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The Covid-19 Pandemic Crisis on the Brazilian Stock Exchange: An Application of the Markov Switching Dynamic Regression Model

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Abstract: **This article provides a quantitative analysis using the Markov Switching Dynamic Regression (MS-DR) model, in order to highlight the dynamics presented by Ibovespa during the period from January 2005 to December 2020, in which the subprime crisis occurred and the COVID-19 crisis started. In particular, it used two regimes (regime 1- low volatility and regime 2-high volatility) in the model so that the market parameters (Ibovespa) behave differently during economic crises with the regimes representative. The Ibovespa remained on regime 1 (low volatility) for three periods, totaling 186 months. In regime 2 (high volatility - 2008 and 2020 crises), it remained for about 6 months, that is, 4 months in the 2008 crisis and 2 months in the COVID-19 crisis. In addition, regime 1 is more persistent, that is, the probability of staying on this regime at a later period is approximately 98,38%, and that of switching to regime 2 is 45,11%. In regime 2, the probability of continuing this regime in the period t + 1 is 54,89%, while the probability of changing to regime 1 is 1,62%.**

Keywords: *Markov Switching Dynamic Regression, Covid-19 Pandemic, Brazilian Stock Exchange.*

I. INTRODUCTION

The econometric works on the estimation of regressions subject to regime changes that follow a Markov chain were developed by Quandt (1972), Goldfeld and Quandt (1973). Hamilton (1989) made important advances in the method developed by Goldfeld and Quandt (1973), by specifying that changes in regimes follow an auto-regressive process. In this sense, he developed a non-linear and smoothed estimation algorithm to find the high and low regimes of the economic series, seeking to maximize the likelihood function in relation to the parameters estimated in the model. This methodology allowed statistical inferences to be made about the different regimes not observed in the series. The model endogenously estimates the dates of the structural changes in the series. Hamilton (1989) applied the method to investigate the nonlinear behavior of the growth of the United States economy and the results showed that the model can be used as an important tool for measuring business cycles.

Hamilton and Susmel (1994) use a model with changes, with respect to volatility. According to the authors, the regime change model, applied to the returns of the American stock market, fits the data better than the ARCH models without regime change.

Ang and Bekaert (2002) applied using a non-linear model to interest rates in the USA, Germany and the United Kingdom. Thus, the authors showed that interest rate regimes correspond reasonably well with US economic cycles, being extremely important to study the effects of monetary policy shocks on the economy.

Ismail and Isa (2006) used regime change testing in their study to detect non-linear characteristics in the exchange rates of three Asian countries. They found that the null hypothesis of linearity is rejected and there is evidence of structural breaks in the exchange rate series.

Júnior and Zuanazzi (2014) tested the hypothesis of non-linearity of the sensitivity of the return on assets of companies from Rio Grande do Sul under different Markovian risk regimes: periods of crisis and stability. They considered three assets of Rio Grande do Sul companies tradable on the São Paulo Stock Exchange (Bovespa). The results showed that the non-linear model (MS-CAPM) is the most suitable. In addition, evidence that assets are more susceptible to macroeconomic changes in times of crisis than in periods of stability.

Mahjoub and Chaskmi (2019) applied the Markov Switching model with two regimes, to identify periods of speculative bubble formation and explosion in the Iranian capital market. Regimen 1 is bubble growth and the explosion stage and regime 2 identifies bubble loss. The result of the research shows that the stock index of the Iranian capital market in the analyzed period

Panda et al. (2017) examine the changing behavior of the dynamic Markov regime between the spot and the futures market in relation to interest rates in India. The study uses daily data on volumes, weighted average price, weighted average yield for the spot market and total values, open interest, settlement price from January 21, 2014 to October 30, 2014. All data come from Clearing Corporation of India Ltd. (CCIL) and the National Stock Exchange (NSE). The authors used regime change regression to capture the behavior of changes, as well as the estimated probability and estimated duration of each regime.

Peira and Soledad (2002) implemented a regime change framework to study speculative attacks against EMS currencies during 1979–1993. To identify speculative episodes, we model exchange rates, reserves and interest rates as time series subject to discrete regime changes between two possible states: "quiet" and "speculative". We allow the odds of switching between states to be a function of fundamentals and expectations. The regime change framework improves the ability to identify speculative attacks vis-à-vis the speculative pressure indices used in the literature. The results also indicate that fundamentals (mainly budget deficits) and expectations drive the likelihood of moving to a speculative state.

