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Advanced Mathematical Modeling of 3-Dimensional Solids

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ABSTRACT

This paper discusses the mathematical modeling of 3-Dimensional objects. Ananalytical description of a 3D object (in this case, a pear) is modeled in a suitable way. We will be comparing the volumes of the pear via two methods-the water displacement method and mathematical modeling with the help of polynomial regression after which we will graph it in 3-Dimensional coordinates. Having done so, we can then calculate the percentage error margin with the theoretical and experimental volume.

INTRODUCTION

In this paper I will calculate the volume and surface area of a pear-an irregular-object using calculus. First, I will find out the volume of the pear by immersing it in a gradu-ated beaker containing water and then after slicing it in half I will trace the longitudinal section of the pear on a graph paper. Once all the coordinates are noted down, they will be input into a graphing software and polynomial regression will be carried out to find the equation that best fits these coordinates and matches the shape of the pear. Once an appropriate function is found, the volume and surface area of the pear will be calculated using volumes of revolution (integral calculus). Post this, the results will be compared to the original findings and the entire investigation will be evaluated and limitations will be reflected upon.[1]

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Step : 1

A pear (*Pyrus*) is selected as the fruit used for this experiment since it contains pre dominant lines of symmetry, and is exhibits axial symmetry which allows it to be modelled around the x-axis. When sliced in half, the cross-section of the pear can be easily traced on a graph and a Cartesian function could be modelled after its general shape using polynomial regression, which is essential for the calculations in this investigation.

In fluid mechanics, a fluid is displaced when an object is largely immersed in it. An

((a)) 400mlwithoutpear ((b)) 540ml with pear

Fig. 1: Water-displacement method

Object that sinks displaces an amount of fluid equal to the object's volume. This allows the water displacement method to be used to experimentally determine the volume of even irregular solids.

The pear is placed inside a graduated beaker that contains 400ml of water. After submerging the pear in the water, the level then rises to 540ml.

It is important to note that due to the lockdown restrictions, proper equipment could not be obtained and a measuring cylinder of least count 0.1l (or $10cm^3$) had to be used.

Thus, based on the water displacement method,

$Volume_{Pear}$=Final level of water - Initial level of water in cm^3

$\therefore Volume_{Pear} = 540ml - 400ml = 140ml = 140cm^3$Thus, experimental volume of the pear = $140cm^3$

Step: 2

The pear is cut into 2 pieces exactly at the middle through its axis of symmetry. The halves were similar enough to not cause any issues during the modelling, and the pear was cut finely so that the half could lay flat on a graph paper.

Fig. 2: Pear Sliced in Half

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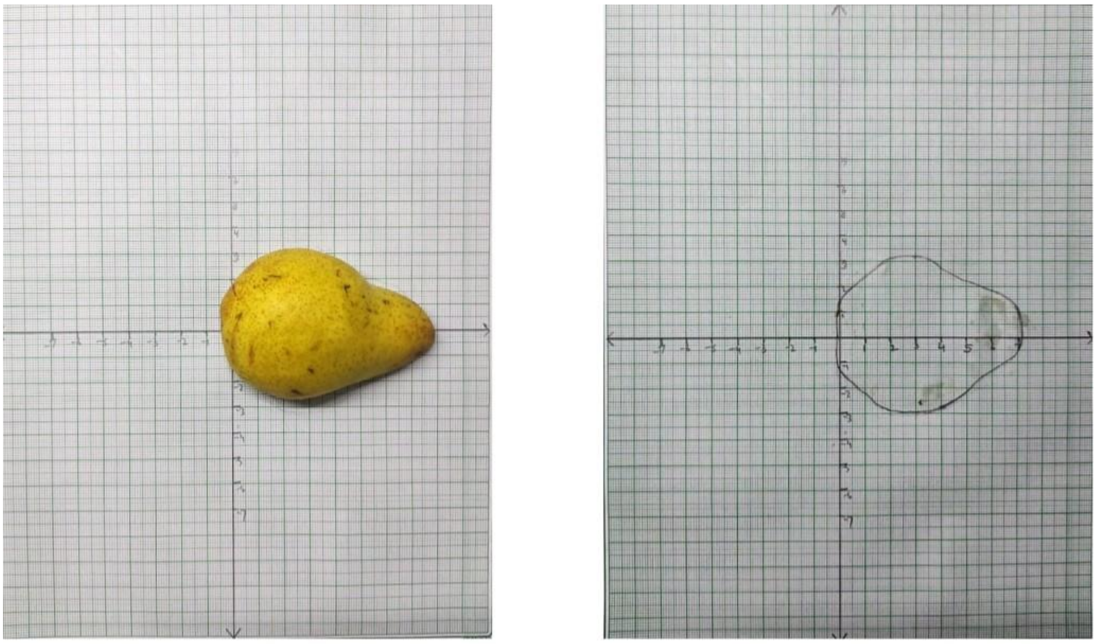
Step: 3

One of the halves of the pear was then placed on a sheet of graph paper and was outlined against a graph with a scale of 1 cm = 1 unit on both the x-axis andy-axis.

