ISSN No:-2456-2165

Path Planning of Unmanned Aerial Vehicles using Game Theory

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Abstract:- The aim of the paper is to investigate path planning of an unmanned aerial vehicle with consideration of game theory. The benefit of this method is lack of detection and finding the optimum trajectory between terrains. Hence, by defining the permissible flight bands, and combining the kinematic and dynamic equation with wind gusts in the optimal control variables, the air vehicles can path through the terrain in minimum time and altitude. However, in final, two agents were considered by defining new cost function as a game theory. The results illustrated the combination of kinematic and dynamic were promising and the new cost function in two agents; optimal control has a good effect on tracking the agents in terrain.

Keywords:- *aerial vehicle, game theory, multi-agents, path planning, terrain mapping.*

I. INTRODUCTION

This Stealth and lack of radar detection is one of the requirements of the Unmanned Aerial Vehicles (UAVs) on certain missions. One way for UAVs is beingundetectedby terrain and mountain. To go stealth, the air vehicles must have a trajectory and a good controller to reach the target. The main objective of optimum path is to generate the Trajectory with consideration of terrain following and terrain avoidance. This requirement allows the unmanned vehicles without the crash have their mission such as petrol, relief, pursuit and evader, rescue and landing into the airport near to the mountain, and etc. However, for these missions the modern controller will cost a fortune and generation of the optimal path is easier to use the usual controller. To determine the optimal path, the offline digital points of the searching area are received from the satellite and simulated by a mathematics model.

So far, different modeling of terrain following and terrain avoidance (TF-TA) such as[1-4] has been studied. In these papers, the model of terrain based on Particle Swarm Optimization (PSO) and Genetic Algorithm (GA) investigated .The results were promising. In [5], the compelling of fuzzy control has been studied. The results showed the significant performance of the controller in the face of terrain. In another case [6], the Wiener-Khinchin (W-K) filter was designed to investigate the collision of terrain with a new machine based on machine vision. Dealing with the issue of decision-making in natural obstacles in [7] were considered. Hence, the method of this paper for solving the optimization problem was convincing. In [8] studied the space robot exposed to several unknown terrains. The method of this paper has been used for compatibility of nonlinear adaptation controller. The optimal controller deal with terrain and showed the significant performance. To create the flight process, route information is generally given to the flight system as a matrix consisting of discrete points. Having the elevations of the earth, a mathematical function can be fitted to them. Methods such as least square, discrete orthogonally, and so on. The accuracy of the flight mission depends on these methods. In [9], reviewing the Chebyshev curve fitting method for modeling of terrain and performed at low altitude and time with considering of Lagrange equations in the interval [+1 -1] with a weight function. In other hand, the multi-agent, in the terrain following and terrain avoidance are studied. An example in [10] discusses a game theory-based method for developing an automatic maneuver algorithm in air warfare. The matrix algorithm executes its decisions based on differential game theory. The outcome in the matrix means that the flying vehicle is superior. The simulated algorithm of six degrees of freedom illustrates the efficient of this method.

In another paper[11], a new algorithm for controlling a quad rotor using game theory is investigated. In this dynamic model, perturbations are also considered. The simulations indicate the optimal performance of this manner. In another reference [12] with the help of game theory, the scenario of defense, attack, and attack of three flying vehicles has been studied. In this article, the acceleration and target position are not specified. The achievement of this paper advocates the suitability of this method. In [13], the cost function is minimized in the regression-mode method, and the threat areas are investigated with the help of differential games. The results of the optimal performance of this function show the cost to minimize flying vehicles. Based on this fact, the optimal control is a good solution to find the optimal route of aircraft. By considering game theory in this theory, multi-agent optimization can be generalized and all the formulas used in one agent can be applied in game theory. This paper, the optimal flight path of the aircraft in the pursuit of a three-dimensional model is discussed. The important point in these issues is a notice to the compatibility of the geometric properties of these routes with the functional ability and dynamic constraints of the aircraft that wants to follow this route in a limited period. As mentioned, the design process before was natural without regard to restrictions and bounds, and the passage of these bounds was done by control systems. Correcting the error of deviation from the reference path requires the high cost of using highly accurate sensors and other items that could have greatly increased the risk of flying at low altitudes[10]. According to the mentioned cases about geometric paths in the computational process of the optimal path, in this paper, the terrain model in kinematic equations

ISSN No:-2456-2165

of motion is used, which has much fewer calculations than solving the three-dimensional problems of dynamic nonlinear equations. Considering the model of terrain in the kinematic equations of the flying vehicle, the movement path is first introduced for the flying vehicle, and the flight is based on the new route at low altitude. Of course, it should be noted that the definition of the appropriate cost function for the problem is one of the important factors in system performance. Because in creating the cost function, the designer can consider a set of operational and strategic criteria, also include the existing system constraints in the structure of the function. In this paper, it has been tried to include the criteria of time and altitude of mission in the cost function with considering of wind gust. In the continuation of this discussion, first, the kinematic equations and the dynamic dimension of motion on the elevations will be introduced and then the formulation of the optimal problem, simulation, and results will be explained and then these issues will have expanded for two flying vehicles based on a new cost function. By respecting the two agents and establish the optimization method with the help of differential game theory, so it can fly to the point in the shortest possible time due to the low altitude flight of the agent. In the previous studies the combination of dynamic and kinematic equation along with game theory cost function in the present of wind gust never been studied. The paper is separated into five sections, first the introduction, and the kinematic and dynamic equation of one agent based on terrain-following were evaluated and simulated then based on new cost function the game theory is used in pursuit- evader problem, the fourth part would be the simulation of the defined model, In the final section, the conclusion was discussed.

II. MODEL OF TERRAIN

The aim to generate the optimal path in the terrain following problem, is to reach the minimum altitude and time. During the flight, the aircraft has a terrain path limitation that may depict as (1):

$$h = h(x, y) \tag{1}$$

In (1), the (h) is the altitude of a plane in the body coordination system and x,y represent a profile of terrain following. This variable includes the set, thread, etc. It should consider these elements as (h) variable, (2).

$$h = h_{clearance} + h_{Therad} + h_{etc} \tag{2}$$

Where h(x,y) can generate discrete data coming from a satellite. This paper assumed the discrete data has a continuous first and second derivative order. Fig.1, shows the local and inertia coordinate systems.

