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Two-Dimensional Description of Charged Plasma Particle in Electromagnetic Fields of Static Electric and Magnetic Field Potentials

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Abstract:- An analytic approach was used to investigate the two-dimensional motion of charged matter particles of electron plasma in electromagnetic fields. In this study, the Lagrange equation for the total energy of a system of electron plasma particles, with kinetic energy function of the particle moving with a velocity, and a potential energy function derived from the Lorentz force of an electric field scalar and magnetic field vector potentials. The velocity and acceleration of the electron plasma specie were obtained from the equation describing the path of the plasma particle in the *x* and *y* axis. An erratic-looking oscillatory motion of the velocity and acceleration components, with irregularities in the amplitude and wavelengths of their motion as they drift, is shown in the profile of results presented. Our result further shows that the electric field is directly responsible for the amplitude of these chaotic oscillatory wave motion exhibited by electron plasma in electromagnetic field. While the magnetic field influences absolutely the irregularities in the amplitude of the wavelengths motion and the damping of the oscillatory motion as the magnetic field strength increases.

Keywords:- *Plasma, Electromagnetic fields, Lorentz force, Lagrange Equation.*

I. INTRODUCTION

Plasma is generally described as a gaseous mixture of electrons, ions and some fraction of neutral atoms, which generates an electric field from bare electrons and ions, and magnetic field from the currents generated from the motion of these charged particles [1]. Where there is an externally applied electric and magnetic fields, we then consider the total electromagnetic fields of the plasma system. This electromagnetic fields further generates an induced force called the Lorentz force, which drives the particles of plasma to a velocity through the fields [15]. considering the potential energy of the electromagnetic field and the kinetic energy of the moving charged plasma particle, the Lagrange equation relates the generalized coordinates and velocities of this charged plasma particle to the total energy of the system of charged plasma particles.

This study confirmed and present the type of motion exhibited, and the possible trajectories of the motion of charged plasma particle in *x* and *y*-axes, which helps to create a better physical picture and understanding of the dynamics of plasma single particle motion in electric and magnetic field with different fields strength and configuration, as also emphasized in [11].

There is a vast amount of literatures on charged plasma particle motion in electromagnetic field, which has revealed how this study has been approached experimentally, and a number of theoretical methods which include computational, analytic, iteration etc. that has also been used to investigate the problem. In a major advances in 2010, [8] investigated the ions response to external electromagnetic fields, where he selected some signal regimes based on the plasma frequency and showed that electric field is shielded out for low plasma frequency without the total internal electric field been altered, and at frequency near the plasma frequency a resonant behavior was observed which comes with a large internal electric field and induced current. A full orbital simulation of plasma specie in a tokamak with strongly sheared electric field, revealed a strong confinement effect of highly ionized impurity in the presence of sheared electric field as presented by [13]. [6] considered the symmetry of the motion of charged particle in a time independent electromagnetic field, where they established a relationship between the symmetry describing the particle's motion and the symmetry of the magnetic field lines. [7] suggested a model for the dynamics of charged plasma particle in a tokamak, the model shows a region where charged particles are confined near the magnetic field lines of an ABC (stationary solutions of the force free type of magnetohydrodynamic equations) fields. Also, [4] investigated the propagation and reflection of ions of some selected noble gases through a barrier created by a magnetic field of non-uniform configuration and without radial electric field, the reflection was shown to depends on the initial transverse energies and masses of these ions, and results further shows that the model can be used to avoid losses of ions from particle's source. [12] studied the effect of chaotic scattering of the combined gravitational field, and an asymptotically uniform magnetic field on charged particles acceleration. There findings reveal that charged particles are strongly accelerated to a speed comparable to the speed of light preferably along the magnetic field lines. [1] also highlighted the importance of the study of single particle motion in understanding of plasma and its applications, he went further to present the various drift motion charged particle's experience for inhomogeneous, curved and time-independent magnetic field configuration. [2] exerted that according to the magnetic field setup considered in their study, the path traced by positively charged particle is free as

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they drift in the direction of the current, and shows that its velocity does not dependent on the angle of projection. [5] considered an arrangement of magnetic field with cylindrical symmetry, and was able to identify a plane in a direction parallel to the field, and is bounded by a curve from the rotational motion with a constant angular speed along a circular orbit of a large radius.

To the best of our knowledge, after an extensive review of literatures as regards to plasma single particle motion in electromagnetic fields, no investigation of the parameters considered in this study using this approach has been carried out and presented. In this study the total energy of a plasma particle moving under the influence of a Lorentz force of an electric field scalar, and magnetic field vector potentials function was used to develop a mathematical model for the system of electron plasma particles.

The aim of this research work is to broaden the understanding of the motion of plasma single particle motion in electromagnetic field.

The outline of this paper is as follows. our mathematical model is obtained and present in section 2. In section 3, the profile of results obtained was presented and discussed. Finally, we presented our conclusions in section 4.

II. FORMALISM

We considered particles of electron plasma that are driven and directed by the Lorentz force as they enter into the region of space that is entirely dominated by the electric and magnetic forces acting perpendicularly from their respective fields. The field equations as summarized in [20] are the Gauss equation for magnetism and the Faradays equation of changing electric current stated below.

$$\nabla \cdot B = 0 \quad 1$$

$$\nabla \times E = -\frac{\partial B}{\partial t} \quad 2$$

The equation (1) shows that the fields who's divergent vanishes is defined as the curl of a vector potential function A [16], i.e.

$$B = \nabla \times A \quad 3$$

In the region of space that is dominated by the vector potential A, the Faraday's equation as in [20] can be stated and further simplified from the vector relation to

$$E = -\nabla\boldsymbol{\phi} - \frac{\partial \boldsymbol{A}}{\partial \boldsymbol{t}} \quad 4$$

The Lorentz force acting on a charged plasma particle with charge e, moving with a velocity v is given as

$$F = e(E + v \times B) \quad 5$$

where E is the electric field intensity and B is the magnetic induction vector.

