ISSN No:-2456-2165

Inventory Model with Demand as a Polynomial Function of Time and Constant Deterioration

Shelly[1] and **Dr. Rajender Kumar** (Supervisor)[2]

[1]Research Scholar, Department of Mathematics, Indira Gandhi University, Meerpur (Rewari), 122502, (Haryana), India,
[2]Assistant Professor, Department of Mathematics, Indira Gandhi University, Meerpur (Rewari), 122502, (Haryana), India

Abstract:- In the present work, a model of inventory management is generated for deteriorates goods, with shortages which are fully reserved. Demand rate is assumed as polynomial function of time and deterioration rate is independent of time i.e. constant deterioration.

I. INTRODUCTION

Inventory management has become the most important thing in order to minimize the cost related to inventory and to maximize the overall profit. Sometimes companies, or any kind of organization face problems in maintaining their inventory, because of some stocks of goods that deteriorates over time, like dairy products and like fashion goods, they only have sales in the market when they are in trend, otherwise, they are wasted and there are so many products that deteriorate with time. So there is a need for inventory models, which consider the effect of the deterioration of items. From the previous century, many models are generated on this topic. Some of the Cited models are listed below .

Datta & Pal (1988)[3], Lee & Wu (2002)[7], Sharma, Sharma & Ramani (2012)[16] and Sharma & Preeti (2013)[15] considered Power demand pattern for items that deteriorates with time,using varying deterioration in their respective models. Wu (1999) [20], Wu (2002) [19], Lee & Wu (2002)[7], Skouri et. al. (2009)[18], Sharma et. al. (2012)[16] considered Weibull distributed deterioration in their respective models. Sharma et. al. (2012)[16], Karmakar et.al. (2014)[6], Ibe et. al. (2016)[5], Shah (2018) [14] considered time varying holding cost in their respective models. Lee (2004)[8] created model with exponential distributed deterioration and Wu (2002) [19] & Ghosh (2004) [4] created model with time varying quadratic demand. Wu (1999)[20] and Skouri (2009)[18] developed models with ramp type demand rate. Ouyang (2005)[12], Shah (2010)[13] and Aliyu (2020)[1] developed models with exponentially declining demand.

Mukherjee(2010)[11] developed a model in which the time of duration of shortages varies directly with deterioration. Bhowmick(2011) [2] et. al., developed a model with continuous production model for deteriorating items with shortages. Maragatham(2017)[10] et. al., presented Model for Items in a single warehouse and assumed constant lead time . Sharma(2018)[17] developed a model for items that deteriorates with time, such as fruits, vegetables, and foodstuffs by considering demand as time-dependent. Long(2019)[9] demonstrated that structural deterioration affects the value of damage detection information. In the present paper, working is done based on the above papers by taking demand as a function which is polynomial in nature with respect to time and time-independent deterioration i.e. constant deterioration.

II. ASSUMPTIONS AND NOTATIONS

Notations:-

The following are the notations used here:-

1. C1 = Cost per unit of holding inventory per unit time i.e. Holding Cost
2. C2 = Shortage cost per unit per unit time.
3. C3 = Deterioration cost.
4. T = Each cycle length.
5. I(t) = Inventory at any time t.
6. C(t) = Average total cost.
7. D(t) = Demand Rate
8. θ(t) = Deterioration Rate Function
9. S = Initial Inventory

Assumptions:-

The following are the assumptions used here:-

1. Demand Rate D(t) is assumed as polynomial function of time, given by $D(t) = t + 2t^2 + 3t^3 + \ldots + nt^n$.
2. The deterioration rate function, θ(t) is assumed in the form $\theta(t) = \theta_0$.
3. Replenishment size is constant and the replenishment rate is infinite.
4. The Lead time is zero.
5. Shortages are considered and totally reserved.
6. During the period T, neither is replacement nor repair of deteriorated units.