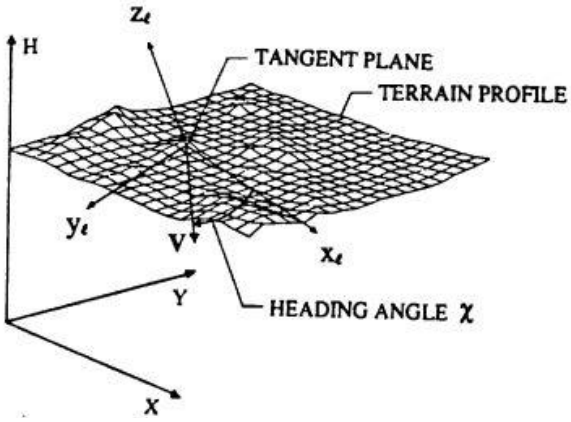

Fig.1: Local and inertia coordinate systems[14]

By defining the equation of movements in the local system and transferring it to the inertia system, we have the field in our problem (3).

$$\dot{x}_l = Vcos(\chi) + u$$
$$\dot{y}_l = Vsin(\chi) + v \tag{3}$$

Where in (3), (V,χ) are the control variables, and (u,v) are the local wind gust speed status. The inertia problem represents as [13]:

$$\dot{x} = \frac{Vcos(\chi)}{\sqrt{1+f_x^2}} + \frac{VF_xF_y\sin(\chi)}{\sqrt{1+f_x^2}\sqrt{1+f_x^2+f_y^2}} + u \tag{4}$$

$$\dot{y} = \frac{V\sin(\chi)\sqrt{1+f_x^2}}{\sqrt{1+f_x^2+f_y^2}} + v \tag{5}$$

$$\dot{h} = Vsin(\gamma) \tag{6}$$

Which in (6) γ is a flight path introduced by (7):

$$\gamma = f_x\dot{x} + f_y\dot{y} \tag{7}$$

These equations represent a kinematic pattern of flight path in the field.

Cost Function Definition

To generate the optimal path, this paper illustrates the optimal control method [14]. The mathematical model of cost function decreases the complex calculations in the optimization algorithm. In this scenario, we represent the complex COST function including vertical acceleration, minimum time, and altitude.

$$J = \int_{t_0}^{t_f} [(1-k) + kF(x,y) + (W1F_{xx}^2 + W_2F_{xy}^2 + W_3F_{yy}^2)] \tag{8}$$

In (8), the minimum time and altitude is the two phrases of the cost function, which we can change with the (0<k<1) variable and the second part is the vertical acceleration, is imported to the flying vehicle.

The Hamilton function To optimize this equation was written [14].The equation of (4,5) is added as a state of Hamiltonian function.

ISSN No:-2456-2165

$$H = \left[(1-k) + kF(x,y) + (W1F_{xx}^2 + W_2F_{xy}^2 + W_3F_{yy}^2)\right] + \lambda_x\dot{x} + \lambda_y\dot{y} \quad (9)$$

What λ_x and λ_y exemplify as a co-status function for an optimal control problem:

$$\dot{\lambda}_x = -\frac{\partial H}{\partial x}, \quad \dot{\lambda}_y = -\frac{\partial H}{\partial y} \quad (10)$$

In this issue, the initial and final points are presented:

$$x(t_0), y(t_0) = given$$
$$x(t_f), y(t_f) = given \quad (11)$$

Considering (χ) as a change in the Hamiltonian function, the values H/χ=0 and χ illustrate:

$$\tan(\chi) = \frac{\lambda_x f_y f_x - \lambda_y(1+f_x^2)}{\lambda_x\sqrt{1+f_x^2+f_y^2}} \quad (12)$$

Since the Hamiltonian function does not depend explicitly on time, the optimal equation is therefore based on free final time:

$$H(t)=0 \quad (13)$$

With the resolution of the border equation of (4), (5), the λ_x, λ_y is evaluated.

$$\lambda_y = \frac{\left(\frac{\sin(\chi)}{B} - \frac{\cos(\chi)D.E}{A}\right).(KF - K + W1(fxx^2) + W2}{\frac{V\cos(\chi)^2}{C} + \frac{V\sin(\chi)^2}{C} - \frac{\sin(\chi).v}{B} + \frac{\cos(\chi).u.B}{\sqrt{C}}}$$

Where:

$$A = \sqrt{(D^2+1).C} \quad (14)$$
$$B = \sqrt{D^2+1}$$
$$C = D^2 + E^2 + 1$$
$$D = fxx$$
$$E = fxy$$

$$\lambda_x = \frac{\cos(\chi).(KF - K + W1(fxx^2) + W2fxy^2 + W}{\sqrt{A}\left(\frac{V\cos(\chi)^2}{A} + \frac{V\sin(\chi)^2}{A} - \frac{\sin(\chi).v}{B} + \frac{\cos(\chi).u.B}{\sqrt{A}} +\right.}$$

Where: (15)

$$A = D^2 + C^2 + 1$$
$$B = \sqrt{D^2+1}$$
$$C = fy$$
$$D = fx$$

If we differentiate between (14), (15) with time and equation (4), (5), we can give the χ :

$$\dot{\chi} = (\lambda_y(v_x + \frac{V\sin(\chi)D.B}{B.F^{\frac{3}{2}}} - \frac{FxFxxV\sin(\chi)}{D\sqrt{F}} - \lambda_x(\frac{FxFxxV\cos(\chi)}{(fx^2+1)^{\frac{3}{2}}} - \frac{fx.Fxy.\sin(\chi)}{\sqrt{E}} - \frac{Fxx.Fy.\sin(\chi)}{\sqrt{E}} + \frac{F_x.F_y.V.\sin(\chi)\left((Fx^2+1).B + 2Fx.Fxx.F\right)}{2E^{\frac{3}{2}}} \quad (16$$

$$F_xK + Fxx.Fxxx.W1 + 2Fxxy.Fxy.W2 + 2FxyyFyyW3)/(\frac{\sin(\chi).D.A}{\sqrt{f}.C} - \frac{\cos(\chi).D\left(\frac{v_w\cos(\chi)}{D} + \frac{u_w\sin(\chi)D}{\sqrt{f}} + \frac{Fx.Fy.\sin(\chi)}{\sqrt{E}}\right)A}{\sqrt{f}.C^2} \quad (17$$

Where:

$$A = W1Fxx^2 + W2\,Fxy^2 + W3\,Fyy^2 - K + FK + 1$$

(18)

$$B = 2\,Fx\,Fxx + 2\,Fxy\,Fy$$

$$C = \frac{V\cos(chi)^2}{\sqrt{f}} - \frac{v_w\sin(chi)}{D} + \frac{V\sin(chi)^2}{\sqrt{f}} + \frac{u_w\cos(chi)\,D}{\sqrt{f}} + \frac{FxFyv\_wcos(chi)}{\sqrt{E}}$$

$$D = \sqrt{Fx^{\ 2} + 1}$$
$$E = (Fx^{\ 2} + 1).f \quad (19)$$
$$f = Fx^2 + Fy^2 + 1$$

By solving the three equations of (4), (5), (16), one can enumerate the optimum trajectory. The Euler algorithm for solving the first-order nonlinear differential equation is considered. In this algorithm with knowledge of initial x(0), y(0), and guessing the χ(0) we start to reach the final x and y.

