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# A Rigorous Experimental Technique for Measuring the Thermal Diffusivity of Metals

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**Abstract:- In this paper, we present a basis for rigorous experimental measurements of the thermal diffusivity α of metals and hence their thermal conductivity K in 3D geometric material objects rather than the 1D objects used in current literature. The proposed experimental technique is based on the numerical method known as the Cairo technique. The key point of this research is that the new numerical method anticipates an exponential cooling curve for material objects and describes the exponent control equation as a function of thermal diffusivity and body dimensions.**

**We measured the time-dependent 3D temperature field during the cooling curve in standard 10 cm cubes of pure Egyptian aluminum and high-grade Russian low-carbon steel. The experimental results obtained confirm the exponential cooling curve and presented precise values of thermal diffusivity of aluminum and steel in good agreement with those of the thermal tables.**

**The experimental measurements of the time-dependent 3D temperature field confirm the validity of the proposed experimental technique and the accuracy of its basis in the numerical method called Cairo technique.**

## I. INTRODUCTION

The object of this work is to numerically solve and experimentally validate the general form of the time-dependent equation of heat diffusion in 3D geometric space, i.e.,

d / dt (partial) U (x,y,z,t) = α Nabla$^2$ U (x,y,z,t) + S (x,y,z,t) .. . . . . . . (1)

Subject to the boundary conditions BC of Dirichlet on the limits of the domain of U and the initial conditions IC of U at t=0 that is U ( x,y,z,0).

Obviously, the thermal energy density U(x,y,z,t) is equal to $K_B$ T(x,y,z,t) where $K_B$ is the Boltzmann constant and T is the absolute temperature in degrees Kelvin.

To our knowledge, the solution of the time-dependent heat partial differential equation (1) in 2D and 3D geometry considering simultaneously BC and IC is absent from the current literature.

The analytical solution is inaccessible or a false analytical solution and the numerical solution is inaccessible or a false numerical solution.

In fact, the solution of the heat partial differential equation (1) was only accessible in one dimension and therefore the thermal diffusivity α is always defined as a scalar α=K/ρ C in normal conventions and therefore its experimental measurements were made in 3 consecutive steps ρ ,C and K in one dimension of space [1,2].

The temperature T in the energy density diffusion equation is strictly defined as absolute in degrees Kelvin and so we have to solve equation 1 for arbitrary boundary conditions BC and initial conditions IC simultaneously. Assuming alternatively that IC or BC is zero is not sensible and is not physics since absolute temperature can never go to zero.

However, we have proposed and described in some previous papers a new numerical method capable of solving the time-dependent heat equation in 3D geometry with arbitrary BC and IC which has proven effective in solving diffusion limit value problems. (heat, electrical potential, sound intensity in audio rooms, etc. [3,4,5].)

The proposed new numerical technique is based on the matrix formalism of the heat diffusion equation with its cornerstone called transition matrix B which is well defined and capable of producing unique, stable and fast convergent solutions.

In fact the matrix formalism of the diffusion problem of Dirichlet boundary conditions with arbitrary IC is essential in itself because it converts complicated and intractable 3D time-dependent phenomena into a much simpler phenomenon because:

The proposed matrix formalism allows passing or transforming the Dirichlet boundary conditions of the 2D and 3D geometry into an adequate 1D boundary condition. It is the same for the initial conditions IC. Moreover, the proposed matrix formalism also allows to pass or transform the 2D and 3D temperature energy density distribution field into an adequate 1D field.

The Schrödinger equation which is at the heart of quantum mechanics can be considered as a diffusion equation with a complex diffusion coefficient ih(bar)/2m [6] and therefore one

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can claim to apply the chains of the matrix B in its solution but the application of the new B matrix technique in QM is still a subject of speculation since the wave, the psi function and the potential energy field in general are complex functions.

## II. THEORETICAL AND EXPERIMENTAL ANALYSIS

In this work, theory and experiment cannot be separated, therefore theoretical and experimental analysis are added together. Throughout this work we carry out a continuation and a validation of the new numerical method known under the name of technique of Cairo [3]. In other words, our objective here is to seek a theoretical and experimental validation of the new numerical method which is explained in the present section II of theoretical and experimental analysis keeping in mind that they cannot be separated and should be summarized together.

