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# A Rigorous Reformulation of Einstein derivation of the special relativity

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**Abstract:- It is true that the conclusions of Einstein SR in 1905 are perfect and incontestable but on the other hand it is also real that his derivation is doubtful and fundamentally fluid or not rigorous on certain physical points.**

**However, while each of his 5 assumptions is impossible to dispute or prove wrong, we assume that not all of them are essential to proving Einstein SR.**

**To be precise, we assume that at least one of them is inaccurate. Namely, combining the relativistic mass transformation with Newton's law of motion which is highly contested in modern quantum mechanical theory, but which was key to arriving at Einstein's own derivation of his famous mass-energy equivalence relationship E=mc^ 2.**

**Einstein's derivation of the relativistic mass transformation using Newton's law of motion to derive the mass-energy equivalence relation E=mc^2 is inaccurate.**

**Moreover, its derivation from the Lorentz transformation LT is useless because LT itself is a universal law of physics that applies to EMW propagation as well as the partial differential equation of diffusion and any physical phenomenon in 4D spacetime. It doesn't need a bypass.**

**We propose a rigorous reformulation of Einstein's derivation of special relativity where we assume the rest mass – energy equivalence relation E(0)=me(0) c^2 a universal law with c^2 a universal transformation constant.**

**Then we prove the mass energy Equivalence relation E=mc^2 by using Maxwell EMW theory instead of passing by the doubtful Newtons law of motion.**

**In other words, we have combined the universal laws of Maxwell's electromagnetic theory with the mass-energy equivalence relation at rest to derive the relativistic mass transformation and thus the mass-energy equivalence relation E=mc^ 2 without going through Newton's law of motion. Newton's law can therefore be a consequence of special relativity and not the reverse.**

**The investigation of the proposed reformulation provided new results and accurate define magnetic field of a moving charge as relativistic kinetic energy of the charged particle itself.**

## I. INTRODUCTION

The derivation of A. Einstein of the special relativity SR, is based on 5 independent and irreducible hypotheses from his own point of view[1].

Einstein was not the kind of scientist to play with assumptions and said over a century ago that none of its 5 independent hypotheses can be disproved by experiment.

However, recently objections have been raised that one or more of its assumptions may be replaced or deleted [2,3].

In other words, it is true that the conclusions of Einstein SR in 1905 are perfect and incontestable but on the other hand it is also real that his derivation is doubtful and fundamentally fluid or not rigorous on certain physical points[2,3].

However, while each of his assumptions is impossible to dispute or prove wrong, we assume that not all of them are essential to proving Einstein SR.

To be precise, we assume that at least one of them is inaccurate.Namely, combining the relativistic mass transformation with Newton's law of motion which is highly contested in modern quantum mechanical theory, but was essential in arriving at Einstein own derivation of its famous mass-energy equivalence relation E=mc^ 2. We think that it can be suppressed and replaced by another adequate hypothesis of the theory of electromagnetic waves of Maxwell and arrive rigorously at the same conclusions.

Einstein himself was aware of this fact and said:
I do not stand on the shoulders of Newton but on those of James Clark Maxwell.

This is the subject of the present article.
In order not to worry too much about the details and the description of the original special theory of relativity in 1905, which originally aimed to prove the Lorentz transformations and the formula for adding velocities, let's go directly to the derivation of the theory and the proposed reformulation.

## II. EINSTEIN THEORY AND PROPOSED REFORMULATION

Many scientists think you can't build an entire theory on a particular case, but then again Einstein was not the kind of scientist to play with assumptions and said over a century ago that none of its 5 independent hypotheses can be disproved by experiment.

All of his assumptions are universal and general cases, but our objection here is to find a better way or

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reformulation of these assumptions that would circumvent the relativistic mass transformation and Newton's law of motion in its general form.

It should be fruitful and promising.

The Einstein Derivation itself [1] is a thought experiment of a hypothetical event called Einstein's luminous sphere propagating through 4D free space and measured in two inertial frames of which one is moving at the uniform velocity v in the x direction called the moving frame or Einstein train [1] with respect to the other considered stationary with its observer attached, called the rest frame or Einstein station Fig. 1.