A. Algorithm Solution

In this schematic, the field pattern is defined by (20).

$$f(x, y) = \sin(x) . \cos(y) \quad (20)$$

This is an easy feature to consider as ground. For the Euler solution we must consider good area for χ(0) and solving the x˙,y˙,χ˙ at the same time till we reach the x= x_f.Then the value of $|y - yf| < e$ must satisfy; Otherwise we have to choose another $x_0$for $\chi_0$we add to reach the final value of (x,y), this method known as the steepest descent[15].

a) Variable of ***V***

As mentioned (V) is the control variable along with χ. In this section as appears by $\frac{\partial H}{\partial V} = 0$, V is linear in the Hamiltonian equation. Maximum speed normally depends on a structure of the flying vehicle and by using the Pontriagen optimal condition for limited law control the optimal control is Bang-Bang.

$$V = \begin{cases} V_{max} if S < 0 \\ V_{min} if S > 0 \\ singular if S = 0 \end{cases} \quad (21)$$

S, defines by:

$$S = \frac{\partial H}{\partial V} = \frac{\{\lambda_x(\sqrt{1 + F_x^2 + F_y^2} \cos(\chi) + F_x F_y \sin(\chi))\}}{\sqrt{((1 + F_x^2 + F_y^2)(1 + F_x^2))}} + \frac{-\lambda_y(1 + F_x^2)\sin(\chi)}{\sqrt{(1 + F_x^2)(1 + F_x^2 + F_y^2)}} \quad (22)$$

By substituting λ_y, λ_x in (22), S change into:

$$S = \frac{[(1 - k) + kF + (W_1 F_{xx}^2 + W_2 F_{yy}^2 + W_3 F_{xy}^2)]}{V} \quad (23)$$

In (23) V is always positive and with the Bang-Bang theory, S needs to be negative to be in the form of optimum control. This algorithm applies before the flight to create the optimal path based on minimum time, altitude and vertical acceleration.The addition of the thread region in the path allows the algorithm to design the path so that it does not pass through the thread region. Due to the fact that proper design of cost functions is very important in the formation of dynamic constraints, but importing all existing constraints is a costly and laborious task that is not cost-effective. This paper aims to combine the dynamic into the kinematic equations in path planning with regard of wind gust. By combining the equations the numerous calculation decrease and we are only deal with kinematic equation which is one of the advantage of this method.

The dynamic equations are:

$$\frac{d\gamma}{dt} = \left(\frac{L + T\sin(\alpha)}{mV}\right)\cos(\emptyset) - (\frac{g\cos(\gamma)}{V}) \quad (24)$$

$$\frac{d\psi}{dt} = \left(\frac{L + T\sin(\alpha)}{mV\cos(\gamma)}\right)\sin(\emptyset) \quad (25)$$

$$\frac{dh}{dt} = V\sin(\gamma) \quad (26)$$

$$\frac{dx}{dt} = V\cos(\psi)\cos(\gamma) \quad (27)$$

$$\frac{dy}{dt} = V\sin(\psi)\cos(\gamma) \quad (28)$$

The equation of (24-28) represent the aerodynamic and propulsion of the flying vehicle that makes the flying vehicle can maneuver with the heading and decent/ climb rate. These equations depends on time, (x,y) are trajectory coordinate in horizontal plane, (h) is the attitude of flying vehicle, ψ,γ,ϕ represent rolling, path, heading angle in order. V is the speed of the plane, (L, D) illustrates the aerodynamics and drag to the air vehicle, and T is the propulsion represents by (29):

$$T = f_T(\eta) T_{max}(M, h) \quad (29)$$

Which in (29), η is the engine throttle and 0<η<1, f_T it depends on engine specification, T_max is maximum propulsion of the engine. Equation (22-24) is confused and can be written as follows:

$$(L + T\sin(\alpha))^2 = (m.V\cos(\gamma)\dot{\psi})^2 + (mV\dot{\gamma} + mg\cos(\gamma))^2 \quad (30)$$

As the (30) illustrates, the left-hand side (LHS) of the equation is Requirements of the Terrain Maneuver Power (RTMP) which includes propulsion and aerodynamic power. The Right Hand Side (RHS) of the equation are limited by terrain avoidance essential. The variables of γ˙(,ψ)˙ in equations of (30) are also a function of the coordinate directions.

As mentioned, the goal of this paper is to blending the dynamic into the kinematic equation. In this section, each component is designed according to the equations of the field and eventually replaced.

$$\dot{\psi} = \psi_x \dot{x} + \psi_y \dot{y} + \psi_\chi \dot{\chi} \quad (31)$$
$$\dot{\gamma} = \gamma_x \dot{x} + \gamma_y \dot{y} + \gamma_\chi \dot{\chi}$$

Equations of (31) are partial derivatives of (x,y,χ) with assuming the (31) are depend on (x,y,χ). To change the existing parameters, it is necessary to change the dynamic parameters. By blending the equations (27, 28), it can be achieving that:

$$y_x = \frac{dy}{dx} = \tan(\psi) \quad (32)$$

With inverting the (32) and taking the derivate in x, y, χ we can wrote:

$$\psi_x = \frac{y_{xx}}{1+y_x^2}$$
$$\psi_y = \frac{y_{xy}}{1+y_x^2} \qquad (33)$$
$$\psi_\chi = \frac{y_{x\chi}}{1+y_x^2}$$

Which in (33) y_x is depending on x, y, χ. On other way by combing the (1, 3) we can obtain the γ :

$$h_x = \frac{\tan(\gamma)}{\cos(\psi)} \qquad (34)$$

By deriving the (34) with x, y, χ (35-37) illustrated:

$$\gamma_x = \frac{h_{xx}.\cos(\psi) - h_x \sin(\psi)(\frac{y_{xx}}{1+y_x^2})}{1 + h_x^2 \cos(\psi)^2} \qquad (35)$$

$$\gamma_y = \frac{h_{xy}.\cos(\psi) - h_x \sin(\psi)(\frac{y_{xy}}{1+y_x^2})}{1 + h_x^2 \cos(\psi)^2} \qquad (36)$$

$$\gamma_\chi = \frac{h_{x\chi}.\cos(\psi) - h_x \sin(\psi)(\frac{y_{x\chi}}{1+y_x^2})}{1 + h_x^2 \cos(\psi)^2} \qquad (37)$$