*A. Theoretical View*

The cornerstone of the new numerical technique is the matrix formalism used in the description of the diffusion problem. In short, the B-Transition matrix is implicitly a time-dependent 3D geometry describing the diffusion phenomena of nature. The 3D cubic transition matrix a B nxn has an allowed number of free nodes n given by n=N^3 where N is a positive integer greater than or equal to 2 (2,3,4,..etc). Therefore, the allowed number of free nodes is 8, 27, 64, etc., but here we will limit our analysis to the case N = 3 and n = 27, as shown in Figure 1.

All matrices are two-dimensional and matrix operations follow the rules of linear algebra. In fact, these matrix operations rules are unique in that they find a way to transform or pass the two and three-dimensional Dirichlet boundary conditions to the nx1 matrix (called vector b). They also find a transform or to go from the 3D initial conditions at the free nodes concerned to the nx1 matrix (called the vector IC), and finally transform the 3D energy density field into the nx1 matrix (called U(x,y, z ,t) vector) by an appropriate arrangement and order of nodes in 3D space, as shown in Figures 1 and 2.

The role of the well-defined transition matrix B is surprising because it simultaneously gathers all vectors b, IC, source term vector S and U (x, y, z, t) in an accurate, stable and fast convergent solution for the time-dependent 3D heat equation Eq 1 as follows,

Assuming that the transfer matrix E is given by the sum of the matrix powers,

E(N)=B^0+B+B^2+B^3+. . . . +B^N . . . . . . . . . . . . . . . . . . (2)

Where B^0=I, the unitary matrix, then the time dependent solution,

U(N) = E(N) (b+S) + B^N (IC) . . . (3)

Where $U^N$(x,y,z,t) is the numerical value of energy density field at geometrical point x,y,z at the time t given by t=N dt with dt time jump or step.

E(N) is the transfer function after N iterations or N time steps.

Equation 3 is of utmost importance because it serves for the time-dependent transient solution and also applies to generate the time-independent steady-state solution for U(N). For a large N the transfer function E(N)=B^ 0 +B+B^2 +…..B^ N is simply found by the relation,

E(N)=1/(I-B) …. (4)
for N large enough.

Note that for N sufficiently large, B^N and therefore the contribution of the initial conditions IC tends towards zero and we end up with the contribution of the boundary conditions (Eq 3).

Now our task of performing theoretical calculations and experimental measurements of the thermal energy density scattering field can be well characterized as shown in the present Section II A, B and C.

Again, the transition matrix B is an nxn square matrix with the n = integer^3 allowed where the integer is greater than or equal to 2. This means that the matrix B must be 8x8,27x27,64x64 ..etc. Obviously, the accuracy of numerical calculations increases as n increases.

Throughout this work and to be specific, we consider without loss of generality the material tested in the form of a standard cube of side L=10 cm divided into a grid of 27 equidistant free nodes as shown in Figure 1.

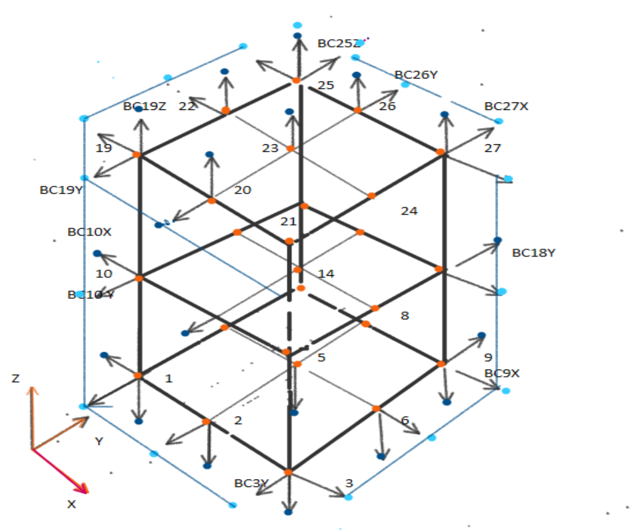


Fig 1. Cube of side L divided into 27 equidistant free nodes with 27 modified Dirichlet boundary conditions.