III. ANALYSIS OF MODEL

Let Inventory level at any time t be I(t). Inventory level slowly decreases during time interval $(0,t_1)$, $t_1< T$ and becomes exactly zero at $t = t_1$. Shortages takes place in the interval $(0,t_1)$, which are totally reserved. Differential equations which governs this inventory system during the interval $0 \le t \le T$ using demand and deterioration rate are

$$\frac{dI(t)}{dt} + \theta_0\, I(t) = -(t + 2t^2 + 3t^3 + \cdots + nt^n) \qquad (1)$$

and

$$\frac{dI(t)}{dt} = -(t + 2t^2 + 3t^3 + \cdots + nt^n) \qquad (2)$$

Solution of differential equation (1) is

$$I(t)e^{\theta_0 t} = -\int (t + 2t^2 + 3t^3 + \cdots + nt^n)e^{\theta_0 t}dt + C$$
$$= -\int (t + 2t^2 + 3t^3 + \cdots + nt^n)(1 + \theta_0\, t\,)dt + C$$
$$= -\int [\,(t + 2t^2 + 3t^3 + \cdots + nt^n) + \theta_0\,(t^2 + 2t^3 + 3t^4 + \cdots + nt^{n+1})\,]dt + C$$
$$= -\left[\left(\frac{1}{2}\,t^2 + \frac{2}{3}\,t^3 + \cdots + \frac{n}{n+1}\,t^{n+1}\right) + \theta_0\left(\frac{1}{3}\,t^3 + \frac{1}{2}\,t^4 + \cdots + \frac{n}{n+2}\,t^{n+2}\right)\right] + C$$

Putting t = 0, I(0) = C. But I(0)=S. Therefore C = S. Thus

$$I(t)e^{\theta_0 t} = S - \left[\left(\frac{1}{2}\,t^2 + \frac{2}{3}t^3 + \cdots + \frac{n}{n+1}t^{n+1}\right) + \theta_0\left(\frac{1}{3}\,t^3 + \frac{1}{2}\,t^4 + \cdots + \frac{n}{n+2}\,t^{n+2}\right)\right];\;\; 0 \le t \le T \qquad (3)$$

Again from (3), $I(t_1)=0$. So

$$0 = S - [\left(\frac{1}{2}\,t_1^2 + \frac{2}{3}\,t_1^3 + \cdots + \frac{n}{n+1}\,t_1^{n+1}\right) + \theta_0\left(\frac{1}{3}\,t_1^3 + \frac{1}{2}\,t_1^4 + \cdots + \frac{n}{n+2}\,t_1^{n+2}\right)$$

Thus

$$S = \left[\left(\frac{1}{2}\,t_1^2 + \frac{2}{3}\,t_1^3 + \cdots + \frac{n}{n+1}\,t_1^{n+1}\right) + \theta_0\left(\frac{1}{3}\,t_1^3 + \frac{1}{2}\,t_1^4 + \cdots + \frac{n}{n+2}\,t_1^{n+2}\right)\right] \qquad (4)$$

Putting the value of S in (3), we get

$$I(t)e^{\theta_0 t} = \frac{1}{2}\,(t_1^2 - t^2) + \frac{2}{3}(t_1^3 - t^3) + \cdots + \frac{n}{n+1}\,(t_1^{n+1} - t^{n+1}) + \theta_0\,(\frac{1}{3}\,(t_1^3 - t^3) + \frac{1}{2}(t_1^4 - t^4) + \cdots + \frac{n}{n+2}(t_1^{n+2} - t^{n+2}))$$

Hence

$$I(t) = \frac{1}{2}\,(t_1^2 - t^2) + \frac{2}{3}\,(t_1^3 - t^3) + \cdots + \frac{n}{n+1}(t_1^{n+1} - t^{n+1}) + \theta_0\left[\frac{1}{6}\,(t^3 - 3t_1^2 t + 2t_1^3) + \frac{1}{6}(t^4 - 4t_1^3 t + 3t_1^4) + \cdots + \frac{n}{(n+1)(n+2)}\left(t^{n+2} - (n+2)t_1^{n+1}t + (n+1)\right)t_1^{n+2}\right] \qquad (5)$$

$$I(t) = \sum_1^n \left[\frac{m}{m+1}\,(t_1^{m+1} - t^{m+1}) + \theta_0\,\frac{m}{(m+1)(m+2)}\,[\,t^{m+2} - (m+2)t_1^{(m+1)}\,t + (m+1)t_1^{m+2}\;\;]\right]$$