Ozdemir (2020) in his study is to assess the feed price driven dynamics of the U.S. wholesale beef prices in which regime switches are induced by transitions between Markov regimes. By allowing the transition probabilities to vary according to some main grain feed prices, we examine if the regime transition probabilities vary over time under two different states of the growth rate of beef prices as “low-mean growth” and “high-mean growth” price regimes. The results show that when the prices are in high-mean growth regime, the probability that it will remain in this regime is greater than that it will switch to low-mean regime. This findings also indicate that livestock feed prices provides some predicted power to the model of beef price regime switching process and supports livestock feed prices contributing to whether the beef price levels remains in low/high-mean regime. By employing Markov switching dynamic regression model, we also find that all types of the feed prices have a significant effect on the beef prices in low-growth regime, but only the prices of hay and sorghum significantly affect the beef prices in the high-growth regime.

Xaba et al. (2019) used a Markov-switching dynamic regression (MS-DR) model to estimate appropriate models for BRICS countries. The preliminary analysis was done using data from 01/1997 to 01/2017 and to study the movement of 5 stock market returns series. The study further determined if stock market returns exhibit nonlinear relationship or not. The purpose of the study is to measure the switch in returns between two regimes for the five stock market returns, and, secondly, to measure the duration of each regime for all the stock market returns under examination. The results proved the MS-DR model to be useful, with the best fit, to evaluate the characteristics of BRICS countries.

Choi and Hammoudeh (2010) use the Markov Switching model with two volatility regimes for the strategic commodity prices of Brent oil, WTI oil, copper, gold and silver, and the S&P 500 index, but with varying high-to-low volatility ratios. The dynamic conditional correlations (DCCs) indicate increasing correlations among all the commodities since the 2003 Iraq war but decreasing correlations with the S&P 500 index. The commodities also show different volatility persistence responses to financial and geopolitical crises, while the S&P 500 index responds to both financial and geopolitical crises.

Moolman (2004) found that Linear models are incapable of capturing business cycle asymmetries. This has recently spurred interest in non-linear models such as the Markov switching regime (MS) technique of modelling business cycles. The MS model can distinguish business cycle recession and expansion phases, and is sufficiently flexible to allow different relationships to apply over these phases. In this study, the South African business cycle is modelled using a MS model. This technique can be used to simultaneously estimate the data generating process of real GDP growth and classify each observation into one of two regimes (i.e. low-growth and high-growth regimes).

II. METHODOLOGY AND DATA

Markov Switching Dynamic Regression Model

Hamilton (1989) proposed MS that is based on the assumption that the development of X_t can be explained by states (or regimes), where a two regime Markov-switching regression model can be expressed as:

Regime 1: $Y_t = \mu_1 + \varphi Y_{t-1} + \varepsilon_t$

Regime 2 : $Y_t = \mu_2 + \varphi Y_{t-1} + \varepsilon_t$

where Y_t is the dependent variable,

μ_1 and μ_2 are the intercepts in each state (regime),

φ is the autoregressive coefficient and ε_t is the error at time t.

In the case where the state (regime) shifts are known, the two regime Markov-switching model can expressed as:

$$Y_t = S_t\mu_1 + (1 - S_t)\mu_2 + \varphi Y_{t-1} + \varepsilon_t$$

where S_t represents the regime and is equal to 1 if the process is in regime 1 and 2 if it is in regime 2. However, in most cases it is not possible to observe in which regime S_t the process is currently in and therefore unknown. In Markov-switching regression models the regime S_t follows a Markov chain. A model with k regime-dependent intercepts, can be expressed as:

$$Y_t = S_t\mu_{st} + \varphi Y_{t-1} + \varepsilon_t$$

Where $\mu_{st} = \mu_1, \mu_2, \ldots\ldots\ldots, \mu_k$ for $S_t = 1, 2, \ldots\ldots.., k$ regimes.