The scalar potential $\boldsymbol{\phi}$ and the vector potential A, respectively expressed in terms of the electric field E and magnetic field B in equation (5) is given by

$$F = e(-\nabla\phi - \frac{\partial A}{\partial t} + v \times \nabla \times A) \quad 6$$

The equation (6) is the Lorentz force resulting from the interaction of electric field scalar function and a magnetic field vector function driving the charged plasma particle through a velocity.

Considering a plasma particle with a velocity dependent potential functions, if the kinetic energy T and potential energy V are expressed in the form

$$T = {}^{1}\!/_{2}\, mv^2 \quad 7$$

$$V = e(\phi - \nabla \cdot A) \quad 8$$

where m is the mass of the particle and v *is* the velocity of the particle. For a charged plasma particle moving with a characteristic kinetic energy potential, in an electromagnetic field with its own potential energy function, we used the relevant Lagrange equation [10, 3]

$$L(r, \dot{r}, t) = T - V \quad 9$$

For a plane of infinite and homogeneous symmetry, because of its invariance in translation, the Euler's-Lagrange equation of motion [3] for a free particle with a position vectors r, can be resolved into the x and y component of the equation of motion

$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{x}_i}\right) - \frac{\partial L}{\partial x_i} = 0 \quad 10$$

And

$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{y}_i}\right) - \frac{\partial L}{\partial y_i} = 0 \quad 11$$

i = 1, 2, 3… n

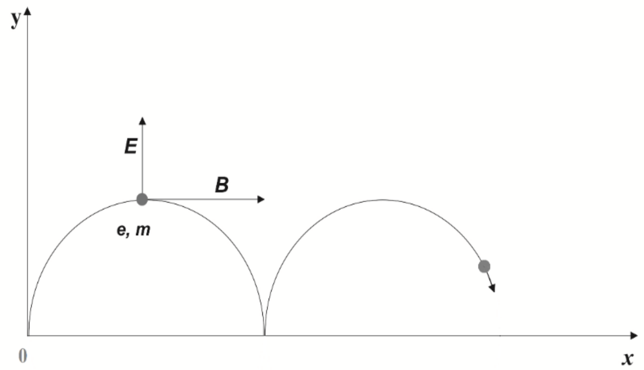

Fig 1: Physical model and geometry of the charged plasma particle.

A charged matter particle of electron plasma with mass m and charge e moving under the influence of uniform electric and magnetic fields which are mutually orthogonal, relative to a fixed frame. If the fields are expressed by $E = Ej$ and $B = Bk$, and the plasma particle is at the origin as shown in the figure 1. Then the scalar and vector potentials which yield the fields under investigation using equations 3 and 4 are

$$\phi = -E_y \qquad 12$$

$$A = {}^1\!/_2\, B(-y_i + x_j) \qquad 13$$

recalling equation 9, the Lagrange equation in two dimensions yield

$$L = \frac{m}{2}(\dot{x}^2 + \dot{y}^2) + eE_y + \frac{1}{2}eB(x\dot{y} + y\dot{x}) \qquad 14$$

Applying equations 10 and 11 respectively, we obtain

$$m\ddot{x} - eB\dot{y} = 0 \qquad 15$$

$$m\ddot{y} + eB\dot{x} = 0 \qquad 16$$

B is the magnetic field, e is the electron matter particle and m is the mass of the particle, $\dot{x}$ velocity in the x coordinate, and $\dot{y}$ velocity in the y coordinate, $\ddot{x}$ acceleration in the x coordinate and $\ddot{y}$ is the acceleration on the y coordinate with the initial conditions

$$x(0) = \dot{x}(0) = 0 \qquad 17$$

$$y(0) = \dot{y}(0) = 0 \qquad 18$$

III. METHOD OF SOLUTIONS

For the x and y directions, we assume a solution of the form

$$(x, y) = e^{\lambda t} \qquad 19$$

where λ is a constant.

The solution assumed in equation 19 is substituted into equation 15 and 16 and we obtain the characteristic equation

$$m^2\lambda^4 + e^2B^2\lambda^2 = 0 \qquad 20$$

Which gives the following eigenvalues

$$\lambda^2 = 0 \ or \ \frac{-e^2B^2}{m^2} \qquad 21$$

The trajectory of the particle is given as

$$x = \frac{E}{B}t - \frac{mE}{eB^2}\sin\frac{eB}{m}t \qquad 22$$

$$y = \frac{mE}{eB^2}\left[1 - \cos\frac{eB}{m}t\right] \qquad 23$$

The x and y components of the velocity are given by

$$v_x = \frac{\partial x}{\partial t} = \frac{E}{B} - \frac{E}{B}\cos\frac{eB}{m}t \qquad 24$$

$$v_y = \frac{\partial y}{\partial t} = \sin\frac{eB}{m}t \qquad 25$$

And the corresponding acceleration in the two-dimensions are

$$a_x = \frac{\partial^2 x}{\partial t^2} = \frac{eE}{m}\sin\frac{eB}{m}t \qquad 26$$

$$a_y = \frac{\partial y^2}{\partial t^2} = \frac{eE}{m}\cos\frac{eB}{m}t \qquad 27$$