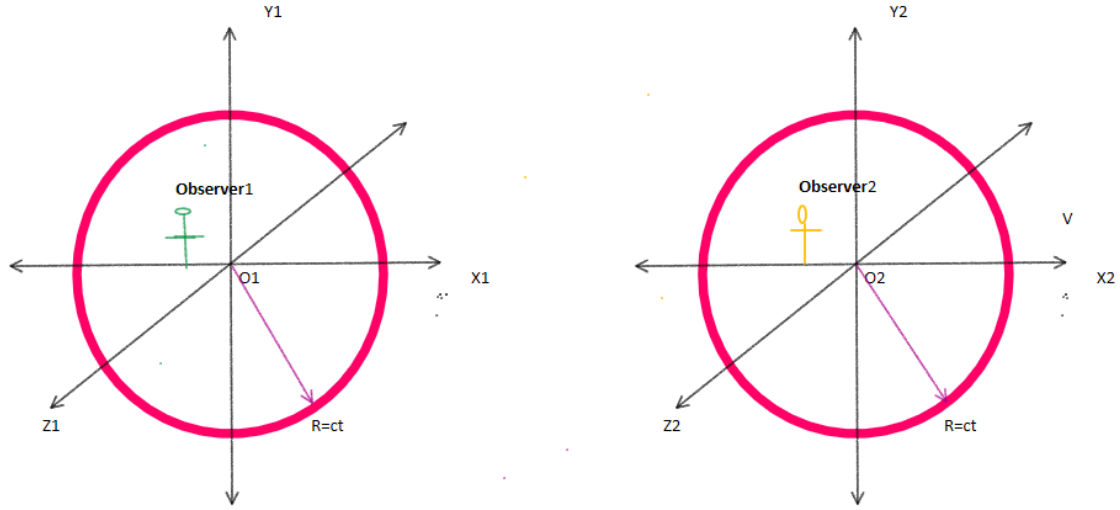


Fig.1: presents the same event in 4D space.

Propagate the luminous sphere of radius R in two inertial frames with center O2 coincident with O1.

Einstein stated that Euclidean geometry such as that in Fig.1 is a branch of physics.

A priori, it is stipulated that the length of the train measured from the embankment may be different from that obtained by measuring in the train itself.

In his thought experiment, Einstein came up with 5 hypotheses that led to some giant main conclusions:

First, it provided theoretical evidence for relativistic changes in time, length, and relative velocity previously known as imperial Lorentz transformations.

Secondly, he also discovered the relativistic mass transformation which was eventually crowned by the most important equation in physics, namely the mass-energy equivalence E = mc^2.

The original derivation of SR a century ago in a straight and understandable picture is simple and based on 5 consecutive irreducible assumptions, but it can be reformed as shown below in parentheses and in bold letters.

- The laws of physics are the same for all inertial frames of reference.
  There is no preferred frame of reference, which is a valid universal law of physics.

**[Obviously, this hypothesis is untouchable.]**

- The speed of light in vacuum c is a universal constant regardless of the uniform motion of the observer. (To agree with the Michelson-Morley interference experiment of 1887).

Bearing in mind that c is the upper limit of speed of any object with mass or even a photon or massless charge.Einstein said it was also the maximum possible speed of information transfer.

**[It is not c in itself which is a universal constant, but the mass-energy equivalence relation E0=m0c^2 which is the original universal law and therefore c^2 is constant. Thus c^2 is a universal constant of transformation of energy and mass at rest, just like the constant k of Boltzmann, the constant h of Planck, the gravitational constant G, etc. They are all defined by imperial laws which have no theoretical proof. The constancy of c is a consequence of this.**

**yet c is constant is Einstein's way of proving Lorentz transformations but again we have reservations about this saying.**

**We'd better assume that the Lorentz transformations are also the universal law of physics and don't need proof.]**

Returning to Fig.1, it follows from i& ii that the two equations describing the same luminous sphere event are identical and their centers O1 and O2 coincide.

The two propagation equations observed by the two frames 1&2 can be written as follows:

x1^2+y1^2 +z1^2 =c^2 t1^2 . . . . . .. . . . . .(1) . . .observed by rest frame (station)

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(x2-vt2)^2+y2^2 +z2^2 =c^2 t2^2 .. . (2) . . .observed by moving frame (train)

Einstein assumed that the two observers are at the center of the same sphere of light! Fig. 1.

Therefore,

x1^2+y1^2 +z1^2 -c^2 t1^2 = (x2-vt2)^2+y2^2 +z2^2 -c^2 t2^2 . . . . . (3)

- Einstein third assumption is that there is no change in space perpendicular to direction of motion x.