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We must first specify the essential theoretical and experimental considerations necessary for the realization of the experimental technique and present its adequate interpretation.

*B. Mathematical View*

The statistical transition matrix B which contains all the information for solving Equation 1 in the time-dependent 3D geometry of the cube in Figure 1 is given by a procedure similar to that followed in Reference 3, where the entries of the matrix B27X27 are expressed in the following form [3],

**27X27 B-Matrix inputs,**

Line1 RO 1/6-RO/6 0.0000 1/6- RO 1/6-RO/6 0.0000 0.0000 0.0000 0.00001/6-RO/6 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.00000.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000

Line 2 1/6-RO/6 RO 1/6-RO/6 0.0000 1/6-RO/6 0.0000 0.0000 0.0000 0.000 0.0000 1/6-RO/6 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.00000.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000

Line 3 0.0000 1/6-RO/6 RO 0.0000 0.0000 1/6RO/6 0.0000 0.0000 0.00000.0000 0.0000 1/6-RO/6 0.0000 0.0000 0.0000 0.0000 0.0000 0.00000.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000

. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . .

Line 14 0.0000 0.0000 0.0000 0.0000 1/6-RO/6 0.0000 0.0000 0.0000 0.00000.0000 1/6-RO/6 0.0000 1/6-RO/6 RO 1/6-RO/6 0.0000 1/6-RO/6 0.00000.0000 0.0000 0.0000 0.0000 1/6-RO/6 0.0000 0.0000 0.0000 0.0000. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . .. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . .

Line 25 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.00000.0000 0.0000 0.0000 0.0000 0.0000 0.0000 1/6-RO/6 0.0000 0.00000.0000 0.0000 0.0000 1/6-RO/6 0.0000 0.0000 RO 1/6-RO/6 0.0000

Line 26 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.00000.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 1/6-RO/6 0.00000.0000 0.0000 0.0000 0.0000 1/6-RO/6 0.0000 1/6-RO/6 RO 1/6-RO/6

Line 27 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.00000.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 1/6-RO/60.0000 0.0000 0.0000 0.0000 0.0000 1/6-RO/6 0.0000 1/6-RO/6 RO

In current thermal tables, K, ρ and C are measured in 3 independent steps, whereas in the proposed experimental technique explained in section III, α is measured directly in one step. In other words, in the current literature, the accepted inadequate scalar definition of thermal diffusivity α is known to be α=K/ρ C .

However, by experimentally measuring the transient temperature field in 3D combined with the theoretical method of the Cairo technique, Eq 3, one can calculate the thermal diffusivity which is contained in the main diagonal element RO of the so-called transition matrix chains B. We assume that this 3D experimental and theoretical technique is advantageous to calculate α in an isotropic and anisotropic material object.

Here, we obtain the appropriate numerical results for T(x, y, z, t) in the energy density scattering time domain with the appropriate value of RO and compare them with the experimental results in real experimental time. This procedure leads to obtaining the correct value of the thermal diffusion coefficient α.

It can be shown that [3,8],

B^N. I = (0.5+0.5 RO)^N .I. .. . . . . . . (5)

I is the unitary 27x27 matrix.

And,

α = Log (0.5+_0.5 RO) . . . . . . . . . .(6)

Equations 5&6 holds for all zero or positive values of N and all values of the diagonal input RO element of [0,1].

Moreover, the proposed numerical technique suggests an exponential cooling curve for material objects and offers a semi-imperial formula for the exponent as a function of thermal diffusivity [3,7,8],

T(t)at center of mass =T(0)Exp(-α .5.Pie.t) ……(7)

It is predicted that Eq 7 is valid for any regular shape, cubic or not, provided that the cooling temperature curve is measured precisely at the center of mass of the object and that the characteristic length L of the shape of the object is correctly evaluated mathematically or experimentally.

It follows from equation 7 that the thermal diffusivity α can be expressed in terms of the half-period $T_{1/2}$ "the time t after which the initial temperature T(0) drops to half its value" as,

α = Log 2./ (0.5 Pie .$T_{1/2}$) . . . . . . . . . . . . . . . . (8)

The equations 5 , 6 , 7 and are of great help in the proposed numerical technique.