IV. RESULTS AND DISCUSSIONS

The problem of the motion of charged plasma particles in electromagnetic fields is investigated and the effects of this changing electric field intensity and magnetic field strength on the x- velocity and acceleration components, y – velocity and acceleration component are examined, and their results presented graphically in Figs. 2 – 26. The profile of results was obtained from the nonlinear equation involving B, E, e, m, and t, developed from a mathematical model for the descriptive analysis of the time dependent motion of the electron plasma species in electromagnetic field. The model which is made-up of some independent variables whose numerical values at every instant determines the *x* and *y* component velocities and accelerations of the constituent particle of the electron plasma, and some other variables whose constant value also influences the motion and behavior of these plasma species in electromagnetic field. The graphical profiles of results were obtained using the Mathematica Computational Software. For physically realistic values for the electron's charge and mass corresponding to -1.602x10^{-19}C and 9.11x10^{-31}kg respectively and for some selected range of values of the electric field strength (i.e. E = 1.0, 3.0, 5.0, 7.0NC^{-1}), magnetic field strength (i.e. B = 1.5, 2.5, 3.5, 4.5T). the following profiles are presented below.

The time series of the effect of magnetic field on the *x* – velocity component of the motion of electron plasma in electromagnetic field for the selected values of electric field is shown in figure 2 – figure 4. They show a chaotic and erratic-looking oscillatory motion with the same irregular pattern for all the trends corresponding to the selected electric field values, which emanates from the origin of the axis and spread to maintain a coherent and persistent chaotic oscillatory motion throughout the selected time frame. The various trends which corresponds to the increasing values of the electric field from bottom to top of figures, indicates that the amplitude of this chaotic oscillatory motion exhibited by the charged electron plasma increases linearly as presented in figure 6, as the electric field intensity increases. Comparing the profiles of result presented in figure 2, figure 3 and figure 4 and figure 5, for magnetic field values corresponding to 1.5T, 2.5T, 3.5T and 4.5T respectively, reveals that the chaotic patterns of the oscillatory motion, and the decrease as shown in figure 7, on the *x* – velocity axis is determined by the magnetic field.

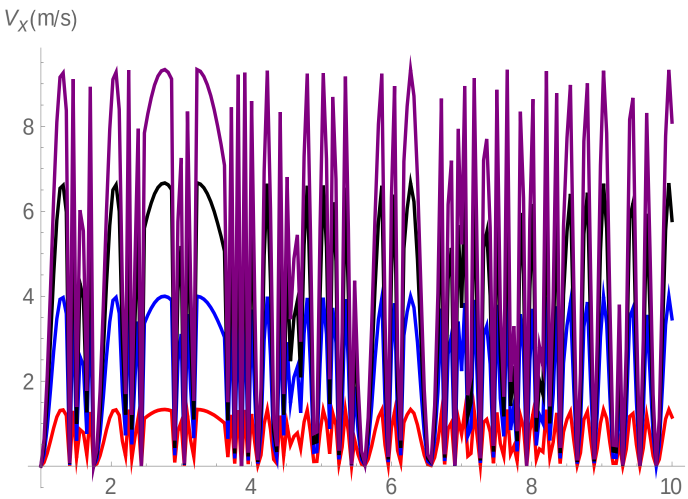

Fig 2: The dependence of Time(t) on the Velocity(m/s) of electron plasma for different values of electric fields.

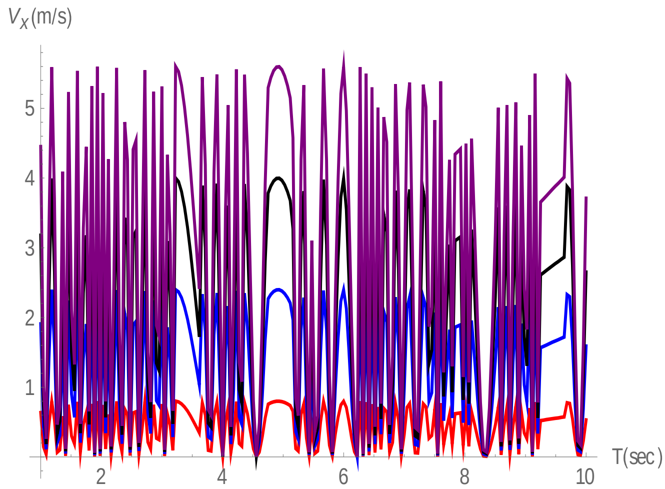

Fig 3: The dependence of Time(t) on the Velocity(m/s) of electron plasma for different values of electric field.

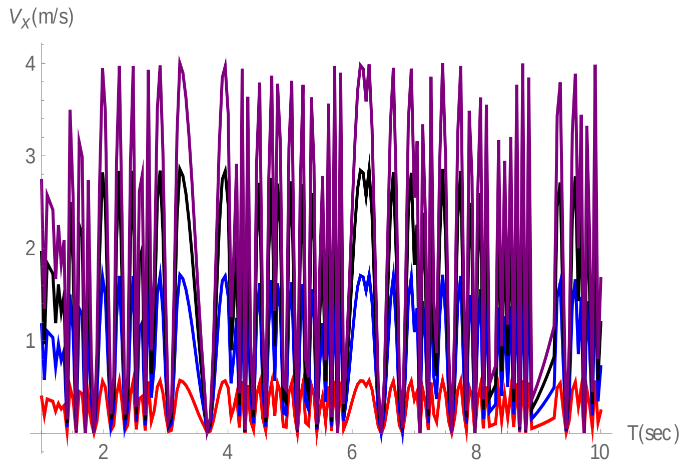

Fig 4: The dependence of Time(t) on the Velocity(m/s) of electron plasma for different values of electric field.