In equation of (35-37), $y_{xy}, y_{x\chi}, y_{xx}$ evaluate by deriving base on (34) but first need to y_x rewrite by terrain.

$$y_x = \frac{1 + F_x^2}{\frac{\sqrt{1+F_x^2+F_y^2}}{\tan(\chi)} + F_x.F_y} \qquad (38)$$

By deriving the (38) with x, y, χ, $y_{xy}, y_{x\chi}, y_{xx}$ represent by:

$$y_{xx} = \frac{vx + \frac{V sin(chi) C.A}{B * E^{\frac{3}{2}}} - \frac{FxFxxV sin(chi)}{C.\sqrt{E}}}{B} + \left(\left(v_w - \frac{V sin(chi).C}{\sqrt{E}}\right)\left(\frac{FxFxxV \cos(chi)}{\sqrt{(Fx^2+1)^{\frac{3}{2}}}}\right) - \left(\frac{FxFxyV \sin(chi)}{\sqrt{D}} - \frac{FxxFyV \sin(chi)}{\sqrt{D}} + \frac{FxFyV sin(chi)\ ((Fx^2\ +\ 1)\ A\ +\ 2\ FxFxxE)}{B.D^{\frac{3}{2}}}\right)\right) / B^2 \qquad (39)$$

Where:

$$A = 2F_x.F_{xx} + 2FxyFy$$

$$B = uw + \frac{V cos(\chi)}{C} + \frac{2F_x.F_{xx} + 2F_y.F_{yx}}{\sqrt{D}}$$
$$C = \sqrt{1 + F_x^2}\ ,$$
$$D = (Fx^2 + 1)E$$
$$E = \sqrt{1 + F_x^2 + F_y^2}$$

$$y_{xy} = \frac{vy + \frac{V \sin(\chi) CA}{2 * E^{1.5}} - \frac{Fx.Fxy.V sin(\chi)}{C sqrt(E)}}{2} - \frac{\left(\left(\frac{v_w - V \sin(\chi) C}{\sqrt{E}}\right)\left((uy) - \frac{FxFxyV \cos(\chi)}{(Fx^2\ +1)^{\frac{3}{2}}} + \frac{FxFyyV \sin(\chi)}{\sqrt{D}} + \right.\right.}{B^2} \qquad (40)$$

Where:

$$A = 2\ FxFxy\ +\ 2\ FyFyy$$

$$B = uw + \frac{V\ cos(chi)}{C} + \frac{Fx\ Fy\ V\ sin(chi)}{\sqrt{D}}$$

$$C = \sqrt{fx^2 + 1}$$
$$D = (fx^2 + 1) * E$$

$$E = fx^2 + fy^2 + 1$$

$$y_{x\chi} = \frac{\left(\left(\frac{V sin(\chi)}{B} - \frac{FxFyV \cos(chi)}{C}\right)\right)}{A^2} + \frac{(v_w - \frac{V sin(chi).B}{\sqrt{Fx^2 + Fy^2\ + 1}}}{A^2} - \frac{V cos(\chi).B}{A\sqrt{fx^2 fy^2 + 1}} \qquad (41)$$

Where:

$$A = u\_w + \frac{V \cos(chi)}{B} + \frac{FxFyV sin(chi)}{C}$$

$$B = \sqrt{fx^2 + 1}$$

$$C = \sqrt{(fx^2 + 1)(fx^2 + fy^2 + 1)}$$

Now the $y_{xy}, y_{x\chi}, y_{xx}$ are obtained and with substituting in (35-37)$\dot{\psi}, \dot{\gamma}$ are illustrated:

$$\dot{\psi} = \frac{-1}{(1+y_x^2)(\frac{P_2}{\tan(\chi)}+F_x.F_y)^2}\Big\{\Big[2P_1P_{1x}\Big(\frac{P_2}{\tan(\chi)} + F_xF_y\Big) - P_1^2\Big(\frac{P_{2x}}{\tan(\chi)} + F_{xx}F_y + F_xF_{xy}\Big)\Big]\dot{x} + \Big[2P_1P_{1y}\Big(\frac{P_2}{\tan(\chi)} + F_xF_y\Big) - P_1^2\Big(\frac{P_{2y}}{\tan(\chi)} + F_{xy}F_y + F_xF_{yy}\Big)\Big]\dot{y} - [A^2 B cosec(\chi)]\dot{\chi}\Big\} \quad (42)$$

$$\dot{\gamma} = \Bigg[\frac{1}{(1+y_x^2)^2(\frac{p_2}{\tan(\chi)}+F_xF_y)^2(1+{h_x}^2\cos(\psi)^2)}\Big\{\Big[h_{xx}.\cos(\psi) - h_x\sin(\psi)\,1+y_x^2 + 2P_1P_{1x}\Big(\frac{P_2}{\tan(\chi)}+F_xF_y\Big) - P_1^2\Big(\frac{P_{2x}}{\tan(\chi)}+F_{xx}F_y + F_xF_{xy}\Big)\Big]\dot{x} + \Big[h_{xy}.\cos(\psi) - h_x\sin(\psi)1+y_x^2 + 2P_1P_{1y}\Big(\frac{P_2}{\tan(\chi)}+F_xF_y\Big) - P_1^2\Big(\frac{P_{2y}}{\tan(\chi)}+F_{xy}F_y + F_xF_{yy}\Big)\Big]\dot{y} + h_{x\chi}.\cos(\psi)\,(1+y_x^2) - h_x\sin(\psi)A^2Bcosec(\chi)]\dot{\chi}\Big\} \quad (43)$$

Where:

$$P_{1x} = \frac{(F_{xx}.F_x)}{P_1},\quad P_{2x} = \frac{(F_{xx}.F_x + F_y.F_{yx})}{P_2},$$
$$P_1 = \sqrt{1+F_x^2},\quad P_1 = \sqrt{1+F_x^2+F_y^2}$$

With replacing the (42, 43) in (31), the equations written as:

$$RTMP^2 = (mVcos(\gamma)[\psi_x\dot{x} + \psi_y\dot{y} + \psi_\chi\dot{\chi}])^2 + (mV[\gamma_x\dot{x} + \gamma_y\dot{y} + \gamma_\chi\dot{\chi}])^2 \quad (44)$$

The (44) is an algebra equation and can be solved based on $\dot{\chi}$, by replacing the $\dot{\chi}_{opt}$ the optimal RTMP can be obtained. In fact RTMP is an evaluation of aerodynamic and propulsion ability of aerial vehicle in pursuing of terrain. The real answer of this equation means the flying vehicle can path through to terrain but if the answer is imaginary the flying vehicle is not allowed to go through in the terrain.

