ISSN No:-2456-2165

Union of 4-Total Mean Cordial Graph with the Star $K_{1,N}$

D. SRIRAM
Department of Mathematics P.S.R Engineering College,
Sivakasi-626140,Tamilnadu, India.

Abstract:- Consider (*p, q*) graph *G* and define *f* from the vertex set $V(G)$ to the set Z_k where $k \in$ N and $k > 1$. Foreache *uv*, assign the label $f(u)+f(v)$ 2, Then the function *f* is called as *k*-total mean cordial labeling of *G* if number of vertices and edges labelled by *i* and not labelled by *i* differ by at most 1, where $i \in \{0, 1, 2, \cdots, k-1\}$. Suppose a graph admits a *k*-total mean cordial labeling then it is called as *k*-total mean cordial graph. In this paper we investigate the 4-total mean cordial labeling of $G \cup K_{1,n}$ where G is a 4-total mean cordial graph.

I. INTRODUCTION

All Graphs in this paper are finite, simple and undirected. In [4] the concept of k-total mean cordial labeling have been introduced. Also, 4-total mean cordial behaviour of several graphs like path, cycle, com- plete graph, star, bistr, comb, crown have been investigated in [4]. In this paper we investigate the 4-total mean cordial labeling of $G \cup K_{1,n}$ where G is a 4-total mean cordial graph. Let x be any real number. Then ⌈x⌉ stands for the smallest integer greater than or equal to x. Terms are not defined here follow from Harray [3] and Gallian [2].

II. K-TOTAL MEAN CORDIAL LABELING

Definition 2.1. Let *G* be a (p, q) graph. Let $f : V(G) \to \{0, 1, 2, \ldots, k-1\}$ be a function where $k \in$ N and $k > 1$. For each edge uv, assign the label $f(uv) = f(u)+f(v)$ 2. f is called *k*-total mean cordial labeling 0if $1 \le i \le t$ 3if $t+1 \le i \le 2t$ Here $tm_f(0) = tm_f(2) = tm_f(3) = t$ and $tm_f(1) = t+1$. Case 2. n is odd. Take $n = 2t+1$. Here $(f(u_i)=$ of G if $|tm_f(i) - tm_f(j)| \le 1$ for all $i, j \in \{0, 1, 2, \cdots, k-1\}$ where $tm_f(x)$ denotes the total number of vertices and the edges labeled with x, $x \in \{0, 1, 2, \cdots, k-1\}$. A graph with a *k*-total mean cordial labeling is called *k*-total mean cordial graph.

➢ *2000 Mathematics Subject Classification. 05C78.*

- Key words and phrases. Star, bipartite graph, union of graphs and 4-total mean cordial graph.

Definition 2.2. A bipartite graph is a graph whose vertex set $V(G)$ can be partitioned into two subsets V_1 and V_2 such that every edge of G joins a vertex of V_1 with a vertex of V_2. If every vertex of V_1 is adjacent with every vertex of V_2, then G is a complete bipartite graph.If $|V_1| = m$ and $|V_2| = n$, then the complete bipartite graph is denotedby $K_{m,n}$.

- Definition 2.3. $K_{1,n}$ is called a Star.
- Definition 2.4. The union of two graphs G_1 and G_2 is the graph $G_1 \cup G_2$ with $V(G_1 \cup G_2) = V(G_1) \cup V(G_2)$ and $E(G_1 \cup G_2) = E(G_1) \cup E(G_2)$.

III. PRELIMINARIES

First we reminisce the following theorem on star graph which admitsa 4-total mean cordial labeling given in [4]. Theorem 3.1. [4] The star $K_{1,n}$ is 4-total mean cordial for all values of n. *Proof.* Let $V(K_{1,n}) = \{u\} \cup \{u_i : 1 \le i \le n\}$ and $E(K_{1,n}) = \{uu_i : 1 \le i \le n\}$. Define a vertex labeling f from the set of vertices of star to the set $\{0, 1, 2, 3\}$ as follows: $f(u) = 1$ and according to the nature of n, to provide vertex labels to the vertices u_i, we may consider the following two cases:

- Case 1. n is even.Let $n = 2t$. In this case

$$f(u_i) = \begin{cases} 0 & \text{if } 1 \le i \le t \\ 3 & \text{if } t+1 \le i \le 2t \end{cases}$$

- Here $tm_f(0) = tm_f(2) = tm_f(3) = t$ and $tm_f(1) = t + 1$.