Moreover, equation 8 predicts an experimental technique to measure the thermal diffusivity α by fitting the experimental cooling curve to an adequate RO to obtain $T_{1/2}$ and hence α. In fact, it is this method that we use here to experimentally find the thermal diffusivity of metals.

Note that equations 5, 6 and 7 are in a way a numerical and experimental validation of the transition matrix B.

*C-Experimental View*

We design a simple transient heat temperature experiment consisting of standard metal cubes of the material under test fitted with holes for temperature measurement Fig 2.

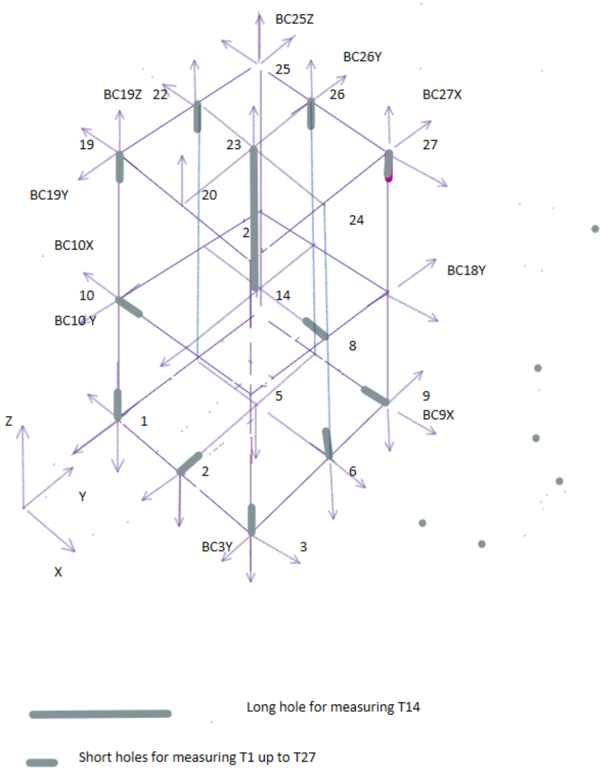


Fig 2. Cube of side L divided into 27 equidistant free nodes with 27 modified Dirichlet boundary conditions and fitted with holes for measurement of T(x,y,z,t)

In addition to the metal cubes, two large water reservoirs are prepared, one kept cold at the temperature $T_c$ and the other at the hot temperature $T_h$.

By instantly transferring the test cube from hot reservoir to cold reservoir or vice versa and measuring the temperature field at different nodes, we get the real-time cooling or heating curve function of real time t.

Note that $T_h$ and $T_c$ must be calculated in degrees Kelvin and one is used as IC and the other as BC when substituting in equation 3.

## III. EXPERIMENTAL SETUP AND EXPERIMENTAL RESULTS

We used a simple preliminary experimental setup that approximates Figure 2 which is photographed in Figure 3.

Fig 3. Experimental setup

We experimentally tested two metals to find the thermal diffusivity of 10 cm cubes of each. The first is high purity Egyptian aluminum alloy (purity equal to or above 98%) and the second is high grade low carbon Russian steel alloy. The resistance thermometers used to measure the temperature were TP 300 for the temperature range -50 to 300 C with an error less than or equal to 0.5 C.

The experimental results are represented in FIG. 4 (in black dots) and the half-period $T_{1/2}$ is deduced therefrom. Meanwhile, the exponential fit curve for the experimental results using Equation 3 (shown in red dots) gives the compatible RO approximately as follows:

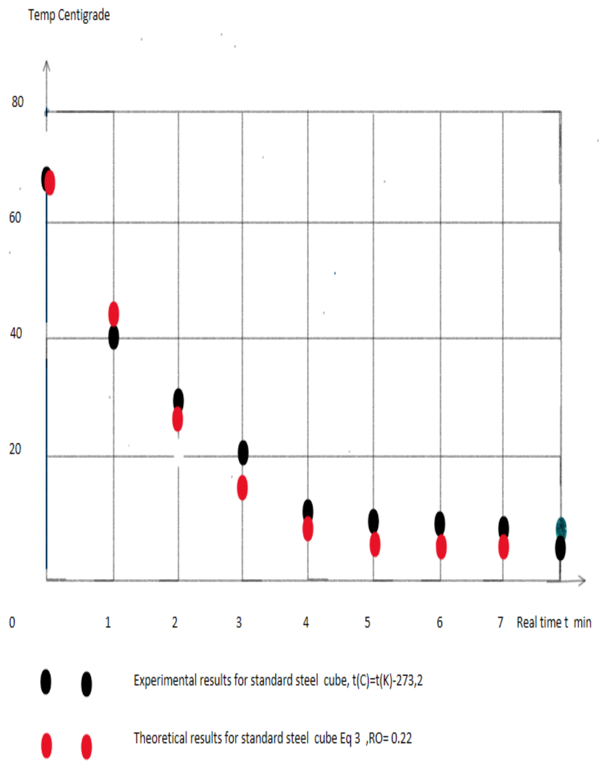


Fig 4a. Curve fitting through Equation 3 for the steel cube.

With RO for steal cube =0.22

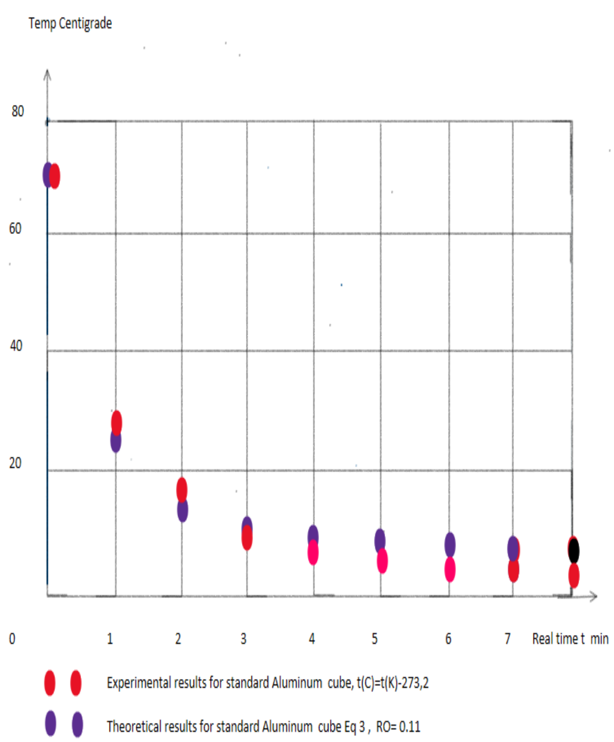


Fig 4b. Curve fitting through Equation 3 for the Aluminum cube with RO=0.13.

We calculate the thermal diffusivity α by using Eq 8,
$\alpha = L^2 . \text{Log } 2./ (0.5 \text{ Pie } .T_{1/2})$ . . . . . . . . . . . . . . . . (8)

Figure 4a shows that $T_{1/2}$ for the standard steel cube = 100 s, so α (steel)=44E-6 m^2 /s.

Figure 4b shows that $T_{1/2}$for the standard aluminum cube = 45 s, so α (aluminum) = 98 E-6 m^2 /s.

Note that:

- The black and red dots in curves of 4a and 4b present excellent fit which is in a way a validation for Cairo technique equations and prediction of exponential cooling curve for any RO element of [0,1]. with RO=0.22 for steal and 0.13 for Aluminum.
- The resulting values of thermal diffusivity α for steal )=44E-6 m^2 /s.and α for Aluminum = 98 E-6 m^2 /s are in good agreement with their values in thermal tables.[9]

## IV. CONCLUSION

We present a new experimental technique to measure the thermal diffusivity α and/or the thermal conductivity K of metals. The new experimental technique is intended to apply to any regular shape, cubic or not, provided that the cooling temperature curve is measured precisely at the center of mass of the object and that the characteristic length L of the shape of the object is correctly evaluated mathematically or experimentally.

We applied the new experimental technique based on the so-called Cairo numerical method to standard cubes of aluminum and steel and the results obtained for the thermal diffusivity of aluminum and steel are in good agreement with those presented. in the heat tables.

It is recommended to apply the use of the new experimental technique to other metals and its extension to non-metallic objects.

*NB. All calculations in this article were produced through the author's double precision algorithm to ensure maximum accuracy, as followed by Ref. 10 for example*

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