B. Control Variable Calculation

In the anterior section the RHS of the RTMP is deliberated. In this section the control variables of T,α,ϕ are evaluated. Then the airspeed of the air vehicle has to be maximum, so:

$$V = V_{max} \Rightarrow \dot{V} = 0 \quad (45)$$

With replacing the (45) into the speed dynamic equation indicate [15]:

$$Tcos(\alpha) - D - mgsin(\gamma) = 0 \quad (46)$$

By rewriting the (30), (47) illustrate:

$$T.\sin(\alpha) + L = RTMP \quad (47)$$

The lift and drag represent by:

$$L = L^\alpha\alpha$$
$$L^\alpha = \frac{1}{2}\rho V^2 SCL_\alpha$$
$$D = D^0 + D^\alpha\alpha^2$$
$$D^0 = \frac{1}{2}\rho V^2 SCD_0 \quad (48)$$
$$D^\alpha = \frac{1}{2}\rho V^2 SKCL_\alpha^2$$

With exchanging the equations of (46-48), two equations accomplish based on (α).

$$Tcos(\alpha) - D - mgsin(\gamma) = 0 \quad (49)$$
$$T.\sin(\alpha) + L = RTMP$$

By solving (49), L, α represent by:

$$T_{req} = \frac{RTMP}{\alpha_{req}} - L^\alpha \quad (50)$$

$$D^\alpha\alpha_{req}^3 - (L + D^0 + mgsin(\gamma))\alpha_{req} + RTMP = 0 \quad (51)$$

Afterward, the calculation of the require propulsion force and angle of attack is completed; however it should be notice that:

If the evaluated $\dot{\chi}$ in the α_{req}equation pass the allowed band, χ must retain to the closest band and α_{req} must calculatedagain.

If T_{req} in (50) becomes more than $T_{available}$ which is determined by ability of engine performance in altitude and speed of flight. It must put the amount of require propulsion with available propulsion and calculate the RTMP again.

$$\alpha_{req} = \frac{RTMP}{T_{req} + L^\alpha} \quad (52)$$

ISSN No:-2456-2165

III. GAME THEORY

In the previous section we dealt with the single flying vehicle (agent) and tried to find the optimal path with terrain following and avoidance. This section deal with two agents which each agent has own path and trying to engage one another.

In this situation this paper aims to use differential game theory for defining the new cost function in the low altitude and time. The differential game has a several implementations, the non-zero-sum games are: Stackelberg and Nash game. Which each game has own performance, Assuming the each player has own information. A differential game follows as:

$$\dot{x} = f(x, \phi, \psi, t) \qquad x(t0) = x0 \tag{53}$$

Where x(t) is a state of equations and the $\phi(t)$ is the control player 1 and ψ is the control player 2. The initial and final constraints are $\mathrm{R(x(t0), t0) = 0}$, $\mathrm{P(x(tf), tf) = 0}$ in order respectively , tf is free. This paper is uses zero-sum game which means two participants has own decision and loss of one decision maker gain equal with wining the other participant gain[14].

$$G_1 = -G_2 = G \tag{54}$$

The cost function defined by:

$$J[(x, \phi, \psi, t)] = minmax(g(x(tf), tf) + \int_{t0}^{tf} L\big((x, \phi, \psi, t)\big)\, dt) \tag{55}$$

Which is in (55) the (L) must be optimized and, g is the final condition. These terms with two agents are in the form of pursuit and evader and assumed the formation is consist during the mission.

A. Formulation of flying path

As stated in the previous section, the assumption that both agents are aware of their beginning and the situation remains the same until the end of the game is resolved. This condition is restrictive, but shows its special capability in some maneuvers. The subject is studied with a simple flight of the agent. In this paper, the mass point model is used to solve issues. As the previous section, it used the terrain following and avoidance with consideration of wind gust

$$h = f(x, y) + h_c \tag{56}$$

Since the equations (4, 5) are for one agent, by generalizing these equations for two agents, the equations are written as follows [14].

$$\dot{x_1} = \frac{V_1 cos(\chi_1)}{\sqrt{1+f_{x_1}^2}} + \frac{V_1 F_{x_1} F_{y_1} \sin(\chi_1)}{\sqrt{1+f_{x_1}^2}\sqrt{1+f_{x_1}^2+f_{y_1}^2}} + u_1 \tag{57}$$

$$\dot{y_1} = \frac{V_1 \sin(\chi_1)\sqrt{1+f_{x_1}^2}}{\sqrt{1+f_{x_1}^2+f_{y_1}^2}} + v_1 \tag{58}$$

$$\dot{x_2} = \frac{V_2 cos(\chi_2)}{\sqrt{1+f_{x_2}^2}} + \frac{V_2 F_{x_2} F_{y_2} \sin(\chi_2)}{\sqrt{1+f_{x_2}^2}\sqrt{1+f_{x_2}^2+f_{y_2}^2}} + u_2 \tag{59}$$

$$\dot{y_2} = \frac{V_2 \sin(\chi_2)\sqrt{1+f_{x_2}^2}}{\sqrt{1+f_{x_2}^2+f_{y_2}^2}} + v_2 \tag{60}$$

Index (1) and (2) is for the evader and pursuit agent respectively. It is assumed the pursuer is faster than the evader for the physical meaning, so that the flying vehicle engages with the target.

$$V_2 > V_1 \tag{61}$$

If the altitude variation considered zero, the problem will be very easy to trap the evader.

B. The game theory condition

Based on the zero-sum game, the game ends when the pursuer reaches the evader or its circular range (d). When the pursuer is in accepting range, then the game is over. The equation of (62) is used to approach.

$$(x_1 - x_2)^2 + (y_1 - y_2)^2 < d^2 \tag{62}$$

In Equation (62), (d) is the approach range of two agents and has a specific value.