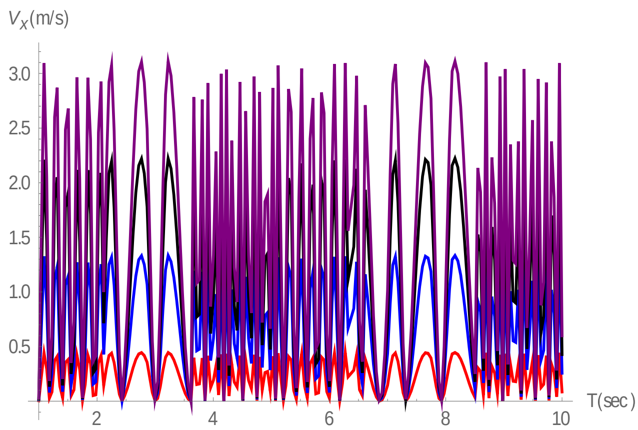

Fig 5: The dependence of Time(t) on the Velocity(m/s) of electron plasma for different values of electric field.

The time series of the effect of magnetic field on the *x* – component acceleration of electron plasma in electromagnetic fields for the selected values of the magnetic field strength is presented in fig 8 - 11. They show an erratic-looking oscillatory motion characterized by irregular oscillatory pattern in all the trends whose motion emanates from the origin and extend to the negative and positive sides of the vertical x – acceleration axis. Each trend of the accelerating electron plasma in the figures, which represents the selected values of the electric field, increases linearly from bottom to top according to the linear relationship presented in the figure 12 and figure 13, for increasing value of the selected electric field intensity on both sides of the vertical axis. Also, by comparing the profiles of figure 8, figure 9, figure 10, and figure 11, for magnetic field values of 1.5T, 2.5T, 3.5 and 4.5T respectively, it was revealed that the oscillatory structure exhibited by each of the figure is determined by the strength of the magnetic field. Also, the constant maximum amplitude observed in these figures show that the dampen effect of the magnetic induction did not extend to the acceleration of charged plasma particles.

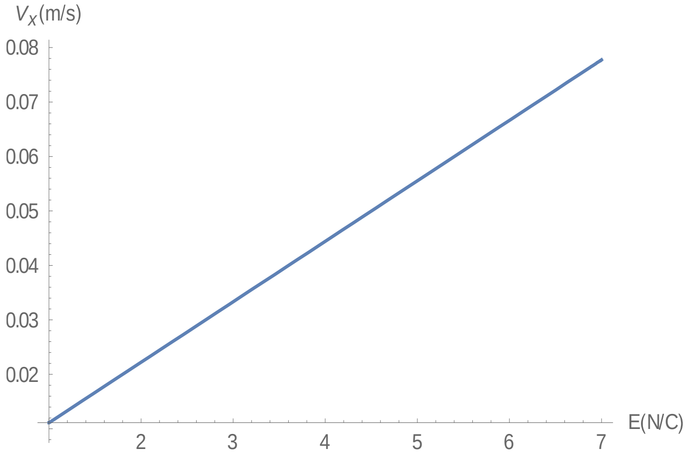

Fig 6: The dependence of oscillatory amplitude of the velocity V(m/s^2) on electric field E(N/C)

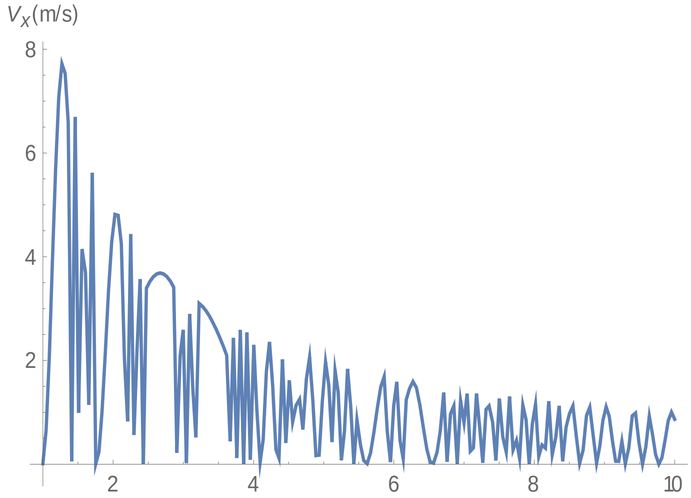

Fig 7: The dependence of oscillatory amplitude of the velocity V(m/s^2) on magnetic field B(T)

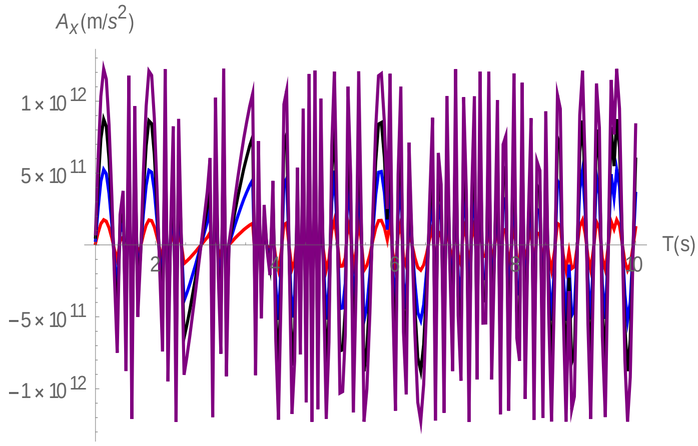

Fig 8: The dependence of Acceleration (m/s^2) of the electron plasma for different Electric field values on Time(t).