In other words, y1=y2 and z1=z2,hence Eq.3 reduces to,

x1^2+ +c^2 t1^2 = (x2-vt2)^2+c^2t2^2 . . . . . (4)

**[Assumption iii and equation 4 are in full agreement with theory and experiment.**

**However, the famous Noble Prize scientist R. Feynman [2] disagreed with hypothesis iii and went ahead assuming spatial relativistic variations normal to motion, i.e. in the y and z directions and arrived at its own SR which we consider inaccurate.]**

- Einstein's fourth hypothesis starts from the proposition that relativistic transformations of space and time must be linear like nature itself and that spatiotemporal transformations keep the volumetric element of 4D space invariant, for two inertial observers measuring the length of the station. x1t1(at the station) =x2t2 (in the train).

In mathematical language with Gamma as a parameter,

x1=x2.Gamma

and,

t1=t2 / Gamma

by solving Eq,4 for Gamma, you simply get,

Gamma=SQRT[1/(1-v^2/c^2)] . . . . . (5)

which is the Lorentz transformation itself.

In other words,

x2(moving frame)=x1(rest frame) /Gamma. (6)called space contraction, and,

t2(moving frame)=t1(rest frame)* Gamma. (7). . called time dilation.

**[Here again we have to agree with Einstein Eqs. 6 and 7 which perform Lorentz transformations resulting from the constant speed of light c. Alternatively, it is also shown in section III, that we regard the Lorentz transformations as imperial universal laws of physics consistent with the constancy assertion of c.**
**Lorentz transformations need no proof.]**

For reasons of clarity and simplicity, in the following analysis we denote the measurements in the rest frame by x0,t0 and m0 rather than x1,t1,m1 in Fig.1 and the moving frame by x,t,m instead of x2, t2 and m2 in Fig. 1.

Therefore,

x=x0/Gamma, known as length contraction. . . .(8)

and

t=t0 *Gamma, known as time dilation. . . . . (9)

Gamma >= 1

Again, in the above relations of length contraction and time dilation, the notations x0, t0 replace x1 and t1 in Figure 1 and represent the measurements of the observer in fixed frame 1 (train station) and x and t replace x2, t2 and represent the measurements of the observer in the moving frame 2 (Einstein's train).

Note that x0 and t0 are conveniently called true length and real time while x and t are called measured length and measured time which converge to x0 and t0 as v tends to zero.

- Einstein fifth hypothesis

Einstein used his Lorentz-derived transformations of time, length, and relative velocity to move forward and eventually find his revolutionary law of relativistic mass transformation.

m=m0/SQRT (1-v^2/c^2) . . . . . (10)

To arrive at eq. 10, he performed another thought experiment which considers that two balls of the same mass m collide vertically and applied the universal laws of conservation of energy and momentum before and after the collision, such as observed both in the frame at rest and in the frame moving with uniform velocity v.Thus he concludes the relativistic mass transformation formula (10).

Then Einstein considered Newton's second law of motion, in its general form, as a universal relativistic invariant law and used it as,

F=d (mv)/dt,

Where m=m0/SQRT (1-v^2/c^2)

It was easy to show that,

d (Work done W) = F.dx =Kinetic Energy EK =dm c^2

dW=v.dp =v d (Gamma m(0)v)= m(0).v^2 d Gamma +m(0) v d v .(11)

By integrating w.r.t both v and Gamma,we arrive at

Relativistic increase in EK=m0 c^2 (Gamma-1)

Therefore,

E(total)=(m0 +dm)c^2,

that is, the total energy E(total)=m c^2. . . . . (12)

Equation 12 is the well-known mass-energy equivalence relation.

This means that mass can be transformed into energy, called mass annihilation, and energy can be transformed into mass, called mass creation. In the numerical calculations of Eq,12, we follow SI units exclusively.

the transformation constant is c^2=9E16 m^2 s^-2.

Experimental mass creation of an electron-positron pair has been successfully achieved in the laboratory using gamma rays of photon energy +1.02 MeV or +1.92E-13 joules. This amount of energy is exactly the same amount predicted by Einstein E=2 me c^2 where me=9.1E-31Kg and c=3E8 ms^-1.

On the other hand, it has also been observed experimentally that the positron and the electron attract each other and collide, producing mass annihilation or transformation into a gamma photon of 1.02 MeV.