- Case 2. n is odd.Take $n = 2t + 1$. Here

$$f(u_i)=\begin{cases} 0 & \text{if } 1 \le i \le t \\ 3 & \text{if } t+1 \le i \le 2t+1 \end{cases}$$

- Note that $tm_f(1) = tm_f(2) = tm_f(3) = t + 1$ and $tm_f(0) = t$.
- Hence f is a 4-total mean cordial labeling of $K_{1,n}$.Q

IV. MAIN RESULTS

With the help of the Theorem 3.1, we can now verify the 4-total mean cordiality of the graph $G \cup K_{1,n}$ where G is any 4-total mean cordial graph.

- Theorem 4.1. Let G be any (p, q)-4 total mean cordial graph. Then $G \cup K_{1,n}$ is 4 total mean cordial if n is even.
- *Proof.* As G is 4 total mean cordial, there exists a 4 total mean cordial labeling, say g. We define a function $h : V(G \cup K_{1,n}) \to \{0, 1, 2, 3\}$ by

$$h(x)=\begin{cases} g(x) & \text{if } x \in V(G) \\ f(x) & \text{if } x \in V(K_{1,n}) \end{cases}$$

➢ *Now we Check the Validity of the Above Mentioned Labeling in four Cases.*

- Case 1. $p + q \equiv 0 \pmod 4$.
- Let $p + q = 4r$. In this case $tm_g(0) = tm_g(1) = tm_g(2) = tm_g(3) = r$.
- Note that, here h satisfies the required condition given by $tm_h(0) =$
- $tm_h(2) = tm_h(3) = r + t$ and $tm_h(1) = r + t + 1$.
- Case 2. $p + q \equiv 1 \pmod 4$.

➢ *Put $p + q = 4r + 1$. Here, the function g should satisfy any one of the following conditions:*

- $tm_g(0) = r + 1$ and $tm_g(1) = tm_g(2) = tm_g(3) = r$.
- $tm_g(1) = r + 1$ and $tm_g(0) = tm_g(2) = tm_g(3) = r$.
- $tm_g(2) = r + 1$ and $tm_g(0) = tm_g(1) = tm_g(3) = r$.
- $tm_g(3) = r + 1$ and $tm_g(0) = tm_g(1) = tm_g(2) = r$.

➢ *Now we Divide this Case into the Following Possible Subcases.*

- Subcase 2(a). $tm_g(0) = r + 1$ and $tm_g(1) = tm_g(2) = tm_g(3) = r$.
- In this case $tm_h(0) = tm_h(1) = r + t + 1$, $tm_h(2) = tm_h(3) = r + t$ and hence h is a 4-total mean cordial labeling.
- Subcase 2(b). $tm_g(1) = r + 1$ and $tm_g(0) = tm_g(2) = tm_g(3) = r$.
- If $n \equiv 2 \pmod 4$, then we reconstruct the vertex labeling h as follows
- $h(x) = g(x)$ for all $x \in V(G)$, $h(u) = 2$ and

$$h(u_i)=\begin{cases} 0 & \text{if } 1 \le i \le 2t+1 \\ 2 & \text{if } 2t+2 \le i \le 3t+1 \\ 3 & \text{if } 3t+2 \le i \le 4t+2 \end{cases}$$

Where $n = 4t + 2$. In this case $tm_h(0) = tm_h(2) = r + 2t + 1$ and $tm_h(1) = tm_h(3) = r + 2t + 2$ and hence h is a 4-total mean cordial labeling. Suppose $n \equiv 0 \pmod 4$, then we redefine the function h as follows $h(x) = g(x)$ for all $x \in V(G)$, $h(u) = 2$ and

$$h(u_i)=\begin{cases} 0 & \text{if } 1 \le i \le 2t \\ 2 & \text{if } 2t+1 \le i \le 3t \\ 3 & \text{if } 3t+1 \le i \le 4t \end{cases}$$

Where $n = 4t$. Here $tm_h(0) = tm_h(3) = r + 2t$ and $tm_h(1) = tm_h(2) = r + 2t + 1$. Hence h is a required vertex labeling.

- Subcase 2(c). $tm_g(2) = r + 1$ and $tm_g(0) = tm_g(1) = tm_g(3) = r$.
- In this case, we have $tm_h(0) = tm_h(3) = r + t$ and $tm_h(1) = tm_h(2) =$
- $r + t + 1$ and hence h is a 4-total mean cordial labeling.
- Subcase 2(d). $tm_g(3) = r + 1$ and $tm_g(0) = tm_g(1) = tm_g(2) =$
- r. Here $tm_h(0) = tm_h(2) = r + t$ and $tm_h(1) = tm_h(3) = r + t + 1$.
- Thus h should be a 4-total mean cordial labeling.Case 3. $p + q \equiv 2 \pmod 4$.