Solution of differential equation (2) is

$$I(t) = -\left(\frac{1}{2}\,t^2 + \frac{2}{3}\,t^3 + \cdots + \frac{n}{n+1}\,t^{n+1}\right) + B \qquad (6)$$

Since $I(t_1) = 0$, we have

$$0 = -\left(\frac{1}{2}\,t_1^2 + \frac{2}{3}\,t_1^3 + \cdots + \frac{n}{n+1}\,t_1^{n+1}\right) + B$$

This implies

$$B = \frac{1}{2}\,t_1^2 + \frac{2}{3}\,t_1^3 + \cdots + \frac{n}{n+1}\,t_1^{n+1}$$

Hence

$$I(t) = \frac{1}{2}(t_1^2 - t^2) + \frac{2}{3}(t_1^3 - t^3) + \cdots + \frac{n}{n+1}(t_1^{n+1} - t^{n+1});\, t_1 \le t \le T \qquad (7)$$

Thus the entire amount of deteriorated units = I(0) – stock loss due to demand

$$= S - \int_0^{t_1}(t + 2t^2 + \cdots. + nt^n)dt$$
$$= S - \left(\frac{1}{2}t_1^2 + \frac{2}{3}t_1^3 + \cdots + \frac{n}{n+1}\, t_1^{n+1}\right)$$
$$= \left(\frac{1}{2}\, t_1^2 + \frac{2}{3}t_1^3 + \cdots + \frac{n}{n+1}\, t_1^{n+1}\right) + \theta_0\left(\frac{1}{3}\, t_1^3 + \frac{1}{2}\, t_1^4 + \cdots + \frac{n}{n+2}\, t_1^{n+2}\right) - \left(\frac{1}{2}\, t_1^2 + \frac{2}{3}t_1^3 + \cdots + \frac{n}{n+1}\, t_1^{n+1}\right)$$
$$= \theta_0\left(\frac{1}{3}\, t_1^3 + \frac{1}{2}\, t_1^4 + \cdots + \frac{n}{n+2}\, t_1^{n+2}\right) \qquad (8)$$

Total value of inventory held in $[0,t_1]$ is

$$I_1 = \int_0^{t_1} I(t)\, dt$$
$$I_1 = \int_0^{t_1}\left[\frac{1}{2}\,(t_1^2 - t^2) + \frac{2}{3}(t_1^3 - t^3) + \cdots + \frac{n}{n+1}\,(t_1^{n+1} - t^{n+1})\,\right] dt +$$
$$\theta_0 \int_0^{t_1}\left[\frac{1}{6}\,(t^3 - 3t_1^2 t + 2t_1^3) + \frac{1}{6}(t^4 - 4t_1^3 t + 3t_1^4) + \cdots + \frac{n}{(n+1)(n+2)}\left(t^{n+2} - (n+2)t_1^{n+1}t + (n+1)\right)t_1^{n+2}\right] \mathrm{dt}$$
$$I_1 = \left(\frac{1}{3}t_1^3 + \frac{1}{2}t_1^4 + \cdots + \frac{n}{n+2}t_1^{n+2}\right) + \theta_0\left(\frac{1}{8}\, t_1^4 + \frac{1}{5}t_1^5 + \cdots + \frac{n}{2(n+3)}t_1^{n+3}\right)$$

Inventory Holding Cost = C_1 * total inventory

$$= C_1\left[\left(\frac{1}{3}t_1^3 + \frac{1}{2}t_1^4 + \cdots + \frac{n}{n+2}t_1^{n+2}\right) + \theta_0\left(\frac{1}{8}\, t_1^4 + \frac{1}{5}t_1^5 + \cdots + \frac{n}{2(n+3)}t_1^{n+3}\right)\right] \qquad (9)$$

Deterioration Cost = C_3 * the entire amount of deteriorated units

$$= C_3\left[\theta_0\left(\frac{1}{3}\, t_1^3 + \frac{1}{2}\, t_1^4 + \cdots + \frac{n}{n+2}\, t_1^{n+2}\right)\right] \qquad (10)$$