The transition of probabilities between the regimes is carried out by a first order Markov process as follows:

$$\rho_{ij} = \Pr\left(S_t = j\right) | s_{t-1} = i)$$

On what ρ_{ij} refers to the probability of being on the regime *j* given that the process is in the regime *i*, where $\sum_{i=1}^{N} \rho_{ij} = 1$ for all $i, j \in (1,2,......N)$.

The transition probabilities in a square matrix of order *N*, known as the transition matrix and denoted by *P*, have the following form:

$$P = \begin{bmatrix} \rho_{11} & \rho_{12} \\ \rho_{21} & \rho_{22} \end{bmatrix}$$

where

$$\rho_{11} = P[s_t = 1; s_{t+1} = 1]$$
$$\rho_{12} = P[s_t = 2; s_{t+1} = 1]$$
$$\rho_{21} = P[s_t = 1; s_{t+1} = 2]$$
$$\rho_{22} = P[s = 2; s_{t+1} = 2]$$
$$\rho_{11} + \rho_{12} = 1 \quad \text{e} \quad \rho_{21} + \rho_{22} = 1$$

Thus, it is assumed that the transition matrix is irreducible and unconditional (if one of the values of the transition matrix is equal to the unit and all other eigenvalues are within the unit circle). With these conditions, there is a stationary probability distribution of the regimes (Krolzig, 1997). Unconditional probabilities can be determined as follows:

$$\rho_1 = (1 - \rho_{11})/(2 - \rho_{11} - \rho_{22})$$
$$\rho_2 = (1 - \rho_{22})/(2 - \rho_{11} - \rho_{22})$$

The probability of being in regime 1 in equilibrium is obtained by ρ_1 and the probability of being in regime 2 is determined by ρ_2.

In estimating the model, the joint distribution y_t and S_t relative to past information is used:

$$f(y_t, S_t | Y_{t-1}) = f(y_t | S_t, Y_{t-1}) f(S_t | Y_{t-1})$$

Where Y_{t-1} represents all information included in the history of the time dependent variable $t-1$ e $f(y_t | S_t, Y_{t-1})$ is the conditional normal density function for the regime $S_t = j$.

The maximum likelihood estimator is used to determine the parameters of the MS-DR. Therefore, the probability function of the model log with two regimes is expressed as follows:

$$\ln L = \sum_{t=1}^{T} \ln \left\{ \sum_{j=1}^{2} f(y_t | S_t, y_{t-1}) \Pr(S_t = j | Y_{t-1}) \right\}$$

Where the term $\Pr(S_t = j | Y_{t-1})$ is the probability of being in each regime. Given away $\Pr(S_{t-1} = i | Y_{t-1})$, i = 1,2 at the beginning of time t, the probabilities of being in each regime are obtained as follows:

$$\Pr(S_t = j | Y_{t-1}) = \sum_{i=1}^{2} \Pr(S_t = j | S_{t-1} = i) \Pr(S_{t-1} = i | y_{t-1})$$

,

where $\Pr(S_t = j | S_{t-1} = i)$, j = 1,2 ; i = 1,2 are transition probabilities of the elements of matrix P, considered constant. The probability of being in one regime or another, can be changed through macroeconomic performance and information obtained from the stock market.

Being Y_t observed at the end of the period of period t, the probabilities are updated using the following equation:

$$\Pr(S_t = j | Y_t) = \frac{f(y_t | S_t = j, Y_{t-1}) \Pr(S_t = j | Y_{t-1})}{\sum_{j=1}^{2} f(y_t | S_t = j, Y_{t-1}) \Pr(S_t = j | Y_{t-1})}$$

,

where $f(y_t | S_t = j, Y_{t-1})$ s the probability density function of a distribution for the regime $S_t = j$.

Finally, from the transition matrix it determines the expected duration of each regime. The closer the probability is to one, the longer it takes to switch from another regime. Thus the expected duration can be expressed as:

$$Expected \;\; duration(D_i) = \frac{1}{1 - \rho_{ij}}$$

The duration time in each of the two regimes can be determined as:

$$D_1 = 1/(1 - \rho_{11}) \qquad D_2 = 1/(1 - \rho_{22})$$

In the view of Doornik (2013) the Markov-switching models can be MS-AR (Markov-switching autoregression) and MS-DR (Markov-switching dynamic regression). The first is characterized by a more gradual adjustment, appropriate to the most stable series, whose autoregressive component is formed by the difference between the lagged endogenous variable and the average estimated for the endogenous variable in the S_{t-1} regime; and the second adjusts immediately to the new regime, with a more accentuated transition, since the autoregressive component covers only the endogenous variable.