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**[Here we disagree with Einstein on the derivation of equations 10,11 and 12 not the equations themselves. These days, many physicists [2,3] object Newtons law as universal one and moreover wish to reserve the term "mass" for the concept of "rest mass.**

**The essential question that arises is how to arrive at the most important equation in physics,**
**E = (m0 + dm). C ^ 2, called Einstein's mass energy equivalence bypassing assumption 5 and Newtons law? This is the main object of the current paper and is answered in details in section III.]**

## III. INTRODUCING SPECIAL RELATIVITY IN CLASSICAL ELECTRODYNAMICS AND CLASSICAL STATISTICS

In classical electrodynamics the following laws are unconditionally valid,

E0=m0 c^2. (universal imperial law)

Do not look for mathematical proof, there isnone.

(B)Energy density of the electric field E(electric)=1/2 Eps $E^2$ and Energy density of the magnetic field E(magnetic)=1/2 Mue $H^2$. (Theoretically and experimentally proven)

C)The Poynting vector P=ExH(vector multiplication of two vectors) is a universal law representing the flow of electromagnetic power per unit area is a universal law. This law applies to both time-varying and stationary electromagnetic fields [4].

Togo further, let's start by answering this question,

What is the effective mass and effective radius of the electron?

In modern quantum mechanics the radius of electron cloud is nearly that of the hydrogen atom estimated as 5.3 E-11m.

In classical electrodynamics and classical Bohr atom, the electron is a solid sphere of radius 2.82E-15 m

In classical electrodynamics, the effective rest mass of the electron is me(0)=9.1 E-31 Kg.

The charge of the electron is -1.6 E-19 C.

Let us now look at a hypothetical solid sphere image for the electron or any other charge which will be very useful in understanding the electric field of an electron which is unaffected by motion relative to the frame of reference and the magnetic field of a moving charge which is proportional to v.

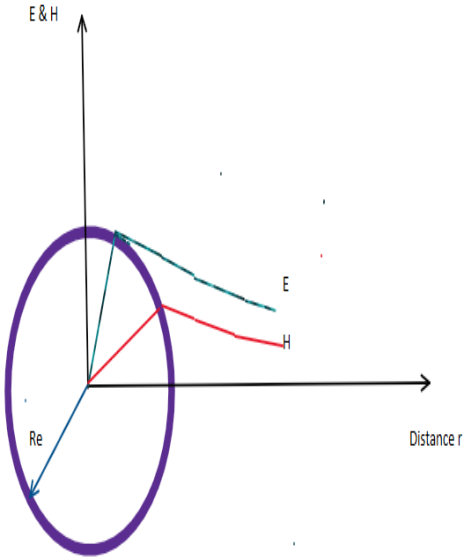
We will study their relation with its relativistic mass for a low speed c >> v.

The total electric field energy of the electron, regardless of its motion, should be me(0) c2 which is its relativistic rest energy for c >> v.

The total magnetic energy of the moving electron should be its relativistic kinetic energy for c >> v.

This means that the magnetic field itself corresponds to the relativistic kinetic energy of the electron.

The hypothetical compact spherical electric charge e is shown in Figure 2.




Hypothetical compact Spherical Charge e

Fig.2. Electric and magnetic fields of a hypothetical compact sphere of charge e and radius Re.

The predicted or expected results of the calculations of the effective radius of the electron Re and its moving mass me should provide us with a solid derivation of the relativistic mass transformation replacing that of Einstein.

The following assumptions are essential:

- Simulation of electron as solid sphere charge similar to classical physics presented by Niels Bohr and Ernest Rutherford in 1913 can be adequate if combined with EMW theory and rest mass energy equivalence.
- The mass-energy equivalence at rest is a universal law (E=m0 c^2), so no loss of generalization.
- The energy of the electric field comes from the electric charge e, stationary or in motion, and the energy of the magnetic field comes from and is equal to the relativistic kinetic energy of the free electric charge as a function of its speed v by relative to the observer's frame of reference.
- laws of electrodynamics are valid for all inertial reference frames.

In Einstein's special theory of relativity, energy is equivalent to mass at rest. We apply this theorem as an imperial universal law to be combined with the above 4 assumptions in the following situation or application (A).

- **Application A: Single electron compact sphere E and H fields**

In modern quantum mechanics, the electron is represented as a cloud of radius .51E-10 m while in classical physics, the electron is represented as a compact spherical charge of radius 2.82E-15 m Fig.2.