Shortage units Quantity = $\int_{t_1}^{T} - I(t)dt$

$$= -\int_{t_1}^{T}\left[\frac{1}{2}(t_1^2 - t^2) + \frac{2}{3}(t_1^3 - t^3) + \cdots + \frac{n}{n+1}(t_1^{n+1} - t^{n+1})\right] dt$$

$$= T\left[\frac{1}{2}\left(\frac{1}{3}T^2 - t_1^2\right) + \frac{2}{3}\left(\frac{1}{4}T^3 - t_1^3\right) + \cdots + \frac{n}{n+1}\left(\frac{1}{n+2}T^{n+1} - t_1^{n+1}\right)\right] + \left[\frac{1}{3}t_1^3 + \frac{1}{2}t_1^4 + \cdots + \frac{n}{n+2}\, t_1^{n+2}\right] \qquad (11)$$

Shortage Cost = C_2 * shortage units quantity

$$= C_2 T\left[\frac{1}{2}\left(\frac{1}{3}T^2 - t_1^2\right) + \frac{2}{3}\left(\frac{1}{4}T^3 - t_1^3\right) + \cdots + \frac{n}{n+1}\left(\frac{1}{n+2}T^{n+1} - t_1^{n+1}\right)\right] +$$
$$C_2\left[\frac{1}{3}t_1^3 + \frac{1}{2}t_1^4 + \cdots + \frac{n}{n+2}\, t_1^{n+2}\right] \qquad (12)$$

The Total Cost per unit time
= Inventory Holding Cost + Deterioration Cost + Shortage Cost

$$= C_1\left[\left(\frac{1}{3}t_1^3 + \frac{1}{2}t_1^4 + \cdots + \frac{n}{n+2}t_1^{n+2}\right) + \theta_0\left(\frac{1}{8}\, t_1^4 + \frac{1}{5}t_1^5 + \cdots + \frac{n}{2(n+3)}t_1^{n+3}\right)\right] + C_3\left[\theta_0\left(\frac{1}{3}\, t_1^3 + \frac{1}{2}\, t_1^4 + \cdots + \frac{n}{n+2}\, t_1^{n+2}\right)\right] +$$
$$C_2 T\left[\frac{1}{2}\left(\frac{1}{3}T^2 - t_1^2\right) + \frac{2}{3}\left(\frac{1}{4}T^3 - t_1^3\right) + \cdots + \frac{n}{n+1}\left(\frac{1}{n+2}T^{n+1} - t_1^{n+1}\right)\right] +$$
$$C_2\left[\frac{1}{3}t_1^3 + \frac{1}{2}t_1^4 + \cdots + \frac{n}{n+2}\, t_1^{n+2}\right]$$

The Average Total Cost per unit time ,

$$C(t_1) = \frac{1}{T}\,[\,Total\ Cost\ per\ unit\ time\,]$$
$$C(t_1) = \frac{C_1}{T}\left[\left(\frac{1}{3}t_1^3 + \frac{1}{2}t_1^4 + \cdots + \frac{n}{n+2}t_1^{n+2}\right) + \theta_0\left(\frac{1}{8}\, t_1^4 + \frac{1}{5}t_1^5 + \cdots + \frac{n}{2(n+3)}t_1^{n+3}\right)\right]$$
$$+ \frac{C_3}{T}\left[\theta_0\left(\frac{1}{3}\, t_1^3 + \frac{1}{2}\, t_1^4 + \cdots + \frac{n}{n+2}\, t_1^{n+2}\right)\right]$$
$$+ C_2\left[\frac{1}{2}\left(\frac{1}{3}T^2 - t_1^2\right) + \frac{2}{3}\left(\frac{1}{4}T^3 - t_1^3\right) + \cdots + \frac{n}{n+1}\left(\frac{1}{n+2}T^{n+1} - t_1^{n+1}\right)\right]$$
$$+ \frac{C_2}{T}\left[\frac{1}{3}\, t_1^3 + \frac{1}{2}t_1^4 + \cdots. + \frac{n}{n+2}\, t_1^{n+2}\right]$$

For minimum average total cost , the necessary and sufficient conditions are $\frac{dC(t_1)}{dt_1} = 0 \; and \; \frac{d^2C(t_1)}{dt_1^2} > 0$.

