Mapping from Subsets of Complex Matrix Spaces Characterized by a Given set of Global Mass and Alignment Factors to the set of Hermitian, Positive Definite and Positive Semi Definite, Unit Trace, Complex Matrices of order 2

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Abstract:- The present research article discuss a mapping scheme from the subsets of the complex Matrix space $M_{m\times n}(C)$ characterized by set of Global Mass factor and Global Alignment factor, denoted as $\bar{M}(r,c)$, to the subset of the Matrix space $M_{2\times 2}(C)$ which consist of all Hermitian, unit trace, positive definite and positive semi definite matrices of order 2, denoted as $\hat{S}[M_{2\times 2}(C)]$. The Mathematical formalism is presented and illustrated through appropriate Numerical examples.

Keywords:- Global Mass Factor of a Matrix, Global Alignment Factor of a Matrix, Effective Global Mass Factor of a Matrix, Hermitian Matrices, Positive Definite Matrices, Positive Semi definite Matrices.

Notations

- $M_{m\times n}(C)$ denotes the Complex Matrix space of Matrices of order m by n
- $M_{2\times 2}(C)$ denotes the Complex Matrix space of Matrices of order 2
- $R(A)$ denotes the Global Mass Factor associated with the matrix $A_{m\times n}$
- $C(A)$ denotes the Global Alignment Factor associated with the matrix $A_{m\times n}$
- $R_0(A)$ denotes the Effective Global Mass Factor associated with the matrix $A_{m\times n}$
- $|c|$ denotes the modulus of the complex number c
- $\{|e_1\rangle, |e_2\rangle, ..., |e_m\rangle\}$ denotes the standard Orthonormal basis in C^m and $\{|f_1\rangle, |f_2\rangle, ..., |f_n\rangle\}$ denotes the standard Orthonormal basis in C^n
- $c^\bullet$ denotes the complex conjugate of the complex number c
- $R^3(R)$ denotes the Real vector space of all real 3-tuplets
- $|\vec{\varepsilon}|$ denotes the magnitude of the real vector $\vec{\varepsilon}$, $\vec{\varepsilon} \in R^3(R)$
- X^H denotes the Hermitian conjugate of the matrix X
- $\bar{M}(r,c)$ Is a subset of the Complex Matrix space $M_{m\times n}(C)$, characterized by the numerical values of the Global Mass factor and Global alignment factor, r and c, respectively.
- $I_{2\times 2} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \Sigma_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \Sigma_2 = \begin{bmatrix} 0 & -i \\ +i & 0 \end{bmatrix}, \Sigma_3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$, $\Sigma_1, \Sigma_2, \Sigma_3$ are the Pauli Matrices of order 2
- $|V\rangle = \begin{bmatrix} v_1 \\ v_2 \\ . \\ . \\ v_p \end{bmatrix}_{p\times 1}, \langle V| = [v_1^\bullet \ \ v_2^\bullet \ \ . \ \ . \ \ v_p^\bullet]_{1\times p}, B = \left[b_{ij}\right]_{p\times p}, \langle V|B|V\rangle = \sum_{i=1}^{p}\sum_{j=1}^{p} b_{ij} v_i^\bullet v_j$

I. INTRODUCTION

The present research article attempts to study the utility of subsets $\bar{M}(r,c)$ of the complex matrix space $M_{m\times n}(C)$ which are characterized by the ordered pair corresponding to the Global Mass and Global Alignment factors, in context of the mathematical structures of relevance in Quantum Information theory and related disciplines. In this article a Mapping scheme is presented that allow us to associate every element of the given subset $\bar{M}(r,c)$ with a Hermitian, unit trace, Positive

ISSN No:-2456-2165

definite/Semi definite matrix belonging to the subset $\hat{S}[M_{2\times2}(C)]$ of the Matrix space $M_{2\times2}(C)$, which has interpretation as Quantum state (Density Matrix) description of a Qubit in Quantum Information theory. This Mapping scheme is so formulated that the individual matrix elements, through their modulus and phase factors play a composite role in determining the Global and Effective Global mass factors and Phase term interrelationships, which in turn determine the diagonal and off diagonal elements of the mapped matrix array belonging to the Matrix space $M_{2\times2}(C)$.