The pear was dried at first to ensure that the graph paper does not get affected. Further more, the outline was highlighted after the pear was traced so that it shows up when the graph is scanned

The Pear half was placed in a manner such that the x-axis would coincide with the axis of symmetry of the fruit.



((a)) Pair placed on the graph ((b)) Outline of pear

Fig. 3: The outline is traced on a graph paper

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Step : 4

The traced graph is imported into the online graphing software ’Desmos’ and the coordinates are mapped. The image was then scaled in order to ensure that the x-axis and y-axis of the image and the graph on ’Desmos’ align accurately.

To ensure the function that is found in the later sections passes the vertical line test, the coordinates of only the upper quarter of the pear are mapped in the graphing software.

17 points aligning with one half of the graph, above the x-axis, were then plotted with high accuracy through zooming into the imported image and taking values accurate to 0.1cmonthey-axisandx-axis.Thesevalueswherethentabledandconsolidatedtogive an outline of the required Cartesian function above thex-axis.[2]

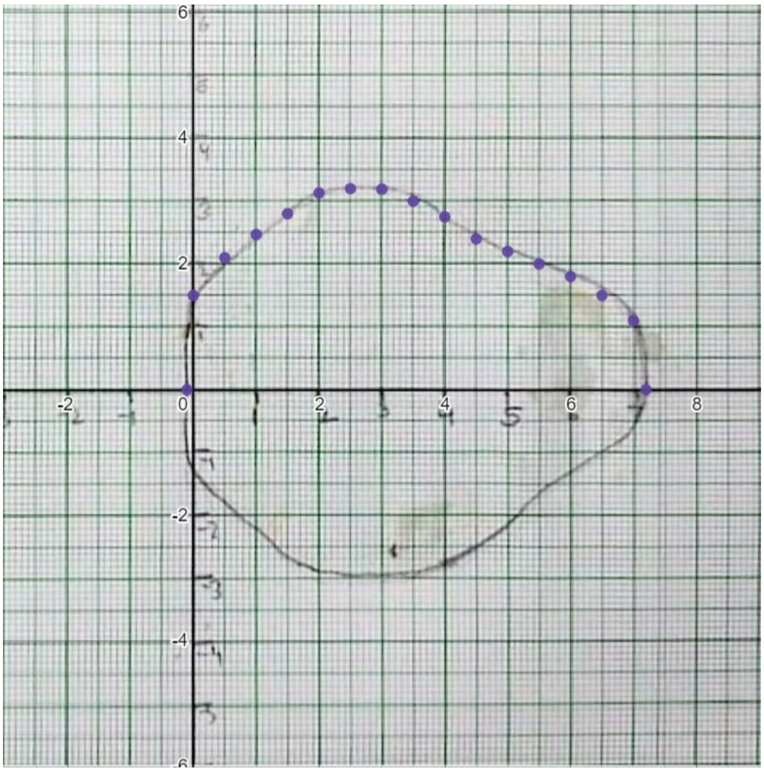


Fig. 4: Mapped coordinates of thepear

Table of coordinates	
x	y
-0.1	0
0	1.5
0.5	2.3
1	2.5
1.5	2.8
2	3.1
2.5	3.2
3	3.1
3.5	2.9
4	2.7
4.5	2.4
5	2.2
5.5	2
6	1.8
6.5	1.5
7	1.1
7.2	0

Table of coordinates mapped on top of the traced pear which will then be used to model polynomial regression functions of the outline of the pear.

Note that the points chosen were closer together near the ends of the half in order to ensure that that the high-magnitude slopes are modelled accurately enough.

Step : 5

Using polynomial regression, the best-fit model for the coordinates of the pear was calculated for the following degrees as shown below.

Best Fit Equation		
Degree	Polynomial Equation	R^2
1	$-1.09766x + 2.47306$	0.07
2	$-1.7500x^2 + 1.1215x + 1.3024$	0.81
3	$0.0261x^3 - 0.4509x^2 + 1.8414x + 1.0527$	0.87
4	$-0.0149x^4 + 0.2356x^3 - 1.3655x^2 + 3.0701x + 0.8971$	0.93
5	$-0.0004x^5 - 0.0075x^4 + 1.9012x^3 - 1.2540x^2 + 2.9802x + 0.9021$	0.93
6	$-0.0031x^6 + 0.0655x^5 - 0.5313x^4 + 2.0896x^3 - 4.2901x^2 + 4.5742x + 0.9371$	0.96

Table 1: Polynomial Regression up to 6th degree

R^2 is the statistical measure to deter- mine the proportion of the variance in the dependent variable that is predictable from the independent variable

As can be seen above, the 6th degree polynomial has the greatest accuracy with an R^2 value of 0.96.