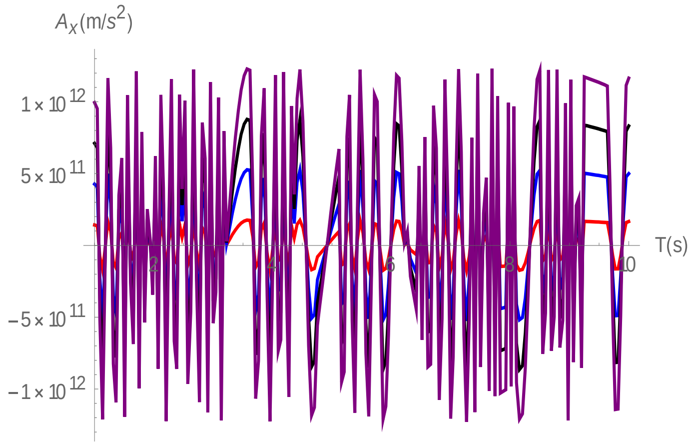

Fig 9: The dependence of Acceleration (m/s^2) of the electron plasma for different Electric field values on Time(t).

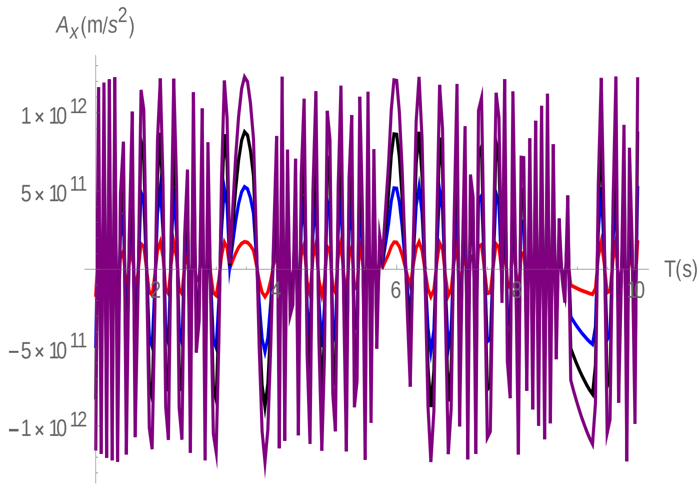

Fig 10: The dependence of Acceleration (m/s^2) of the electron plasma for different Electric field values on Time(t).

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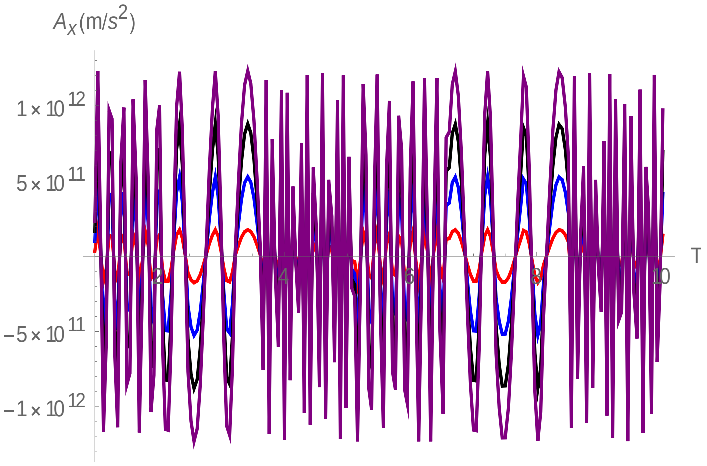

Fig 11: The dependence of Acceleration (m/s^2) of the electron plasma for different Electric field values on Time(t).

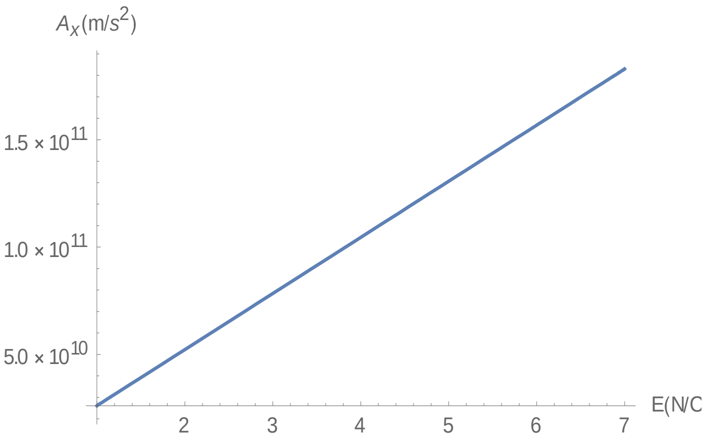

Fig 12: The dependence of oscillatory amplitude of the Acceleration A_x(m/s^2) on magnetic field E(NC^{-1})

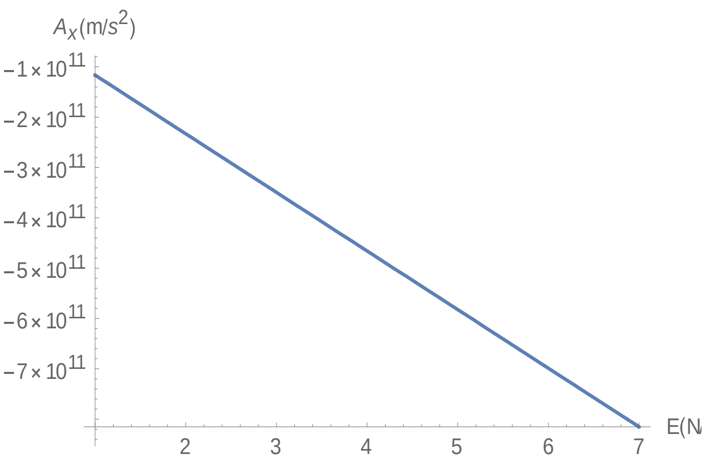

Fig 13: The dependence of oscillatory amplitude of the Acceleration Ax(m/s2) on magnetic field E(NC-1)