➢ *Put $p + q = 4r + 2$. Here g should satisfy any one of the following conditions:*

- $tm_g(0) = tm_g(1) = r + 1$ and $tm_g(2) = tm_g(3) = r$.
- $tm_g(0) = tm_g(2) = r + 1$ and $tm_g(1) = tm_g(3) = r$.
- $tm_g(0) = tm_g(3) = r + 1$ and $tm_g(1) = tm_g(2) = r$.
- $tm_g(1) = tm_g(2) = r + 1$ and $tm_g(0) = tm_g(3) = r$.
- $tm_g(1) = tm_g(3) = r + 1$ and $tm_g(0) = tm_g(2) = r$.f) $tm_g(2) = tm_g(3) = r + 1$ and $tm_g(0) = tm_g(1) = r$.

We define this case into the following possible subcases: Subcase 3(a). $tm_g(0) = tm_g(1) = r + 1$ and $tm_g(2) = tm_g(3) = r$.If $n \equiv 2 \pmod 4$, then we reconstruct h as follows $h(x) = g(x)$ for all $x \in V(G)$, $h(u) = 2$ and

$$h(u_i)=\begin{cases} 0 & \text{if } 1 \le i \le 2t \\ 2 & \text{if } 2t+1 \le i \le 3t \\ 3 & \text{if } 3t+1 \le i \le 4t \end{cases}$$

Where $n = 4t + 2$. In this case $tm_h(2) = r + 2t + 1$ and $tm_h(0) = tm_h(1) = tm_h(3) = r + 2t + 2$. Suppose n

$\equiv 0 \pmod 4$, then we redefine h as follows $h(x) = g(x)$ for all $x \in V(G)$, $h(u) = 2$ and

Where $n = 4t$. Here $tm_h(3) = r + 2t$ and $tm_h(0) = tm_h(1) = tm_h(2) = r + 2t + 1$. Thus h is a 4-total mean cordial labeling.

- Subcase 3(b). $tm_g(0) = tm_g(2) = r + 1$ and $tm_g(1) = tm_g(3) = r$. Here $tm_h(0) = tm_h(1) = tm_h(2) = r + t + 1$ and $tm_h(3) = r + t$ andhence h is a 4-total mean cordial labeling.
- Subcase 3(c). $tm_g(0) = tm_g(3) = r + 1$ and $tm_g(1) = tm_g(2) = r$.
- In this case $tm_h(0) = tm_h(1) = tm_h(3) = r + t + 1$ and $tm_h(2) = r + t$. Thus h satisfies the required condition.
- Subcase 3(d). $tm_g(1) = tm_g(2) = r + 1$ and $tm_g(0) = tm_g(3) = r$.
- Suppose $n \equiv 2 \pmod 4$, then we redefine h as follows $h(x) = g(x)$ forall $x \in V(G)$, $h(u) = 2$ and

$$h(u_i)=\begin{cases} 0 & \text{if } 1 \le i \le 2t+1 \\ 2 & \text{if } 2t+2 \le i \le 3t+1 \\ 3 & \text{if } 3t+2 \le i \le 4t+2 \end{cases}$$

Where $n = 4t + 2$. Here $tm_h(0) = r + 2t + 1$ and $tm_h(1) = tm_h(2) = tm_h(3) = r + 2t + 2$. If $n \equiv 0 \pmod 4$, then we redefine h as follows $h(x) = g(x)$ for all $x \in V(G)$, $h(u) = 2$ and

$$h(u_i)=\begin{cases} 0 & \text{if } 1 \le i \le t \\ 1 & \text{if } t+1 \le i \le 2t \\ 3 & \text{if } 2t+1 \le i \le 4t \end{cases}$$

Here $tm_h(0) = tm_h(1) = tm_h(2) = r + 2t + 1$ and $tm_h(3) = r + 2t$.Thus h satisfies the conditions of a 4-total mean cordial labeling.

- Subcase 3(e). $tm_g(1) = tm_g(3) = r + 1$ and $tm_g(0) = tm_g(2) = r$.
- If $n \equiv 2 \pmod 4$, then we redefine h as follows $h(x) = g(x)$ for all
- $x \in V(G)$, $h(u) = 2$ and

$$h(u_i)=\begin{cases} 0 & \text{if } 1 \le i \le t \\ 1 & \text{if } t+1 \le i \le 2t \\ 3 & \text{if } 2t+1 \le i \le 4t+1 \\ 2 & \text{if } i = 4t+2 \end{cases}$$

Where $n = 4t + 2$. In this case $tm_h(0) = r + 2t + 1$ and $tm_h(1) = tm_h(2) = tm_h(3) = r + 2t + 2$. Suppose $n \equiv 0 \pmod 4$, then vertex labeling given above for $n \equiv 2 \pmod 4$, will satisfy the requirementif we take the same label upto $4t$. It is easy to check that $tm_h(0) = tm_h(1) = tm_h(3) = r + 2t + 1$ and $tm_h(2) = r + 2t$. Hence h is a 4-total mean cordial labeling.