C. Cost function in Game Theory

In most two-player games, the measure of performance is the timing of the chase. It is obvious that the evader is trying to increase his time to avoid getting caught and the pursuer is trying to reduce the time to the lowest altitude. So, the cost function is regarded by (63, 64):

$$J = min.\max\big(Q(tf)\big) + \int_{t0}^{tf} (1 - K + K(1 + w_2 f_2 - W_1 f_1))dt \tag{63}$$

$$Q(tf) = \frac{\nu}{2}\{((x_1 - x_2)^2 + (y_1 - y_2)^2 - d^2)\} \tag{64}$$

In the above equations, the(w_1,w_2) is the weight functions and the negative sign indicate that the evader agent wants to maximize the cost function. The ν parameter is also an indeterminate coefficient in the system that depends on the number of agents.

D. Solving Algorithm of Game theory

Since in optimal control require to create a Hamiltonian function, first need to generalization the function for two agents, the optimal solution for several agents using game theory is expressed by Equation (65).

$$H = 1 - K + K(w_2 f_2 - W_1 f_1) + \lambda_1 \dot{x}_1 + \lambda_2 \dot{y}_1 + \lambda_3 \dot{x}_2 + \lambda_4 \dot{y}_2 \tag{65}$$

Considering the cost function and the continuous equations, the sufficient condition for the game to reach the actual result, the cost function must be extreme. Therefore, to solve these equations, first of all, the quasi-state equations must solve and then with a final condition for quasi-state, can proceed backward to solve the Hamiltonian function.

The final conditions are:

$$\begin{aligned} \lambda_1(tf) &= -\nu(x_2 - x_1) \\ \lambda_1(tf) &= -\nu(y_2 - x_1) \\ \lambda_1(tf) &= \nu(x_2 - x_1) \\ \lambda_1(tf) &= \nu(x_2 - x_1) \end{aligned} \quad (66)$$

(ν)can be determined by substituting the (66) into (65):

$$\nu = \frac{1 + W_2 f_2 - W_1 f_1}{\dot{x_1}(x_2 - x_1) + \dot{y_1}(x_2 - x_1) - \dot{x_2}(x_2 - x_1) - \dot{y_2}} \quad (67)$$

E. Numerical Simulation of Game Theory

Most of the numerical methods used in game theory are related to simple targeting with the help of dynamic programming methods or using the control methods to obtain game points in this theory[10-12]. In this research, the retrospective summation method has been used to produce the extreme path[16-17]. To produce the optimal path, the final coordinate points are first determined. Then, having an escape point, the following points are obtained according to Equation (62). Then we find ν according to the relation (67) and by placing it in the equations of state and quasi-state we will come to the beginning from the last time. However, the Equation of (24) is used for control variables (χ). In the different routes to reach the target point, the tracking point can be used. All the relation and RTMP with consideration of wind effect are evaluated in the game theory.

IV. SIMULATION

The aim of this section is to simulate the path planning of flying vehicle with regard of minimum time and altitude. Since the primary and final points of the path are determined, the algorithm begins with initial heading angle bounded limits. The equations of (4), (5), (16), must solve at the same time to reach the $x(t) = x_f$, with compare of y_f with final y in the limited error we can choose the selected χ before the plane flew.

The steepest descend algorithm is applied in this scenario. The (K)variable is an important because by using this variable the minimum time and altitude can be reached. The most important thing in defining the (F) function is the function must have a first and second derivative order. In order to define the terrain and thread avoidance, for modeling of real 3d points, This paper used the spline fitting method [17]. The aerodynamic coefficient of unmanned aerial vehicle is mass-point data and the flying vehicle attends to reach the $xf = 9000(m), yf = 6000$. Fig.2, represent the model of the terrain.

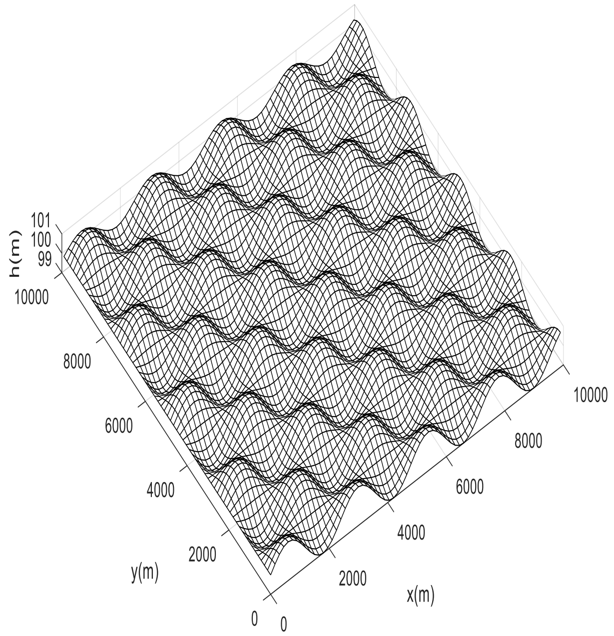

Fig. 2: The model of terrain

Fig.3,4 illustrates the (x,y) variable base on χ. The primary angle of heading is represented as a control variable in optimal path. We can show that with a setting of proper angle can be reach to the final point. Behave of y_f with $W = 0, V_{max} = 200(m/s)$ and $K = 0,1$ shows in fig.3,4.

Table 1 represents the specification of the unmanned aerial vehicle.

$V\ (speed)$	$50(m/s)$
W (weight)	1000
T_{av}	$10000(N)$
x_f, y_f	$9000{,}6000\ (m)$
$u($ Down range wind gust$)$ $v($cross range wind gust$)$	$u = 60\cos(x)\ \ (m/s)$ $v = \sin(x).\cos(y)\ \ (m/s)$
$d(diameter)$	$10\ (m)$