The time series of the effect of magnetic field on the *y* – velocity component of electron plasma motion in electromagnetic field is presented in fig 14, fig. 15, fig. 16 and fig 17. The profiles presented show's a chaotic oscillatory motion with the same irregular oscillatory pattern for all the trends in each figure. Each of the trend from bottom to top of the horizontal axis which is separated by a distance represents the increasing values of the selected electric fields according to the linear increase in figures 18 and 19 for both sides of the axis. The distance between each of the trend in each figure increases as the axial velocity increase and attain a maximum distance at the peak of each trough and crest. The chaotic oscillatory patterns characterized by irregularities in the wavelengths and amplitude shown in the profiles presented in the figures 14, 15, 16 and 17 for magnetic field values of 1.5T, 2.5T, 3.5T and 4.5T respectively, indicates that each of the pattern exhibited is determined by the strength of the magnetic induction, and observation from the figures also shows that the amplitude of the oscillatory motion decreases with time as shown in fig 20, as the magnetic field values increase, which implies a dampen effect of the magnetic induction on the velocity of charged plasma particle.

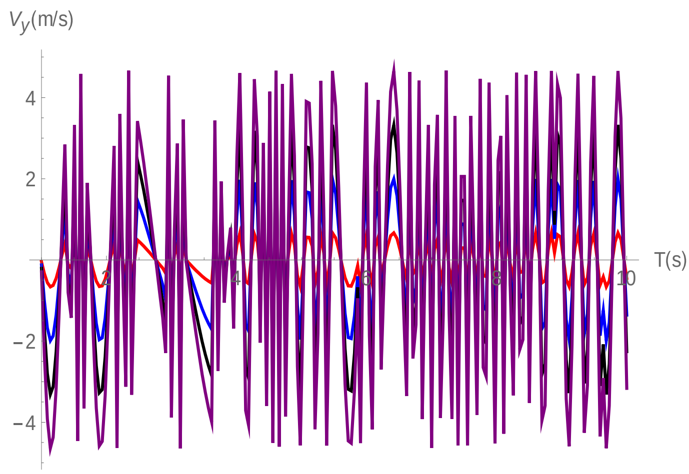

Fig 14: The dependence of Velocity(m/s) of the electron plasma for different Electric field values on Time(t).

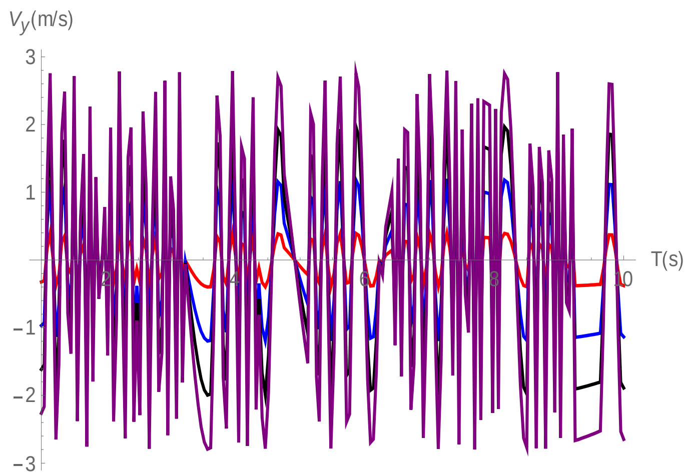

Fig 15: The dependence of Velocity(m/s) of the electron plasma for different Electric field values on Time(t).

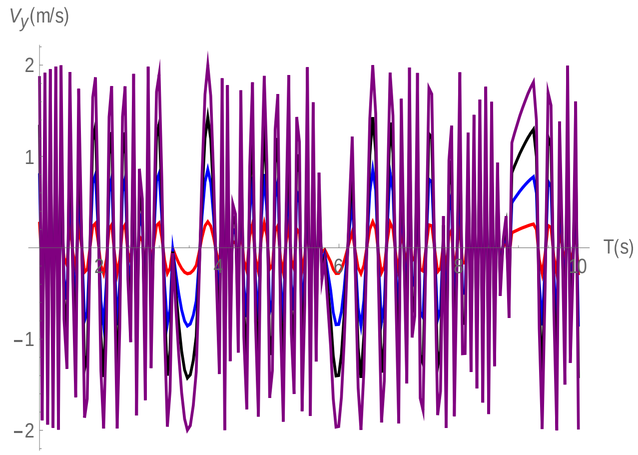

Fig 16: The dependence of Velocity(m/s) of the electron plasma for different Electric field values on Time(t).

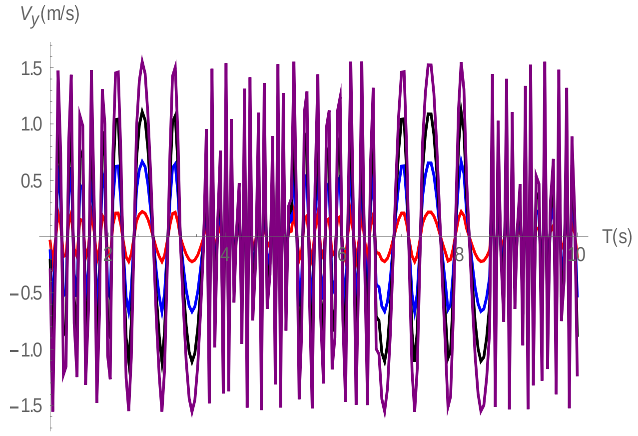

Fig 17: The dependence of Velocity(m/s) of the electron plasma for different Electric field values on Time(t).