In the present article, the series data are monthly, which chose to use the MS-DR model as an estimation method to identify regime changes, the number of periods, the duration and the probability of transition from one regime to another.

The MS-DR model can be specified as:

$$y_t = v(S_t) + \alpha y_{t-1} + \varepsilon_t, \qquad \varepsilon_t \sim N[0, \sigma^2]$$

Doornik (2013) adds that the MS-DR model with a structural component is important for analyzing time series that present alternations of values in the mean and variance.

Linearity Test (BDS)

Once it is detected that the distribution is not normal, it is necessary to test the model for linearity. This test was developed by Brock, Dechert, and Scheinkman (1987) used to test if the random variables that compose a series are independent and identically distributed (IID), that is, it can verify several situations in which the variables are not IID, such as non-stationarity, nonlinearity and deterministic chaos. The test is based on the concept of spatial correlation of chaos theory and according to the authors the BDS statistic is formulated through the Equation:

$$W_m^n(\varepsilon) = \frac{\sqrt{N}\,(C_m^n(\varepsilon) - (C_1^n(\varepsilon))^m}{\sigma_m(\varepsilon)}$$

Where $W_m^n(\varepsilon)$ it converges to a normal distribution N (0, 1) as n tends to infinity.

Thus, hypothesis tests are:

H_0: the series follows an iid (independent and identically distributed) process.

H_1: the series does not follow an iid (independent and identically distributed) process.

Data

The data used in this study refer to the monthly Bovespa indices, covering the period from January 2005 to December 2020, in a total of 192 monthly observations. The data were obtained from the Yahoo finance website.

III. EMPIRICAL RESULTS

Preliminary Analysis

The daily returns were calculated using the formula: $r_t = \ln(P_t) - \ln(P_{t-1})$. This P_t represents the number of points at closing on day t and P_{t-1} the number of points at closing on the previous day (t-1). Figures 1 and 2 show the behavior of the Ibovespa daily quotation and return series in the period considered.

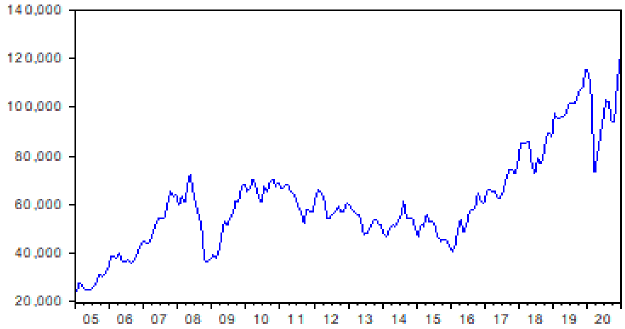

Figure 1. Ibovespa monthly quotes (points)

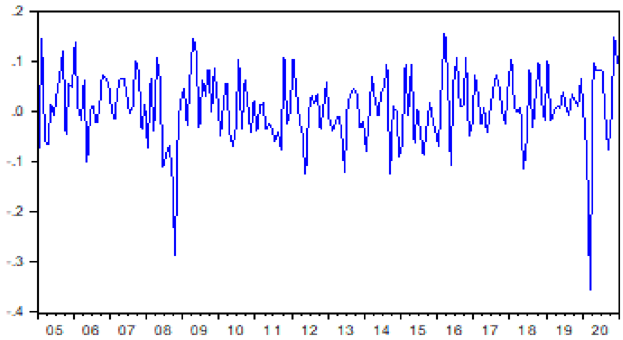

Figure 2. Ibovespa monthly returns.

In the visual inspection of Figure 2, within the analysis period, there is a marked volatility in returns. Thus, it was necessary to test the normality and stationarity of the Ibovespa returns series for application of the MS-DR model.