Now $\frac{dC(t_1)}{dt_1} = 0 \; gives$

$$(t_1 + 2t_1^2 + 3t_1^3 + \cdots + nt_1^n)\left[\frac{C_1\theta_0}{2T}t_1^2 + \frac{(C_1+C_{2+C_3\theta_0})}{T}t_1 - C_2\right] = 0$$

Which further implies

$$\left[\frac{C_1\theta_0}{2T}t_1^2 + \frac{(C_1+C_{2+C_3\theta_0})}{T}t_1 - C_2\right] = 0 \tag{13}$$

Since (13) is a quadratic equation in t_1 having last term negative, thus it has atleast one positive root. Also $\frac{d^2C(t_1)}{dt_1^2} > 0$. Let t_1^* be the positive root of (13). So optimum value of t_1 is t_1^*. Substituting it in (4), the optimized value of S is

$$S^* = \left[\left(\frac{1}{2}\,t_1^{*2} + \frac{2}{3}\,t_1^{*3} + \cdots + \frac{n}{n+1}\,t_1^{*(n+1)}\right) + \,\theta_0\left(\frac{1}{3}\,t_1^{*3} + \frac{1}{2}\,t_1^{*4} + \cdots + \frac{n}{n+2}\,t_1^{*(n+2)}\right)\right] \tag{14}$$

Minimum value of $C(t_1)$ is

$$\begin{aligned} C(t_1^*) = \frac{C_1}{T}&\left[\left(\frac{1}{3}t_1^{*3} + \frac{1}{2}t_1^{*4} + \cdots + \frac{n}{n+2}t_1^{*(n+2)}\right) + \,\theta_0\left(\frac{1}{8}\,t_1^{*4} + \frac{1}{5}t_1^{*5} + \cdots + \frac{n}{2(n+3)}t_1^{*(n+3)}\right)\right] \\ &+ \frac{C_3}{T}\left[\theta_0\left(\frac{1}{3}\,t_1^{*3} + \frac{1}{2}\,t_1^{*4} + \cdots + \frac{n}{n+2}\,t_1^{*(n+2)}\right)\right] \\ &+ \,C_2\left[\frac{1}{2}\left(\frac{1}{3}T^2 - t_1^{*2}\right) + \frac{2}{3}\left(\frac{1}{4}T^3 - t_1^{*3}\right) + \cdots + \frac{n}{n+1}\left(\frac{1}{n+2}T^{n+1} - t_1^{*(n+1)}\right)\right] \\ + \frac{C_2}{T}&\left[\frac{1}{3}\,t_1^{*3} + \frac{1}{2}t_1^{*4} + \cdots . + \frac{n}{n+2}\,t_1^{*(n+2)}\right] \end{aligned} \tag{15}$$

Thus equation (15) gives optimal value of total average cost per unit time. These equations can be further solved for different values of variables used here, using softwares like Matlab and Mathematica.

IV. CONCLUSION

In this paper, a model of inventory management is generated for deteriorating goods by taking demand as a function which is polynomial with respect to time and deterioration is taken as time-independent i.e. constant deterioration.

REFERENCES

[1]. **Aliyu,I. and Sani,B**., An Inventory Model for Deteriorating Items with a Generalised Exponential Increasing Demand, Constant Holding Cost and Constant Deterioration Rate, Vol. 14, no.15, pp.725-736,2020.

[2]. **Bhowmick,J. and Samanta,G.P.,** A Deterministic Inventory Model of Deteriorating Items with Two Rates of Production, Shortages and Variable Production Cycle, ISRN Applied Mathematics, Volume 2011, Article ID 657464,16 pages.

[3]. **Datta, T.K. and Pal, A.K.,** Order level inventory system with power demand pattern for items with variable rate of deterioration, Indian J. Pure Appl. Math., Vol.19, No.11, pp.1043-1053, November 1988.