The time series of the effects of some selected values of magnetic field on *y* – acceleration component of electron plasma in electromagnetic is presented respectively in figures 21, 22, 23 and 24. They also show a chaotic oscillatory motion with the same irregular oscillatory pattern for all the trends representing the selected values of the electric field in each of the figures. In this case the oscillatory motion extends to the negative and positive sides of the acceleration axis. The trends in each of the figures from bottom to top corresponding to the linear increase in both axes as shown in figure 25 and 26, implying that the amplitude of the oscillatory motion exhibited by the electron plasma is determined by electric field strength. Also, the different oscillatory structure exhibited by each of the figure representing the selected values of the magnetic field, which reveals that the chaotic oscillatory pattern of the wave motion of electron plasma in electromagnetic field is determined by the magnetic induction.

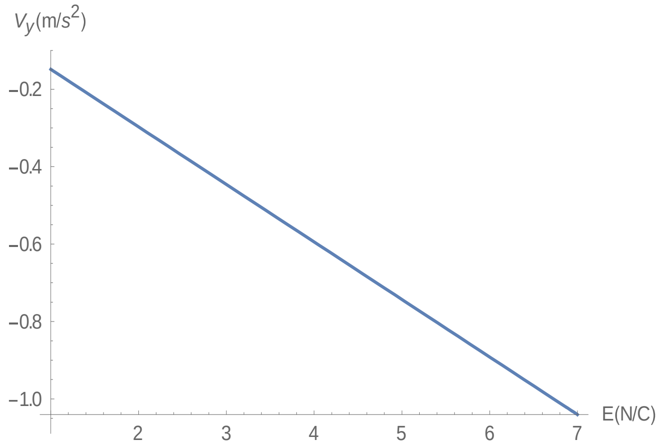

Fig 18: The dependence of Time(t) on the Velocity(m/s) of electron plasma for different values of electric field.

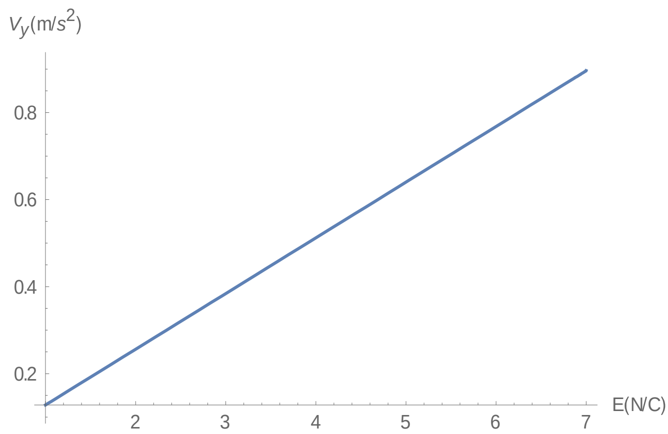

Fig 19: The dependence of Time(t) on the Velocity(m/s) of electron plasma for different values of electric field.

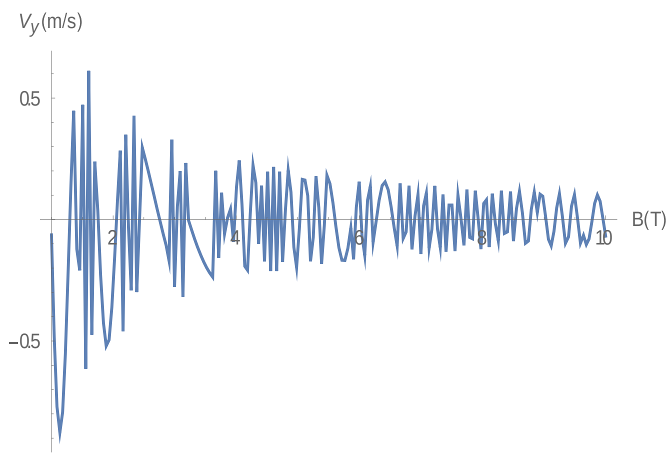

Fig 20: The dependence of oscillatory amplitude of the velocity V(m/s^2) on magnetic field B (T)

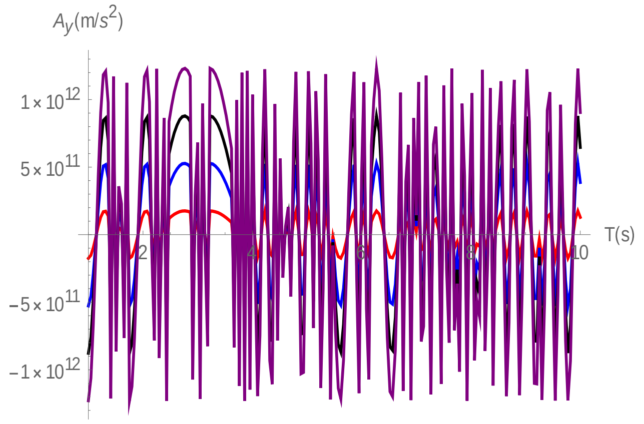

Fig 21: The dependence of Acceleration (m/s^2) of the electron plasma for different Electric field values on Time(t).

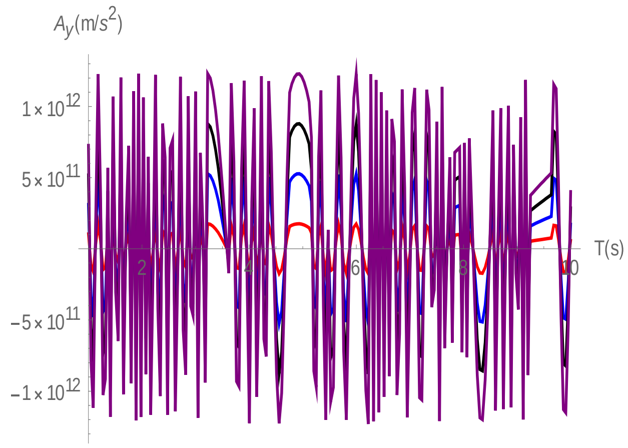

Fig 22: The dependence of Acceleration (m/s^2) of the electron plasma for different Electric field values on Time(t).