Some basic descriptive statistics are presented in Table 1. It can be observed that the monthly returns of the Ibovespa present a leptocurtic distribution due to the excess of kurtosis (7,178233) in relation to the normal distribution (3.0), that is, it has heavier tail. It is also verified that the series is negatively asymmetrical which would indicate that stock market lows are more likely than market highs. The analysis of the results shows that both the mean (0.007904) and the median (0.007561) presented values close to zero. The variation between the minimum value (-0,355310) and the maximum value (0.156724) shown by the series can be explained due to some significant oscillations in the index returns. The low value of the standard deviation (0.069264) indicates that, in general, the high variations in the series occurred in a few occasions, that is, in periods of positive and negative peaks. The statistics of Jarque - Bera (1987) indicated the rejection of the normality of the distribution of the series, with p-value equal to zero.

Table 1. Statistical summary of Ibovespa returns

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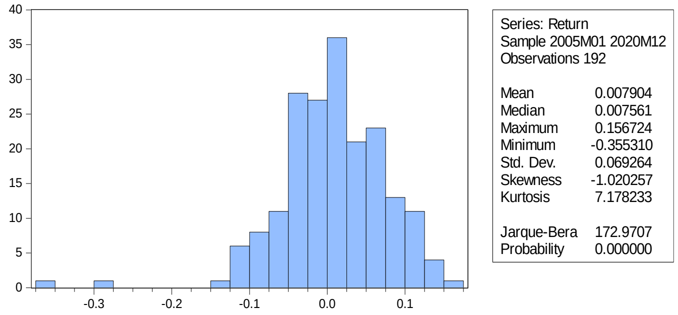

The Q-Q Plot represents one of the most used graphic methods to verify the normality of time series. The procedure used consists of graphically comparing the theoretical amounts of the normal distribution with the amounts of the sample data. Figure 3 shows a non-linear relationship between the theoretical and empirical quantiles, which is quite pronounced in the tails of the distributions, indicating heavier tails in the empirical distribution. Therefore, all tests rejected the hypothesis of normality of the analyzed series.

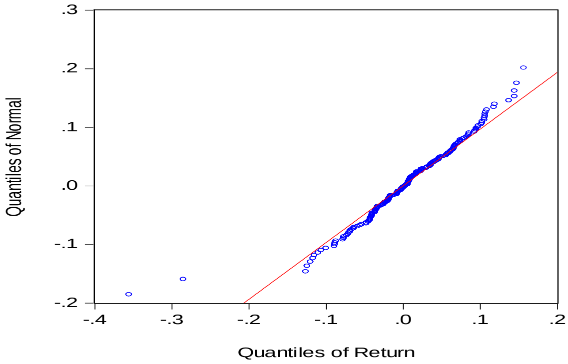

Figure 3. Plot Q-Q of Ibovespa returns.

The Dickey and Fuller (1981); Phillips and Perron (1988); tests and Kwiatkowski, Phillips, Schmidt, and Shin (1992) tests with constant and trend, identified that the Ibovespa returns series are stationary and do not contain unitary roots, as presented in the Table 2.

Table 2. Stationary test for the Ibovespa returns series.

Variable	ADF	Critical value (5%)	PP	Critical value (5%)	KPSS	Critical value (5%)
Ibovespa	-11,9060	-3,4335	-11,8551	-3,4335	0,0929	0,1460

Before the estimation of the Markov Switching Dynamic Regression (MS-DR) model, a nonlinearity test may be necessary to describe the characteristics of the historical series of the Ibovespa returns. Thus, in Table 3 shows that the results presented indicate the nonlinearity effect, that is, that the probabilities are less than 5% at the significance level, implying a rejection of the null hypothesis that the returns series is linearly dependent.

Table-3. Test to the time independence of the Ibovespa (BDS)

Dimension	BDS Statistics	Statistics Z	Probability
2	0,0091	1,9940	0,0407
3	0,0142	1,9962	0,0453
4	0,0205	2,3353	0,0195
5	0,0217	2,3698	0,0178
6	0,0229	2,6011	0,0093

Source: Prepared by the author based on the research.

Markov-switching dynamic regression (MS-DR) model

Table 4 shows the model estimates using the maximum likelihood method, using the OxMetrics 6.0 software (PcGive14). The adjusted model refers to the MS (2) -DR , the mean and variance change according to the state regime. The regime (1) expresses a positive average of the Ibovespa returns together with a low volatility. In regime (2), it shows a negative average result and high volatility in Ibovespa returns. In regime 1, the estimated average monthly return is 1,34% with a variance of 0,059. The regime 2 identifies a negative average monthly return of -13,5% with a variance of 0,128.