II. MATHEMATICAL FRAMEWORK AND ASSOCIATED ANALYSIS

The following results, stated in [1], are used to provide the groundwork for the formalism described in this research article:

- $A \in M_{m\times n}(C), A = \sum_{i=1}^{m}\sum_{j=1}^{n} a_{ij}\left|e_i\right\rangle\left\langle f_j\right|$, $a_{ij} = r_{ij}c_{ij}$, $r_{ij} = \left|a_{ij}\right|$, $c_{ij} \in C$, $\left|c_{ij}\right| = 1$, we consider the following convention that in the case of zero matrix elements of matrix A: $a_{ij} = 0 \Rightarrow r_{ij} = 0, c_{ij} = 1$
- $R(A) = \sum_{i=1}^{m}\sum_{j=1}^{n} r_{ij}$,

  $R_0(A) = \sum_{i=1}^{m}\sum_{j=1}^{n} r_{ij}(1-\exp(-r_{ij}))$
- $C(A) = c_{11}c_{12}....c_{1n}c_{21}c_{22}....c_{2n}.........c_{m1}c_{m2}....c_{mn} = \prod_{i=1}^{m}\prod_{j=1}^{n} c_{ij}$, $C(A) \in C, \left|C(A)\right| = 1, \forall A \in M_{m\times n}(C)$
- $\bar{M}(r,c) \subset M_{m\times n}(C), \bar{M}(r,c) = \{A \in M_{m\times n}(C) \mid A \neq 0_{m\times n}, R(A) = r, C(A) = c\}$ where we have the condition: $r > 0, c \in C, \left|c\right| = 1$
- $\lambda_{ij} = (\frac{r_{ij}}{r_0})(1-\exp(-r_{ij}))$, where $r_0 = \sum_{i=1}^{m}\sum_{j=1}^{n} r_{ij}(1-\exp(-r_{ij}))$, is the numerical realization of the Effective Global Mass factor $R_0(A)$
- $\mu_{ij} = (\frac{r_{ij}}{r})$, where $r = \sum_{i=1}^{m}\sum_{j=1}^{n} r_{ij}$, is the numerical realization of the Global Mass factor $R(A)$
- $\lambda_{ij} \geq 0, \forall i = 1,2,..m;\ j = 1,2,....,n$ and we have : $\sum_{i=1}^{m}\sum_{j=1}^{n} \lambda_{ij} = 1$
- $\mu_{ij} \geq 0, \forall i = 1,2,..m;\ j = 1,2,....,n$ and we have : $\sum_{i=1}^{m}\sum_{j=1}^{n} \mu_{ij} = 1$

We define the following:

- $\alpha_i = c_{i1}c_{i2}.....c_{in}$, $\beta_j = c_{ij}c_{2j}.....c_{mj}$, Therefore, $\alpha_i\beta_j = c_{i1}c_{i2}.....c_{in}c_{ij}c_{2j}.....c_{mj}$ $i = 1,2,..m;\ j = 1,2,....,n$, we have: $\alpha_i\beta_j \in C, \left|\alpha_i\beta_j\right| = 1, \forall i = 1,2,....m;\ j = 1,2,....,n$
- $\left|\theta_{ij}\right\rangle = \begin{bmatrix} \sqrt{\lambda_{ij}}(\alpha_i\beta_j) \\ \sqrt{1-\lambda_{ij}}(\alpha_i\beta_j)^{\bullet} \end{bmatrix}$,