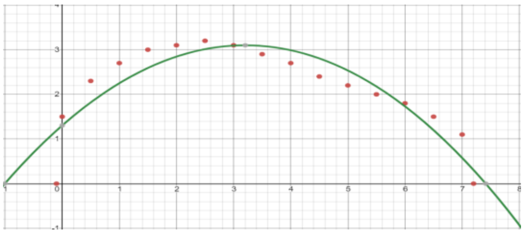
Thus, the best-fit function obtained was:

$y = -0.0030976x^6 + 0,0655217x^5 - 0.5313x^4 + 2.08966x^3 - 4.2901x^2 + 4.5742x + 0.937168$

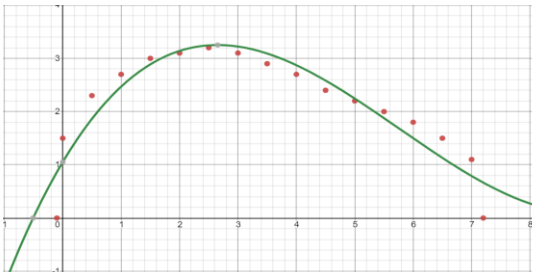
where $x \in R \mid x \in [-0.174, 7.227]$

It should also be noted that the function has not been rounded to 3 s.f. because that would cause drastic changes to the generated graph.

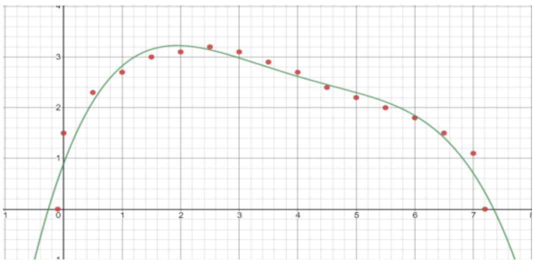
((a)) 1st degree polynomial



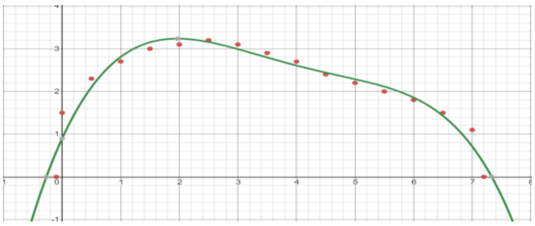
((b)) 2nd degree polynomial



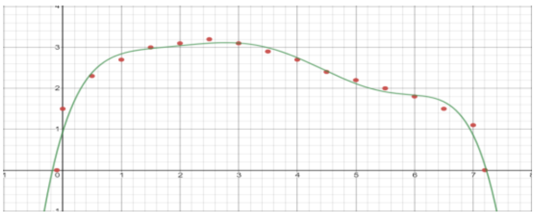
((c)) 3rd degree polynomial



((d)) 4th degree polynomial



((e)) 5th degree polynomial



((f)) 6th degree polynomial

Fig. 5: Polynomial Graphs

Step : 6

In order to find the volume of the pear using the function, we rotate a slice of the function about the x-axis. Now, Volume of the slice is given by

$$V_{slice}= \pi(f\,(x))^2dx$$

This slice of a function, which is the cross-sectional area of the graph within a rangeis found based on the formula for the area of a circle, where the radius of said circle is instead replaced by the desiredfunction.

When a function is rotated about the x axis, based on the formula of volumes of revolution, its volume is calculated by the formula where a and b refer to the domain in which the function lies:

$$V_{total}= \int_a^b \pi(f\,(x))^2dx$$

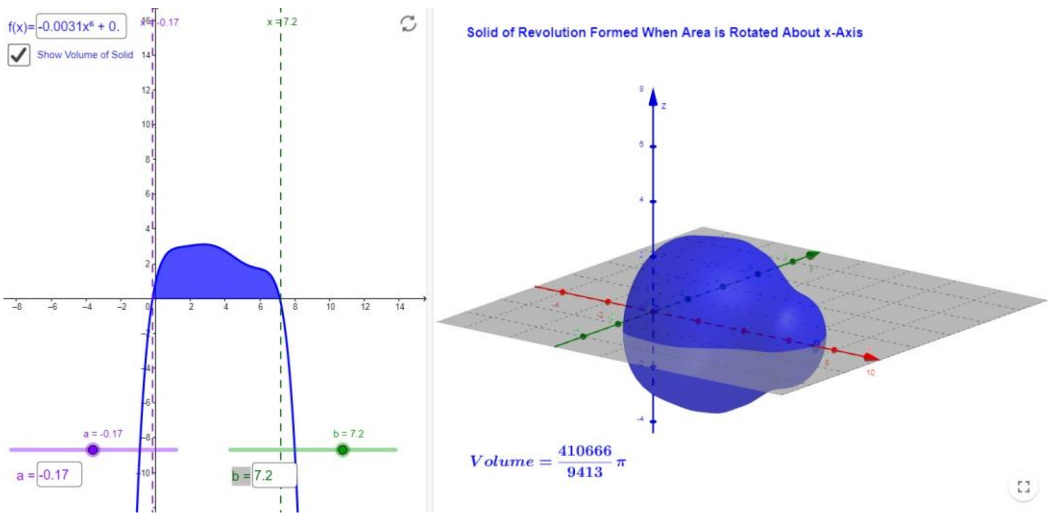


Fig. 6: The same function graphed in 3 dimensional coordinates

Which is the surfacedenoted by=S={ $y^2+z^2=(\mathbf{f}(x))^2; x\in[-0.174,7.227]$

$$V_{total}= \int_{-0.174}^{7.227} \pi(-0.0030976x^6+0.0655217x^5-0.5313x^4+2.08966x^3-4.29x^2+4.5742x+0.937168)^2dx$$

Thus, $V_{total}= 137.064cm^3$

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Step : 7

The surface area of revolution about the x-axis is given by the formula:

$$S = \int_a^b 2\pi f(x) ds \tag{1}$$

where a and b is the domain between which the function is defined and ds is a surface differential which is denoted using

$$ds = \sqrt{1+\left(\frac{df(x)}{dx}\right)^2} dx \tag{2}$$

Which gives,

$$S = \int_a^b 2\pi f(x) \sqrt{1+\left(\frac{df(x)}{dx}\right)^2} dx \tag{3}$$

Thus,

$$S = \int_{-0.174}^{7.227} 2\pi f(x) \sqrt{1+\left(\frac{df(x)}{dx}\right)^2} dx \tag{4}$$

where,

$$f(x) = -0.0030976x^6+0,0655217x^5-0.5313x^4+2.08966x^3-4.29x^2+4.5742x+0.937168$$

Thus, Surface Area of the Pear = $106.468 cm^2$

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Step : 8

The Volume of the pear obtained through the water displacement method was 140cm3 and the Volume obtained through the volume of solids of revolution method was 138.064cm3 Thus,

Theoretical Value=137.064cm3Experimental Value=140cm3

In order to compare the results, we calculate the percentage error margin using the following formula

$$Percentage\ Error=\frac{|Theoretica\ lvalue–Experimental\ Value|}{Experimental\ Value}\ 100\times$$

$$=\frac{|137.064-140|}{140}\ 100\times$$

$$= 2.10\%$$

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LIMITATIONS

There are several limitations in the modelling. These are listed below.

- When the volume of the pear was calculated using the water displacement method, the uncertainty for the method was 10%. This add as considerable margin of error.
- The pear was cut into 2 halves and one of the halves was outlined on the graph paper. However, due to the lack of complex machinery, there may have beenslight variations in the size of the halves and they may not have been completely symmetric.
- The volume and surface area formulas are based on slight approximations, which might lead to slight changes from the actual value of the volume and surfacearea. This is because the pear is not perfectlysymmetric.
- The regression analysis to find a best-fit model naturally has some inaccuracy which is evident through the R^2values for the polynomials.

AREAS OF IMPROVEMENT

Some ways through which some of the limitations may be averted are

- If a more accurate beaker with a lower least count would be used, a more accurate experimental volume would have been obtained through the water displacement method
- The scan may be imperfect and this can be corrected through a scanner machine to ensure that the alignment is at exactly 0degrees.
- A much higher R^2value could be found by using a higher power for the regression polynomial modelling. Other regression methods like exponential and trigonometric modeling were also used but it was observed that polynomial regression yielded the highest accuracy.
- The pear could be sliced into smaller sections that could be individually modelled, yielding better polynomial regression equations. However, the piece wise function obtained should be differential at the end points of each individual sections model.

CONCLUSION

As we can see from the calculations done above, we can say that the method of mathematical modeling of pear, a 3D object is a suitable method for calculating the volume, with the percentage error being only 2.10%

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[2.]"3DModelingasMethodforConstructionandAnalysisofGraphicObjects," IOP Conference Series, vol. 262, pp. i –22, 2017.