Table 1: Specification of aerial vehicle

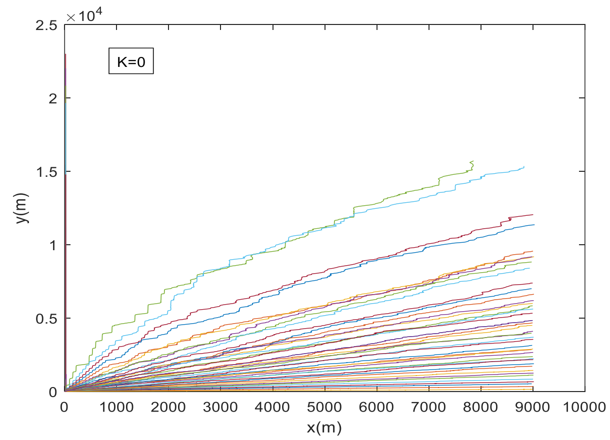

Fig. 3: tracks of final point with different heading angle in K=0

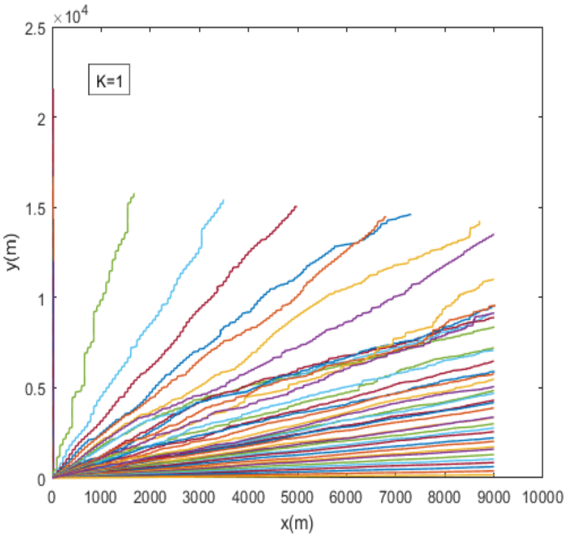

Fig.4. The tracks of optimal point with lowest altitude K=1

As the fig.3, and fig.4, demonstrate, when the (x) reaches to the final point with minimum error then algorithm will try to execute the (yf) with minimum error. Fig.5, illustrates the vertical forces with the weight off$W = 1000, W_1 = W, W_2 = 2W, W_3 = W$. As the results represent there are fewer maneuver with considering the acceleration, because the aerial vehicle cannot follow the terrain in the limited area. Fig.5,shows the best path according with or without the vertical acceleration.

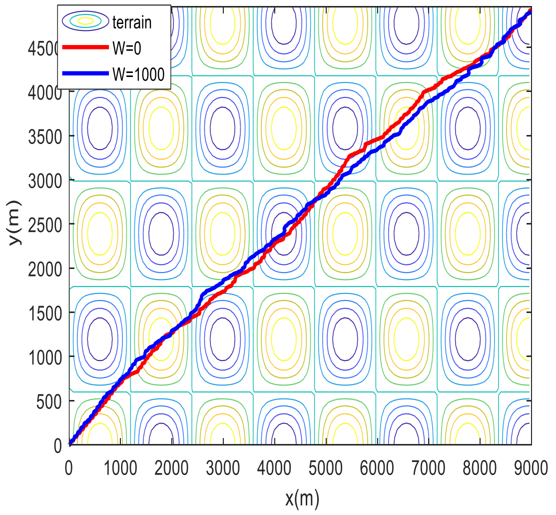

Fig. 5: Difference between with or without vertical acceleration in K=0, W=1000

Fig.6, Represent the surface data with K= 0, 1. In minimum time, the algorithm tries to find the closest path without considering the altitude; however, in minimum altitude the closest path following regardless of considering the time, the plane will go to the lowest altitude. Table.1 illustrate the time of reaching to the target regarding the minimum time and altitude in the present of vertical acceleration and wind gust. The table.1 shows when $(K = 0)$ the time of reaching to final point is in minimum time and when the perturbation like wind gust approach the time increase, as for vertical acceleration.

Table.2 illustrates the different situation of problem in the present of wind gust and vertical acceleration. As the data representation, the time of reach point when these constrains add to the problem is influenced.

K	W	Wind gust	Time(sec)
0	0	0	93
0	1000	0	91
0	0	$u = 60\cos(x)$ $v = \sin(x).\cos(y)$	667
1	0	0	96
1	0	$u = 60\cos(x)$ $v = \sin(x).\cos(y)$	759
1	1000	0	97

Table 3: Different types of constraints

Fig.6, demonstrates the minimum time and altitude in the present of wind effect. As the result shows with consideration of wind gust, the time of path following in the final point is bigger than the result of fig.6, as expected but the new cost function capable to solve the problem.

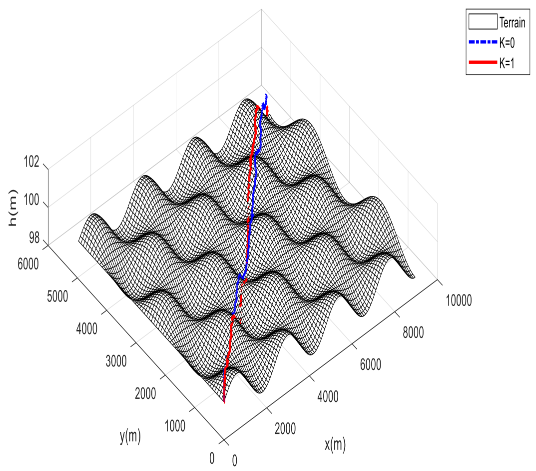

Fig. 6: The difference of minimum altitude and time in terrain following.

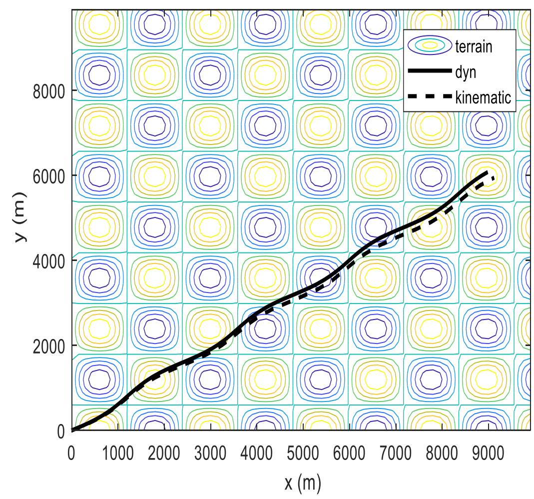

Fig. 7: Terrain following with consideration of wind gust

ISSN No:-2456-2165

To analyze the control variable and state of the system the dynamic system must follow the terrain, it must evaluate the aerodynamic and propulsion force with consideration of RTMP in the present of vertical acceleration, minimum time and altitude, however, the LHS must be bigger than RHS. With this condition the aerial vehicle has the strength to follow the terrain. Fig.8, represent the χ optimal angle for minimum time and altitude. As the figure illustrates when the aerial vehicle wants to follow the terrain in minimum time, the plane has to follow the tip of the terrain to reach the final point, however, when the plane wants to put through in low altitude the plane will follow the lowest point of terrain which doesn't need to change its local heading.