The electron rest mass, symbolized me(0) , is the mass of an electron as measured when its speed is zero relative to an observer.

Again, in Einstein's special theory of relativity, energy equals mass. We apply this theorem to prove that the electric

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field energy of the electron is equivalent to its rest mass me(0) while the magnetic field energy is equivalent to its relativistic kinetic energy which is the increase in its moving mass symbolized me.

Equations and calculations are performed for electrons at non-relativistic velocities v <<cin spherical coordinates without losing generality.

Here we use the agreed experimental numerical values in the SI system of units for the following physical quantities:

Rest mass of electrons me(0)=9.1E-31Kg

Free electronic charge e=-1.6E-19 C

Free space permeability Mue=4 Pi E-7 Kg m C^-2

[Mue] = [M L T^0 Q^-2]

Permittivity of free space Eps=(E-9/36 Pi ) Kg^-1m^-3s^2C^2

[Psi] = [M ^-1L^-3 T^2 Q^2]

The electric field intensity of a single electron = e/ (4Pi Eps r^2)

Outside the electron, r >= Re (electron effective radius)

Obviously, r= SQRT (x ^ 2 + y ^ 2 + z ^ 2).

The electric field energy density U(E)=(1/2) Eps E^2 = e^2/(32 Pi^2 Eps r^4)

Outside electron radius (r=>Re)

And

e^2/(32 Pi^2 Eps r^2) *r^4/Re^4 inside electron(r =< Re).

The total electric field energy in the whole space is,

Ut(E)= Integral from zero to infinity U(E)*4 Pi r^2 dr

Ut(E) =e^2/8 Pi Eps Re +e^2/8 Pi Eps Re=e^2/4 Pi Eps Re . . A(1)

Easy to show that,

Ut(E) = e^2/(4 Pi Eps Re) =me(0) c^2 . . . . . .A(2)

Note that ifwe substitute the agreed experimental values ofe,me(0),c and Eps in Eq A(2) we get Re=2.82 E-15 meter which is the generally accepted value for Re in classical ED picture.

A striking adequate response.

another striking result is that when the electron moves with a speed v (vector), it creates a 3D magnetic field according to H = e. vxr (vector cross product)/r^3 and therefore a magnetic energy density of:

U(H)= (½) Mue H^2

resulting in a relativistic increase in its mass at rest.

Similarly, the magnetic field intensity(H) of a moving electron with velocity v is,

H = e v / (4 Pi Mue r^2)

The magnetic field energy density U(H)=(1/2)Mue H^2

Therefore, the magnetic field energy density U(H) is,

U(H)= Mue. e^2 v^2/(32 Pi^2 r^4)

The total energy of the magnetic field in all space results from the movement or more precisely from the relativistic kinetic energy of the electron Ut(H)is,

Ut(H)= Integral from Re to infinity of U(H))*4 Pi r^2 dr

Ut(H)= Mue. e^2 v^2/(8 Pi Re)

Ut(H) corresponds to the increase in the relativistic kinetic energy of the electrons or, in relativistic language, to the increase in the mass of the electrons due to the movement.

Therefore,

Mue. e^2 v^2/(8 Pi Re)=d(me) c^2. . . . . A(3)

Comparing equations A(2) and A(3) we find that,

d(me)=½.[v^2 me(0) ] MueEps

Knowing that 1/Mue Eps =c^2,then

d(me)=me(0)(1/2) v^2/c^2

That is,

Me =me(0)[1+(1/2) v^2/c^2] . . . . . A(4)

Equation A(4) is the relativistic mass transformation (Eq 10)for v<< c

- **Application B: Chains of transition matrices B in 4D space**

In a previous article, we introduced and defined the so-called transition matrix chains B [5]. They succeeded perfectly in solving the heat diffusion equation as well as the Laplace and Poisson equations in bounded 4D space [6].

It is therefore expected that these transfer chains respect the rules of special relativity or more precisely the rules of Lorentz transformations.

We prove that this is the case if we keep in mind that the main diagonal element of the b-matrix named RO which by definition can take any value in the interval [0,1[ represents the term ½ v^2/c^ 2 for diffusion in 3D geometric space.

RO= ½ v^2/c^2 . . . . . B(1)

In Reference 6, we presented different numerical values for RO versus the resulting time exponent alpha in the case of the 3D cube of 8 free nodes shown in Fig. 3. These results are shown in Table I (Ref.6).