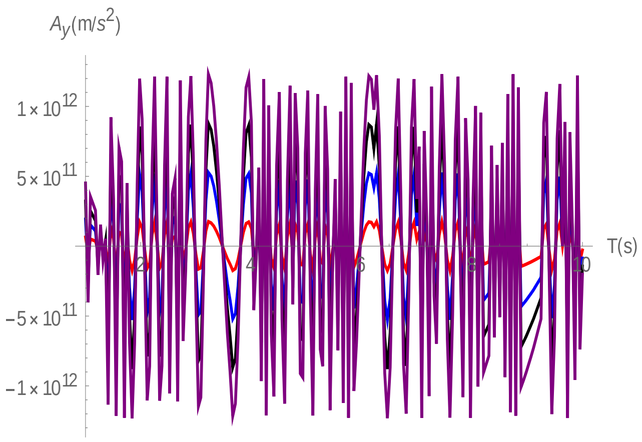

Fig 23: The dependence of Acceleration (m/s^2) of the electron plasma for different Electric field values on Time(t).

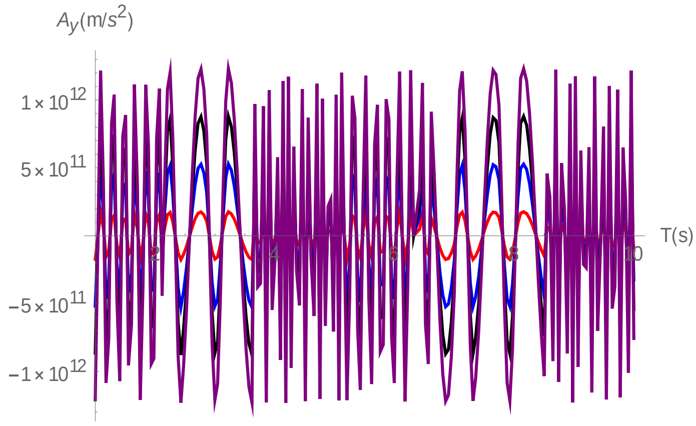

Fig 24: The dependence of Acceleration (m/s^2) of the electron plasma for different Electric field values on Time(t).

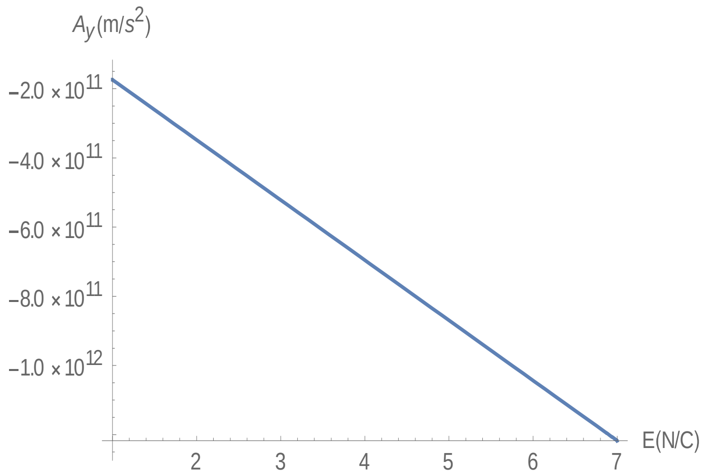

Fig 25: Acceleration A_y(m/s^2) dependency on electric field (N/C)

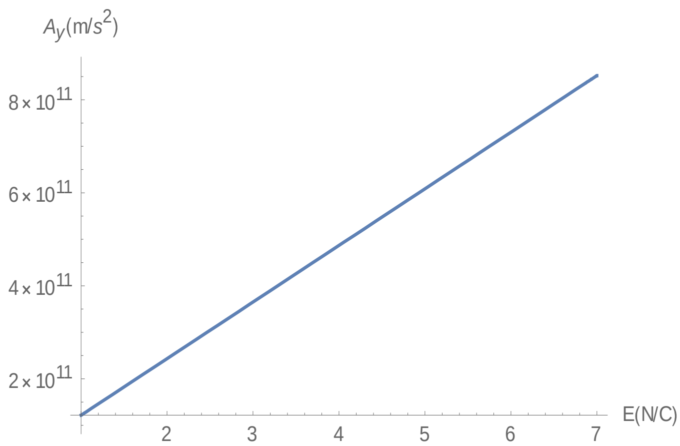

Fig 26: Acceleration A_y(m/s^2) dependency on electric field (N/C)

V. CONCLUSIONS

The analysis of the motion of charged plasma particle in electromagnetic fields of a magnetostatic and electrostatic potential derived from the J. C. Maxwell's equation of the dynamical theory of electromagnetism. Obtaining the Lorentz force due to these potentials and using the Lagrange formalism to obtain the nonlinear equations for the and y components of the velocity and acceleration. The study shows that

- The motion of charged plasma particle in electromagnetic field is an oscillatory motion similar to the spiral motion in [9], but is characterized by a chaotic wavelengths and amplitudes, the same chaotic pattern of motion exhibited by particles in [15], which agrees with the observation of [16, 13] and the motion observed in [16]
- The amplitude of the chaotic oscillatory motion of this charged particle of plasma in electromagnetic field corresponds proportionately by the strength of the electric field.
- The chaotic pattern of the oscillatory motion is determined by the strength of the magnetic field, which is also shown to dampen the oscillatory motion of this particle as seen in [12].

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