[4]. **Ghosh,S.K. and Chaudhuri,K.S**., An Order-Level Inventory Model for a Deteriorating item with Weibull Distribution Deterioration, Time-Quadric Demand and Shortages, Advanced Modelling and Optimization , Volume6, Number 1,2004.

[5]. **Ibe,C.B. and Ogbeide**, D.O., An Inventory Model for Deteriorating Items with Exponential Increasing Demand and Time Varying Holding Cost under Partial Backlogging, Centrepoint Journal (Science Edition), Volume 22, No. 2, pages 69-75,2016.

[6]. **Karmakar,B. and Choudhury,K.D**., Inventory Models with ramp -type demand for deteriorating items with partial backloggingand time-varing holding cost, Yugoslav Journal of Operations Research, Vol 2, pp. 249-266, 2014.

[7]. **Lee,W.C. and Wu,J.W**., An EOQ model for items with Weibull distributed deterioration, Shortages and power demand pattern, Information and Management Science, Vol.13, No.2, 19-34, 2002.

[8]. **Lee,W.C. and Wu,J.W**., A Note on EOQ Model for items with mixtures of exponential distribution, shortages and time-varying demand, Kluwer Academic Publishers, Vol.38, pp. 457-473,2004.

[9]. **Long.,L**. et. al., The e_ects of deterioration models on the value of damage detection information, Taylor and Francis Group, London,2019.

[10]. **Maragatham,M. and Palani,R.,** An Inventory Model for Deteriorating Items with Lead Time price Dependent Demand and Shortages, ISSN 0973-6107 Volume 10, Number 6(2017) pp. 1839-1847.

[11]. **Mukherjee,B. and Prasad,K**., A Deterministic Inventory Model of Deteriorating items with stock and time dependent demand rate, 2010.

[12]. **Ouyang,L.Y., Wu,K.S. and Cheng,M.C**., An Inventory Model for Deteriorating items with Exponential Declining Demand and Partial Backlogging, Yugoslav Journal of operations research,15(2005), Number 2, pp.277-288.

[13]. **Shah,N.H. and Mohmmadraiyan,M.,** An Order-Level Lot-Size Model for Deteriorating items for two storage facilities when demand is Exponentially Declining, Revista Investigation Operacional, Vol.31, No.3, pp.193-199,2010.

[14]. **Shah,N.H. and Naik,M.K**., Inventory Policies for Price-Sensitive Stock-Dependent and Quantity Discounts, International Journal of Mathematical,Engineering and Management Sciences, Vol. 3,No.3, pp.245-257, 2018.

[15]. **Sharma,A.K. and Preeti**, An Inventory model for deteriorating items with power pattern demand and partial backlogging with time dependent holding cost, IJLTEMAS, Vol.2(3), pp.92-104,2013.

[16]. **Sharma,A.K., Sharma,M.K. and Ramani,N.,** An Inventory model with Weibull distribution deteriorating item with power pattern demand with shortages and time dependent holding cost, American Journal of Applied Mathematical Sciences, Vol. 1, No. 1-2, pp.17-22, 2012.

[17]. **Sharma,V. and Chaudhary,R.R**., An Inventory Model for Deteriorating items with Weibull deterioration with time dependent demand and shortages, Research Journal of Management Sciences, Vol.2(3), pp.1-4, March 2018.

[18]. **Skouri,K., Konstantaras,I., Papachristos,S. and Ganas,I**., Inventory Models with ramp type demand rate, partial backlogging and Weibull deterioration rate, European Journal of Operational Research, Vol. 192, pp.79-92,2009.

[19]. **Wu,K.S.,** Deterministic Inventory Model for items with time varying demand, Weibull Distribution deterioration and Shortages, Yugoslav Journal of Operations Research, Vol. 12, pp. 61-71, 2002.

[20]. **Wu,J.W., Lin,C., Tan,B. and Lee,W.C.,** An EOQ Inventory Model with ramp type demand rate for items with Weibull deterioration, International Journal of Information and Management Sciences, Vol. 10, No. 3, pp. 41-51, 1999.