  $\left|\varphi_{ij}\right\rangle = \begin{bmatrix} \sqrt{\mu_{ij}}(\alpha_i\beta_j) \\ \sqrt{1-\mu_{ij}}(\alpha_i\beta_j)^{\bullet} \end{bmatrix}$, therefore, we have the following:

$\left\langle\theta_{ij}\middle|\theta_{ij}\right\rangle = 1$, $\left\langle\varphi_{ij}\middle|\varphi_{ij}\right\rangle = 1$,

$$\left|\theta_{ij}\right\rangle\left\langle\theta_{ij}\right| = \begin{bmatrix} \lambda_{ij} & \sqrt{\lambda_{ij}(1-\lambda_{ij})}(\alpha_i\beta_j)(\alpha_i\beta_j) \\ \sqrt{\lambda_{ij}(1-\lambda_{ij})}(\alpha_i\beta_j)^{\bullet}(\alpha_i\beta_j)^{\bullet} & (1-\lambda_{ij}) \end{bmatrix}$$

,

$$\left|\varphi_{ij}\right\rangle\left\langle\varphi_{ij}\right| = \begin{bmatrix} \mu_{ij} & \sqrt{\mu_{ij}(1-\mu_{ij})}(\alpha_i\beta_j)(\alpha_i\beta_j) \\ \sqrt{\mu_{ij}(1-\mu_{ij})}(\alpha_i\beta_j)^{\bullet}(\alpha_i\beta_j)^{\bullet} & (1-\mu_{ij}) \end{bmatrix}$$

Where $i = 1,2,..m;\ j = 1,2,....,n$

- $\hat{S}[M_{2\times2}(C)] \subset M_{2\times2}(C)$, such that :
- $\hat{S}[M_{2\times2}(C)] = \{Q \in M_{2\times2}(C) \mid Q^H = Q, Trace(Q) = 1, \left\langle v\right|Q\left|v\right\rangle \geq 0, or, \left\langle v\right|Q\left|v\right\rangle > 0, \forall \left|v\right\rangle \in C^2, \left|v\right\rangle \neq 0_{2\times1}$
- $\rho = (\frac{1}{2})\sum_{i=1}^{m}\sum_{j=1}^{n}\mu_{ij}\left|\theta_{ij}\right\rangle\left\langle\theta_{ij}\right| + (\frac{1}{2})\sum_{i=1}^{m}\sum_{j=1}^{n}\lambda_{ij}\left|\varphi_{ij}\right\rangle\left\langle\varphi_{ij}\right|$, therefore, we have the following :

$\rho = \begin{bmatrix} \rho_{11} & \rho_{12} \\ \rho_{21} & \rho_{22} \end{bmatrix}$, where we have:

$$\rho_{11} = \sum_{i=1}^{m}\sum_{j=1}^{n}\lambda_{ij}\mu_{ij} \ , \ \rho_{22} = 1-\rho_{11} = 1-(\sum_{i=1}^{m}\sum_{j=1}^{n}\lambda_{ij}\mu_{ij})$$

$$\rho_{12}=(\frac{1}{2})\sum_{i=1}^{m}\sum_{j=1}^{n}[\mu_{ij}\sqrt{\lambda_{ij}(1-\lambda_{ij})}+\lambda_{ij}\sqrt{\mu_{ij}(1-\mu_{ij})}](\alpha_i\beta_j)(\alpha_i\beta_j)$$

$$\rho_{21}=\rho_{12}^{\bullet}=(\frac{1}{2})\sum_{i=1}^{m}\sum_{j=1}^{n}[\mu_{ij}\sqrt{\lambda_{ij}(1-\lambda_{ij})}+\lambda_{ij}\sqrt{\mu_{ij}(1-\mu_{ij})}](\alpha_i\beta_j)^{\bullet}(\alpha_i\beta_j)^{\bullet}$$

Clearly, $\rho\in\hat{S}[M_{2\times2}(C)]$

- The complete mapping scheme is described in terms of the Transformation $\hat{\Lambda}$:

$\hat{\Lambda}:\bar{M}(r,c)\mapsto\hat{S}[M_{2\times2}(C)]$, such that:

$$\hat{\Lambda}(\sum_{i=1}^{m}\sum_{j=1}^{n}r_{ij}c_{ij}\left|e_i\right\rangle\left\langle f_j\right|)=$$

$$\begin{bmatrix}\sum_{i=1}^{m}\sum_{j=1}^{n}\lambda_{ij}\mu_{ij} & (\frac{1}{2})\sum_{i=1}^{m}\sum_{j=1}^{n}[\mu_{ij}\sqrt{\lambda_{ij}(1-\lambda_{ij})}+\lambda_{ij}\sqrt{\mu_{ij}(1-\mu_{ij})}](\alpha_i\beta_j)(\alpha_i\beta_j) \\ (\frac{1}{2})\sum_{i=1}^{m}\sum_{j=1}^{n}[\mu_{ij}\sqrt{\lambda_{ij}(1-\lambda_{ij})}+\lambda_{ij}\sqrt{\mu_{ij}(1-\mu_{ij})}](\alpha_i\beta_j)^{\bullet}(\alpha_i\beta_j)^{\bullet} & 1-(\sum_{i=1}^{m}\sum_{j=1}^{n}\lambda_{ij}\mu_{ij})\end{bmatrix}$$

Numerical Examples

1) $A_{2\times3}=\begin{bmatrix}1&0&0\\0&1&0\end{bmatrix}$, we have the following Numerical Results:

- $r(A)=2$, $c(A)=1$, $r_0(A)=2(1-\exp(-1))=1.2642$ .... (up to 4 decimal places)
- $\lambda_{11}=\frac{1}{2},\lambda_{12}=0,\lambda_{13}=0,\lambda_{21}=0,\lambda_{22}=\frac{1}{2},\lambda_{23}=0$
- $\mu_{11}=\frac{1}{2},\mu_{12}=0,\mu_{13}=0,\mu_{21}=0,\mu_{22}=\frac{1}{2},\mu_{23}=0$
- $\alpha_i\beta_j=1,\forall i=1,2,....,m;\ j=1,2,....,n$
- $\hat{\Lambda}(A_{2\times3})=\rho_{2\times2}=\begin{bmatrix}\frac{1}{2}&\frac{1}{2}\\\frac{1}{2}&\frac{1}{2}\end{bmatrix}=\left|+\right\rangle\left\langle+\right|$, where $\left|+\right\rangle=\frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix}$

2) $B_{2\times3}=\begin{bmatrix}2&0&0\\0&0&0\end{bmatrix}$, we have the following Numerical Results:

- $r(B)=2$, $c(B)=1$, $r_0(B)=2(1-\exp(-2))=1.7293$ .....(up to 4 decimal places)
- $\lambda_{11}=1,\lambda_{12}=0,\lambda_{13}=0,\lambda_{21}=0,\lambda_{22}=0,\lambda_{23}=0$
- $\mu_{11}=1,\mu_{12}=0,\mu_{13}=0,\mu_{21}=0,\mu_{22}=0,\mu_{23}=0$
- $\alpha_i\beta_j=1,\forall i=1,2,....,m;\ j=1,2,....,n$
- $\hat{\Lambda}(B_{2\times3})=\rho_{2\times2}=\begin{bmatrix}1&0\\0&0\end{bmatrix}=\left|0\right\rangle\left\langle0\right|$, where $\left|0\right\rangle=\begin{bmatrix}1\\0\end{bmatrix}$

3) $C_{2\times3}=\begin{bmatrix}-i&0&0\\0&+i&0\end{bmatrix}$, we have the following Numerical Results:

- $r(C)=2,c(C)=1,r_0(C)=2(1-\exp(-1))=1.2642$ .....(up to 4 decimal places)
- $\alpha_1\beta_1=-1,\alpha_1\beta_2=1,\alpha_1\beta_3=-i,\alpha_2\beta_1=1,\alpha_2\beta_2=-1,\alpha_2\beta_3=+i$
- $\lambda_{11}=\frac{1}{2},\lambda_{12}=0,\lambda_{13}=0,\lambda_{21}=0,\lambda_{22}=\frac{1}{2},\lambda_{23}=0$
- $\mu_{11}=\frac{1}{2},\mu_{12}=0,\mu_{13}=0,\mu_{21}=0,\mu_{22}=\frac{1}{2},\mu_{23}=0$
- $\hat{\Lambda}(C_{2\times3})=\rho_{2\times2}=\begin{bmatrix}\frac{1}{2}&\frac{1}{2}\\\frac{1}{2}&\frac{1}{2}\end{bmatrix}=\left|+\right\rangle\left\langle+\right|$

4) $D_{2\times3}=\begin{bmatrix}1&0&\frac{-i}{2}\\0&\frac{+i}{2}&0\end{bmatrix}$, we have the following Numerical Results:

- $u=(1-\exp(-1))=0.6321$ and $w=(\frac{1}{2})(1-\exp(-\frac{1}{2}))=0.1967$ .....(up to 4 decimal places)
- $r(D)=2,c(D)=1,r_0(D)=u+2w$
- $\lambda_{11}=(\frac{u}{u+2w}),\lambda_{12}=0,\lambda_{13}=(\frac{w}{u+2w}),\lambda_{21}=0,\lambda_{22}=(\frac{w}{u+2w}),\lambda_{23}=0$

- $\mu_{11}=\frac{1}{2},\mu_{12}=0,\mu_{13}=\frac{1}{4},\mu_{21}=0,\mu_{22}=\frac{1}{4},\mu_{23}=0$
- $\alpha_1\beta_1=-i,\alpha_1\beta_2=1,\alpha_1\beta_3=-1,\alpha_2\beta_1=+i,\alpha_2\beta_2=-1,\alpha_2\beta_3=1$
- $\hat{\Lambda}(D_{2\times3})=\rho_{2\times2}=\begin{bmatrix}\rho_{11} & \rho_{12}\\ \rho_{21} & \rho_{22}\end{bmatrix}=(\frac{1}{2})(I_{2\times2}+\varepsilon_1\Sigma_1+\varepsilon_2\Sigma_2+\varepsilon_3\Sigma_3)$

  , where we have the following:

$\rho_{11}=(\frac{2u+2w}{4u+8w})=0.404087$,

$\rho_{22}=(\frac{2u+6w}{4u+8w})=0.595913$,

$\rho_{12}=(\frac{w\sqrt{3}-u+\sqrt{uw+w^2}-\sqrt{2uw}}{4u+8w})=-0.094158$

,

$\rho_{21}=\rho_{12}^{\bullet}=(\frac{w\sqrt{3}-u+\sqrt{uw+w^2}-\sqrt{2uw}}{4u+8w})=-0.094158$

..... (up to 6 decimal places)

Eigenvalues of $\rho_{2\times2}$: 0.634407 , 0.365593 ..... (up to 6 decimal places)

$$\vec{\varepsilon}=\begin{bmatrix}\varepsilon_1\\ \varepsilon_2\\ \varepsilon_3\end{bmatrix}=(\frac{1}{2u+4w})\begin{bmatrix}w\sqrt{3}-u+\sqrt{uw+w^2}-\sqrt{2uw}\\ 0\\ -2w\end{bmatrix}$$

, $|\vec{\varepsilon}|=0.268813$, $\vec{\varepsilon}\in R^3(R)$

5) $E_{2\times3}=\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix}$, we have the following Numerical Results:

- $r(E)=6,c(E)=1,r_0(E)=6(1-\exp(-1))=3.7927$

  .....(up to 4 decimal places)
- $\lambda_{ij}=(\frac{1}{6})$, $\mu_{ij}=(\frac{1}{6})$, $\forall i=1,2,....,m;\ j=1,2,....,n$
- $\alpha_i\beta_j=1,\forall i=1,2,....,m;\ j=1,2,....,n$
- $\hat{\Lambda}(E_{2\times3})=\rho_{2\times2}=\begin{bmatrix}\frac{1}{6} & \frac{\sqrt{5}}{6}\\ \frac{\sqrt{5}}{6} & \frac{5}{6}\end{bmatrix}=\begin{bmatrix}\frac{1}{\sqrt{6}}\\ \frac{\sqrt{5}}{\sqrt{6}}\end{bmatrix}\begin{bmatrix}\frac{1}{\sqrt{6}} & \frac{\sqrt{5}}{\sqrt{6}}\end{bmatrix}$
- $\vec{\varepsilon}=\begin{bmatrix}\varepsilon_1\\ \varepsilon_2\\ \varepsilon_3\end{bmatrix}=\begin{bmatrix}\frac{\sqrt{5}}{3}\\ 0\\ \frac{-2}{3}\end{bmatrix}$, $|\vec{\varepsilon}|=1,\vec{\varepsilon}\in R^3(R)$

6) $F_{2\times3}=\begin{bmatrix}1 & +i & -i\\ -i & 1 & +i\end{bmatrix}$, we have the following Numerical Results:

- $r(F)=6,c(F)=1,r_0(F)=6(1-\exp(-1))=3.7927$

  .....(up to 4 decimal places)
- $\lambda_{ij}=(\frac{1}{6}),\mu_{ij}=(\frac{1}{6}),\forall i=1,2,....,m;\ j=1,2,....,n$
- $\alpha_1\beta_1=-i,\alpha_1\beta_2=+i,\alpha_1\beta_3=1,\alpha_2\beta_1=-i,\alpha_2\beta_2=+i,\alpha_2\beta_3$
- $\hat{\Lambda}(F_{2\times3})=\rho_{2\times2}=\begin{bmatrix}\frac{1}{6} & \frac{-\sqrt{5}}{18}\\ \frac{-\sqrt{5}}{18} & \frac{5}{6}\end{bmatrix}$

Eigenvalues of $\rho_{2\times2}$: 0.855729 and 0.144271

- $\vec{\varepsilon}=\begin{bmatrix}\varepsilon_1\\ \varepsilon_2\\ \varepsilon_3\end{bmatrix}=\begin{bmatrix}\frac{-\sqrt{5}}{9}\\ 0\\ \frac{-2}{3}\end{bmatrix}$, $|\vec{\varepsilon}|=0.711458,$

  $\vec{\varepsilon}\in R^3(R)$

III. DISCUSSION AND CONCLUSION

The Mapping scheme described in this research article maps a non-zero, complex Matrix belonging to the Matrix space $M_{m\times n}(C)$, to unit trace, Hermitian, Positive definite/semi definite

Matrices belonging to the matrix space $M_{2\times2}(C)$, such that both the modulus terms and the phase terms play intricate roles in determining the diagonal and off-diagonal elements of the mapped matrix of order 2, the modulus terms alone determine the Global mass factor and the Effective Global mass factor, together they determine the weightage terms λ_{ij}'s and μ_{ij}'s and thereby, the diagonal terms of the matrix $\rho_{2\times2}$. The off-diagonal terms, are determined by both the weightage terms λ_{ij} 's and μ_{ij} 's as well as the composite phase terms $\alpha_i\beta_j$'s, thus, the structure of the mapped matrix array is determined by the composite effect of the individual modulus term contributions and phase factor interrelationships.