In the Markovian regime change model, it was possible to identify a regime with negative returns and high variance (high volatility or low market) and another regime with positive returns less variance (low volatility or high market).

Portmanteau indicate that there is no presence of autocorrelation of residues. The results of the ARCH-LM tests suggest the acceptance of the model homoscedasticity hypothesis. As for the normality tests Jarque-Bera (1987) does not reject the hypothesis of normality. Thus, the model presents a positive diagnosis and an adequate adjustment demonstrated in the results of the various tests carried out in the present study.

In the transaction and persistence matrix of the regimes, it appears that the current regime 1 is more persistent, that is, the probability of remaining in this regime in a later period is approximately 98,4%, and that of changing to regime 2 is on the order of 45,11%. In regime 2 the probability of continuing in this regime in the period t + 1 is 54,89%, while the probability of switching to regime 1 is 1,62%. Thus, for the period from January 2005 to December 2020, the expected duration of the current regime 1 is 62 months. In regime 2, the estimated duration is 3 months. The unconditional probability in periods of low volatility is 96,88% and 3,12% in periods of high volatility.

Table 4. Estimation of the MS(2)-DR model.

Regime 1 (low volatility)		Regime 2 (high volatility)	
Parameter	Coefficient	Parameter	Coefficient

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$\mu(s_1)$	0,01345	(0,00463)***	$\mu(s_2)$	-0,13482	(0,06562)**
σ^2	0,05912	(0,00328)***	σ^2	0,12841	(0,04294)***
ρ_{11}	0,9838	(0,01259)***	ρ_{12}	0,5489	(0,30610)*
Descriptive statistics					
Log-*likelihood*	254.7942				
Linearity test $(\chi^2)(4)$	28.246 (0,0000)[1]				
Normality test (χ^2)	2,2278 (0,3283)[1]				
ARCH test (1-1)	0,05895 (0,8084)[1]				
Permanteau test - $\chi^2(36\,lags)$	36.3870 (0,4506)[1]				

Transition probability matrix			Average duration period of regimes		
	Regime 1	Regime 2		Unconditional probability	Duration period
Regime 1	0,9838	0,0162	Regime(1)	0,9688	62
Regime 2	0,4511	0,5489	Regime(2)	0,0312	3

Notes: ***, **, * denote statistical significance at the 1%, 5%, 10% , respectively.
Standard errors are in parentheses. p value (1).
Source: Prepared by the author based on the research.

Figure 4 shows the behavior of the series of indices, returns, smoothed and predicted probabilities for the Ibovespa state 1 and 2 regimes. The upper panel presents the series of Ibovespa returns, and the middle and lower panels trace the smoothed probabilities for the market in regime 1 (low volatility) and regime 2 (high volatility), respectively.

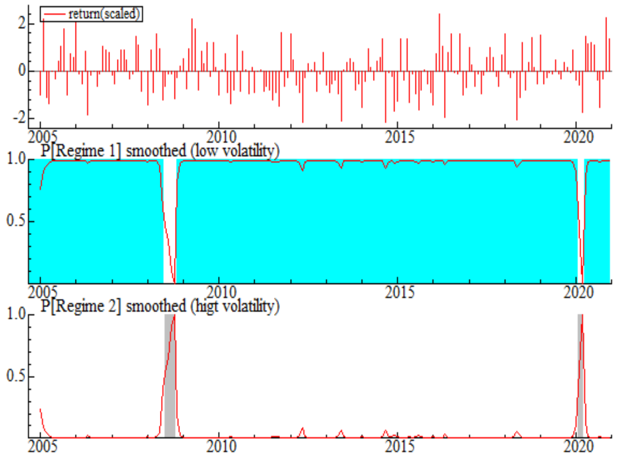

Figure 4. Smoothed probabilities of regimes 1 and 2 obtained from the MS(2)-DR model for Ibovespa returns in the period from January 2005 to December 2020.