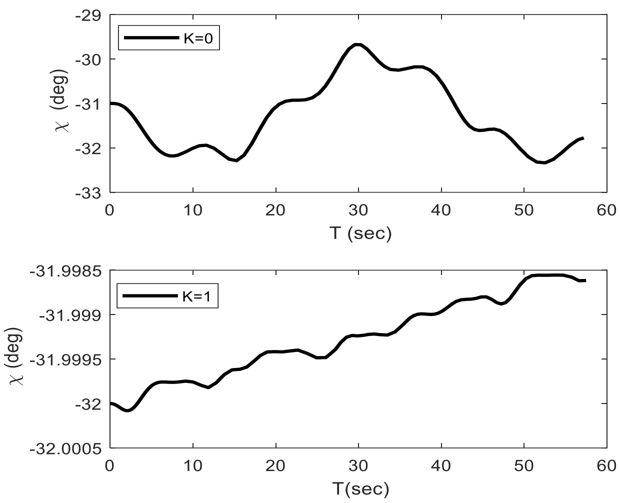

Fig. 8: the local heading angle with present of wind gust

Fig.9, represent the (γ, ψ) angles for minimum time and altitude in the present of wind gust. Since in $(K = 1)$, the aerial vehicle will go to the straight path with regard of the terrain so the χ angle doesn't changing too much.

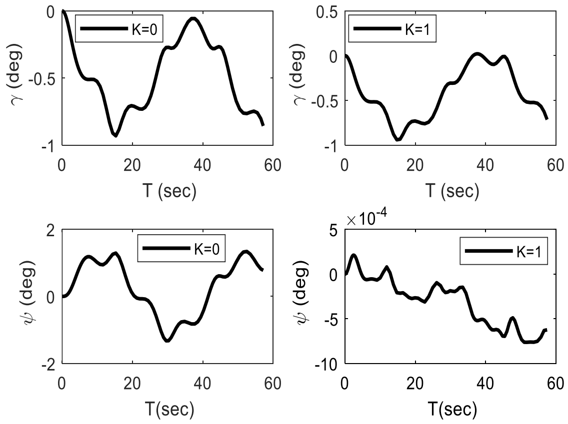

Fig.9. Difference between the γ, ψ angle in lowest time and altitude with wind gust

Fig.10, represent the require thrust with consideration of minimum time and altitude. As the result indicates, the thrust require the aerial vehicle in the higher altitude is less than the lowest altitude because of air density.

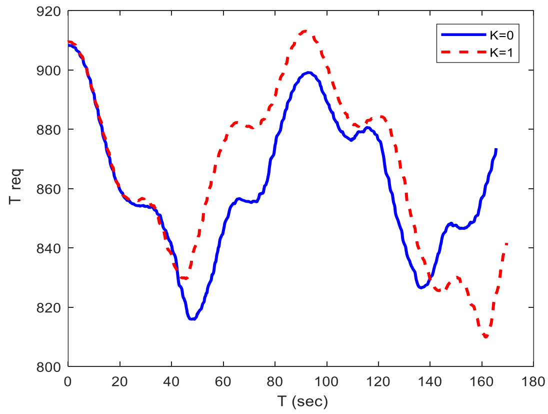

Fig.10: The require thrust for terrain following

The next figures endeavor to discuss two agents.Fig.11, demonstrate the two agent in the case of pursuer and evader. Based on game theory algorithm, the pursuer will try to reach the evader. As the result shows when the evader flying between the terrains, the pursuer wills follow the evader until reaches to the limited area. Fig.12, illustrate the pursuing of two aerial vehicles and they targeting another as the result represented, the twoobject reached within acceptable range.

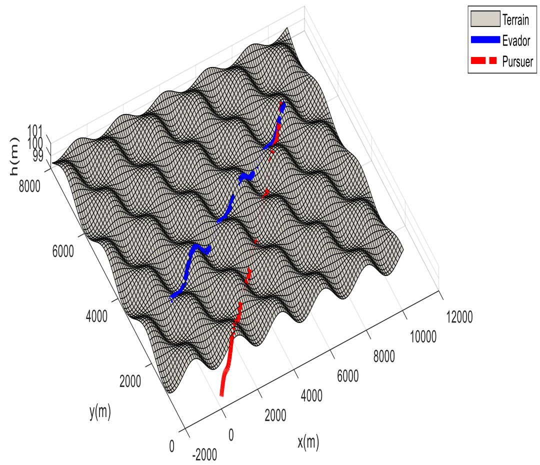

Fig. 11: Two agent in the form of pursuer and evader

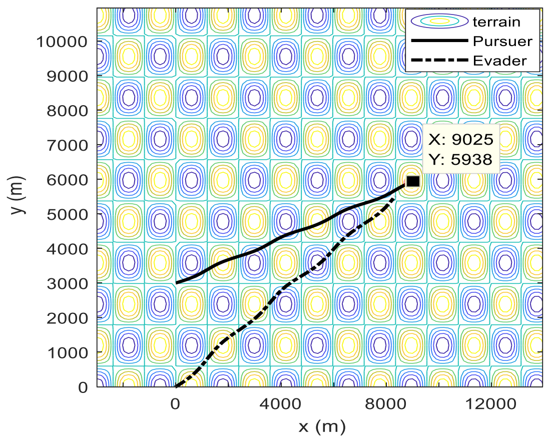

Fig.12. The contour of terrain with pursuer and evader

Fig.13: Shows the following of two agents without the wind gust

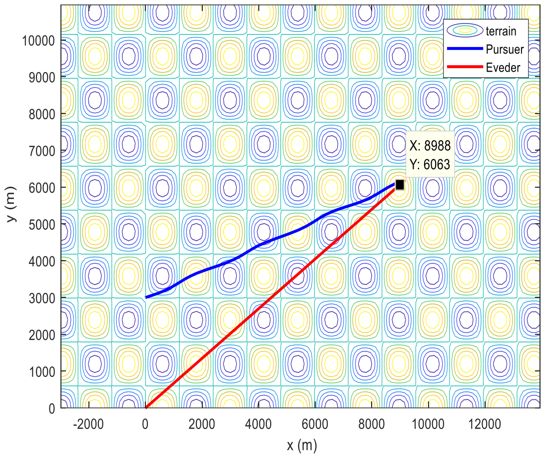

Fig.13: Two agent regardless of wind gust

As the result represents this algorithm has ability to pursue the target in terrain following consideration of wind gust and vertical acceleration.

V. CONCLUSION

In this paper has been tried to reach the final point in terrain following and avoidance with consideration of defining combination of kinematic and dynamic equation. As the result showed this combination has the ability to consider constrains as vertical acceleration and wind gust in minimum time and altitude. In the second part the game theory in terrain following a combination of kinematic and dynamic was studied, and the results illustrated the benefit of this method in the present of wind gust and vertical acceleration.

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