In the Numerical examples 1 through 4, we observe that the matrices $A_{2\times3}, B_{2\times3}, C_{2\times3}, D_{2\times3}$ belong to the subset $\bar{M}(r=2, c=1)$ of the Complex Matrix space $M_{2\times3}(C)$, $A_{2\times3}$ and $C_{2\times3}$ are mapped to the same positive semi definite matrix even though they have different phase interrelationship structures, $B_{2\times3}$ is mapped to a positive semi definite matrix while $D_{2\times3}$ is mapped to a positive definite matrix.

In the Numerical examples 5 and 6, we observe that the matrices $E_{2\times3}$ and $F_{2\times3}$ belong to the subset $\bar{M}(r=6, c=1)$, However, owing to differences in their phase factor interrelationships we observe that both possess the same set of diagonal elements but different off-diagonal terms, such that $E_{2\times3}$ is mapped to a positive semi definite matrix while $F_{2\times3}$ is mapped to a positive definite matrix. In subsequent studies, the mapping formalism will be analyzed in more depth to understand its intricacies and its scope of applicability in real life Theoretical/Computational problems.

REFERENCES

Books:

[1]. Arfken, George B., and Weber, Hans J., *Mathematical Methods for Physicists*, 6th Edition, Academic Press
[2]. Biswas, Suddhendu, *Textbook of Matrix Algebra*, 3rd Edition, PHI Learning Private Limited
[3]. Hassani, Sadri, *Mathematics Physics A Modern Introduction to its Foundations*, Springer
[4]. Hogben, Leslie, (Editor), *Handbook of Linear Algebra*, Chapman and Hall/CRC, Taylor and Francis Group
[5]. Jordan, Thomas. F., *Quantum Mechanics in Simple Matrix Form*, Dover Publications, Inc.
[6]. Meyer, Carl. D., *Matrix Analysis and Applied Linear Algebra*, SIAM
[7]. Nakahara, Mikio, and Ohmi, Tetsuo, *Quantum Computing: From Linear Algebra to Physical Realizations*, CRC Press.
[8]. Rao, A. Ramachandra., and Bhimasankaram, P., *Linear Algebra*, 2nd Edition, Hindustan Book Agency
[9]. Sakurai, J. J., *Modern Quantum Mechanics*, Pearson Education, Inc.
[10]. Steeb, Willi-Hans, and Hardy, Yorick, *Problems and Solutions in Quantum Computing and Quantum Information*, World Scientific
[11]. Strang, Gilbert, *Linear Algebra and its Applications*, 4th Edition, Cengage Learning

Research Articles

[1]. Ghosh, Debopam, *A Matrix Property for Comparative Assessment of subsets of Complex Matrices characterized by a given set of Global Mass and Global Alignment Factors,* (Preprint) DOI: 10.13140/RG.2.2.26871.24484 (2020)
[2]. Aerts, Diederik, De Bianchi, Sassoli Massimiliano, *The extended Bloch representation of quantum mechanics and the hidden-measurement solution to the measurement problem,* Annals of Physics, Vol.351, p. 975-1025 (2014)
[3]. Dorai, Kavita, Mahesh, T. S., Arvind and Anil Kumar, *Quantum computation using NMR*, Current Science, Vol.79, No. 10, p 1447-1458 (2000)
[4]. Hardy, Lucien, *Quantum Theory from Five Reasonable Axioms*, arXiv: quant-ph/0101012v4 (2001)
[5]. Macklin, Philip A., *Normal matrices for physicists*, American Journal of Physics, 52, 513(1984)
[6]. Paris, Matteo G A, *The modern tools of Quantum Mechanics : A tutorial on quantum states, measurements and operations*, arXiv: 1110.6815v2 [quant-ph] (2012)
[7]. Roth, W. E., *On Direct Product Matrices*, Bull. Amer. Math. Soc., No. 40, p 461-468 (1944)
[8]. Uskov, D, Rau, P.R.A, *Geometric phases and Bloch-sphere constructions for SU(N) groups with a complete description of the SU(4) group*, Phys. Rev. A, 78, 022331 (2008)