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Note that RO and Alpha are dimensionless numbers.

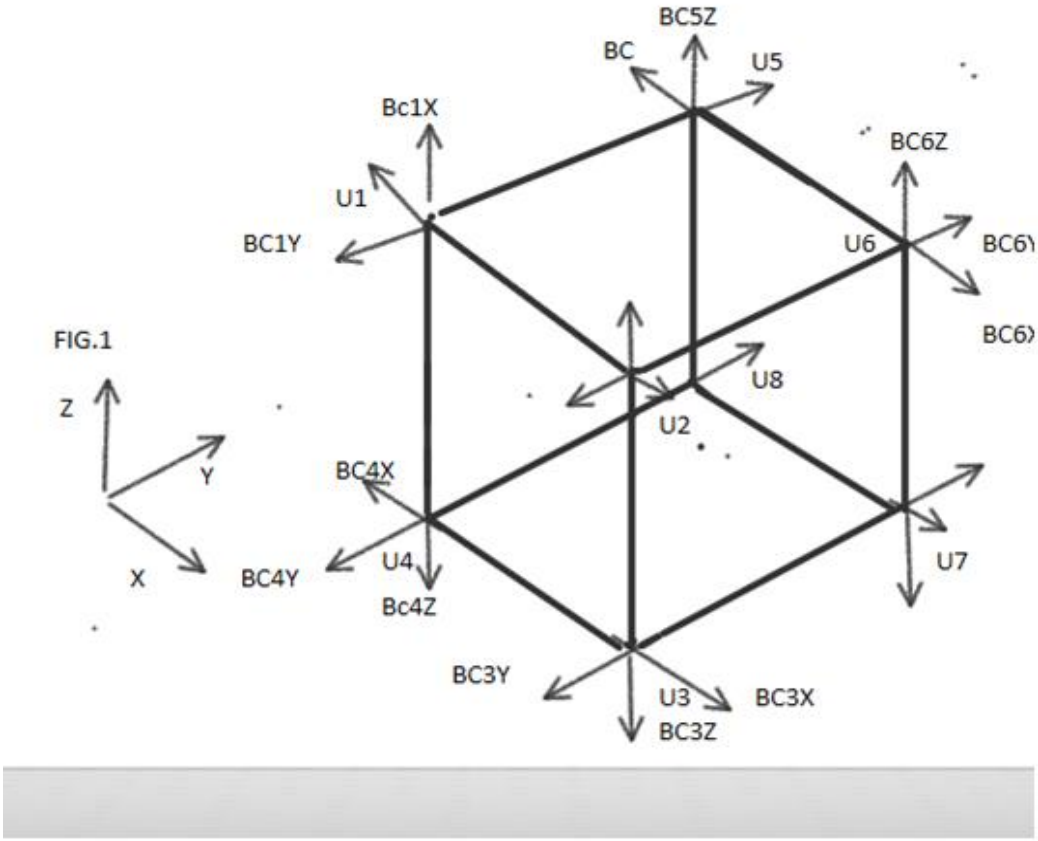


Fig. 3: Transient heat diffusion equation in a 3D cube of 8 free nodes with 8 Dirichlet BC.

Table I. Numerical results for RO vs alpha up to equation 7 for the 3D cube of 8 free nodes Fig.3.

| RO | 0. | 0.2 | 0.4 | 0.6 | 0.8 | 1.0 |
|---|---|---|---|---|---|---|
| ALPHA | 0.693 | 0.511 | 0.357 | 0.223 | 0.105 | 0.000 |

From

Table I, Alpha(0)=0.376 corresponding to dt0

Where Alpha is the exponent of the exponential time of increase/decrease of the temperature curve, time t = N dt time step where N is the number of iterations.

The temperature T(x,t) in separation of variables mode is given by:

T(x,t)=A Exp(-Alpha t ) f(x) . . . . . B(2)

t is the discrete time-independent variable and is given by t=N dt where dt is the time step and N is the number of iterations.

We now move on to a more complicated geometry other than the simple cube, namely theparalleloid of Fig.4.of 27 free nodes and 27 Dirichlet boundary conditions.Fig.4.