From the estimated probabilities, the specific dates of the low volatility (1) and high volatility (2) regimes can be obtained, which are shown in Table 5. The Ibovespa remained under the low volatility regime for three periods, totaling 186 months. In the high volatility regime (crises of 2008 and 2020), Ibovespa remained for about 6 months, that is, 4 months in the crisis of 2008 and 2 months in the crisis of 2020 (period from February 3 to March 31).

Table 5 - Specific dates of the regimes: MS(2)-DR model

Regime 1 (low volatility)			**Regime 2 (high volatility)**		
Period	**Months**	**Probability**	**Period**	**Months**	**Probability**
2005(1) - 2008(6)	42	0,976	2008(7) - 2008(10)	4	0,770
2008(11) - 2020(1)	135	0,990	2020(2) - 2020(3)	2	0,780
2020(4) - 2020(12)	9	0,978			

Source: Prepared by the author based on the research.

In the first period of crisis, beginning in September 2008, there was a significant drop in the Bovespa index, caused by the subprime crisis triggered by the bankruptcy of one of the US investment banks, Lehman Brothers, triggering a crisis in the stock exchanges international standards. After the bank's bankruptcy, the shares started to price an economic crisis, with a strong exit of foreign investors from Brazil. The Ibovespa had a reduction of approximately 60% in 3 months, and it took 14 months for its recovery with the same value before the crisis, after government economic measures. In the second period of crisis, beginning in January 2020, Ibovespa had a negative impact due to the covid19 pandemic, which has been generating strong turbulence in world markets and isolation policies to contain the pandemic progress, reflecting on the economy the effects of the shutdown of several economic activities (commerce, industry, aviation and tourism).

The pandemic crisis of the new coronavirus affected the Brazilian economy still fragile, which had not fully recovered from the recession from 2014 to 2016. The historical fall in Brazil PIB in the second quarter with a retraction of 5.5% (negative growth) was pulled by the industry. The sector decreased 12.3% in relation to thefirst quarter, that is, deepened by the transformation industry, which registered a decrease in the activities of car manufacturers, textile industries and machinery and equipment factories.

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IV. CONCLUSION

The objective of the study was to analyze the changes in Ibovespa returns between January 2005 and December 2020, using the Markov-switching dynamic regression (MS-DR) model.

In the adjusted model, the mean and variance are modified according to the state regime. The regime (1) expresses a positive average of the Ibovespa returns together with low volatility. In regime (2), it shows a negative average result and high volatility in Ibovespa returns. In regime 1, the estimated monthly average return is 1,34% with a variance of 0,059. Regime 2 identifies a negative average monthly return of -13,48% with a variance of 0,128.

In early January 2020, the Ibovespa had a negative impact due to the covid-19 pandemic, which has been generating strong turbulence in world markets and isolation policies to contain the pandemic's progress, reflecting in the economy the effects of the paralysis of several economic activities (commerce, industry, aviation and tourism). Although the downward trend of the stock exchanges is a pattern observed worldwide due to the effects of the covid-19 pandemic, it can justify the sharp percentage of the fall of the Brazilian stock exchange, when compared to other countries, the mass migration of the capital invested in Brazil for US securities and gold, considered safer in times of crisis.

The sharp crisis in the oil sector, which took shape at the beginning of March 2020, caused a drop of 31% in the prices of the commodity in Asian markets. The effects in Brazil can be measured by the devaluation 54,4% of Petrobras preferred stock prices between March 2 and April 1, 2020. In this way, because it has an economy strongly dependent on the export of commodities, among them, oil, and Brazil suffers more significant financial falls than other more developed countries and with less dependence on capital from exports.

The excessive and simultaneous devaluation of Brazilian stocks, reflected in the expressive fall of the Ibovespa, is largely due to pessimistic future expectations, especially in the macroeconomic scenario, as well as the specific situation of each company in its industry affected by the covid-19 pandemic. In the matrix of transaction and persistence of the regimes, it appears that the current regime 1 is more persistent, that is, the probability of remaining in this regime in a later period is approximately 98,38%, and that of moving to regime 2 is on the order of 45,11%. In regime 2, the probability of continuing this regime in the period t + 1 is 54,89%, while the probability of changing to regime 1 is 1,62%. Thus, for the period from January 2005 to December 2020, the expected duration of the current regime 1 is 62 months. In regime 2, the estimated duration is 3 months.

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