The numerical results for RO vs alpha for the free 27-node 3D paralleloid shown in Fig. 4 are shown in Table II

(Ref.6,7)

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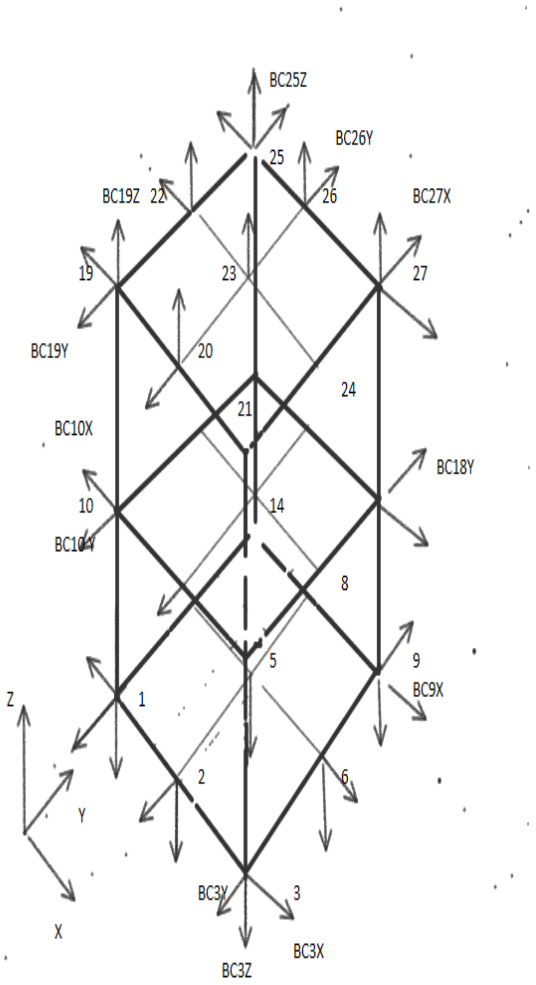


Fig. 4: Heat diffusion in 3D paralleloid of 27 equidistant free nodes and 27 Dirichlet boundary conditions.

Table II. Numerical results for RO vs alpha up to equation 7 (Ref.6) for the 3D paralleloid27 free nodes Fig.4.

| RO | 0. | 0.2 | 0.4 | 0.6 | 0.8 | 1.0 |
|---|---|---|---|---|---|---|
| ALPHA | 0.376 | 0.282 | 0.206 | 0.129 | 0.0636 | 0.0 |

From Table II, Alpha(0)=0.376 corresponding to dt0

Inspecting thoroughly tables I &II we can arrive at the rule of time dilation without the loss of generality,

It is no coincidence that for all RO values equal to or greater than zero and less than unity, in Table I, Table II or any other table, the following formula applies:

Alpha/Alpha(0)=(1-RO) . . . . . B(3),
In other words,dt=dt0/(1-RO)
Substituting for RO=1/2 v^2/c^2,we get,
dt=dt0/(1-1/2 v^2/c^2) . . . . . B(4)
Eq.B(4) is the Lorentz relativistic time dilation for c >>v.

Meaning is that Laplace and Poisson PDE are subjected to time dilation and space contraction for an observer moving with uniform velocity v with respect to a rest frame of experience reference.

We assume that the Lorentz transformation is a universal law of nature that applies to any space-time physical phenomenon.

## IV. CONCLUSION

Einstein's derivation of the relativistic mass transformation using Newton's law of motion to derive the mass-energy equivalence relation E=mc^2 is inaccurate.

Moreover, its derivation from the Lorentz transformation LT is useless because LT itself is a universal law of physics that applies to the partial differential equation of diffusion and to any physical phenomenon in 4D spacetime. It doesn't need a derivation.

We propose a rigorous reformulation of Einstein's derivation of special relativity where we assume the rest mass – energy equivalence relation E(0)=me(0) c^2 a universal law with c^2 a universal transformation constant.

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Then we prove the mass energy Equivalence relation E=mc^2 by using Maxwell EMW theory instead of passing by the doubtful Newtons law of motion.

In other words, we have combined the universal laws of Maxwell's electromagnetic theory with the mass-energy equivalence relation at rest to derive the relativistic mass transformation and thus the mass-energy equivalence relation E=mc^ 2 without going through Newton's law of motion. Newton's law can therefore be a consequence of special relativity and not the reverse.

The applications of the proposed reformulation provided new results and accurate define magnetic field of a moving charge as relativistic kinetic energy of the charged particle itself.

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