Subcase 3(f). $tm_g(2) = tm_g(3) = r + 1$ and $tm_g(0) = tm_g(1) = r$. In this case $tm_h(0) = r + t$ and $tm_h(1) = tm_h(2) = tm_h(3) = r + t + 1$.Thus h is a 4-total mean cordial labeling. Case 4. $p + q \equiv 3 \pmod 4$.

Put p + q = 4r + 3. As g is a 4-total mean cordial labeling of G, itshould satisfy any one of the following conditions:

- $tm_g(0) = r$ and $tm_g(1) = tm_g(2) = tm_g(3) = r + 1$.
- $tm_g(1) = r$ and $tm_g(0) = tm_g(2) = tm_g(3) = r + 1$.
- $tm_g(2) = r$ and $tm_g(0) = tm_g(1) = tm_g(3) = r + 1$.
- $tm_g(3) = r$ and $tm_g(0) = tm_g(1) = tm_g(2) = r + 1$. Consider the following subcases: Subcase 4(a). $tm_g(0) = r$ and $tm_g(1) = tm_g(2) = tm_g(3) = r + 1$.

Suppose $n \equiv 2 \pmod 4$, then we redefine h as follows $h(x) = g(x)$ forall $x \in V(G)$, $h(u) = 2$ and

$$h(u_i)=\begin{cases} 0 & \text{if } 1 \le i \le 2t+2 \\ 2 & \text{if } 2t+3 \le i \le 3t \\ 3 & \text{if } 3t+1 \le i \le 4t+2 \end{cases}$$

Where n = $4t + 2$. Here $tm_h(0) = tm_h(1) = tm_h(2) = tm_h(3) = r + 2t + 2$. Thus h satisfies the conditions of a 4-total mean cordial labeling. If $n \equiv 0 \pmod 4$, then we change h as follows $h(x) = g(x)$ for all $x \in V(G)$, $h(u) = 0$ and

$$h(u_i)=\begin{cases} 0 & \text{if } 1 \le i \le t \\ 1 & \text{if } t+1 \le i \le 2t \\ 3 & \text{if } 2t+1 \le i \le 4t \end{cases}$$

Here $tm_h(0) = tm_h(1) = tm_h(2) = tm_h(3) = r + 2t + 1$. Thus h satisfiesthe required property.

Subcase 4(b). $tm_g(1) = r$ and $tm_g(0) = tm_g(2) = tm_g(3) = r + 1$. Note that $tm_h(0) = tm_h(1) = tm_h(2) = tm_h(3) = r + 2t + 1$ and hence h is a 4-total mean cordial labeling.

Subcase 4(c). $tm_g(2) = r$ and $tm_g(0) = tm_g(1) = tm_g(3) = r + 1$.

If $n \equiv 2 \pmod 4$, then we change the map h as follows $h(x) = g(x)$for all $x \in V(G)$, $h(u) = 0$ and

$$h(u_i)=\begin{cases} 0 & \text{if } 1 \le i \le t \\ 1 & \text{if } t+1 \le i \le 2t \\ 3 & \text{if } 2t+1 \le i \le 4t+1 \\ 2 & \text{if } i = 4t+2 \end{cases}$$

Here $tm_h(0) = tm_h(1) = tm_h(2) = tm_h(3) = r + 2t + 2$. If $n \equiv 0 \pmod 4$, then we redefine h as follows $h(x) = g(x)$ for all $x \in V(G)$,$h(u) = 2$ and

$$h(u_i)=\begin{cases} 0 & \text{if } 1 \le i \le 2t \\ 2 & \text{if } 2t+1 \le i \le 3t \\ 3 & \text{if } 3t+1 \le i \le 4t \end{cases}$$

ISSN No:-2456-2165

In this case $tm_h(0) = tm_h(1) = tm_h(2) = tm_h(3) = r + 2t + 1$. Thus h is a 4-total mean cordial labeling.

Subcase 4(d). $tm_g(3) = r$ and $tm_g(0) = tm_g(1) = tm_g(2) = r + 1$.

Suppose $n \equiv 0 \pmod 4$, then we change the map h as follow.

$$h(u_i)=\begin{cases} 0 & \text{if } 1 \le i \le 2t \\ 2 & \text{if } 2t+1 \le i \le 3t \\ 3 & \text{if } 3t+1 \le i \le 4t \end{cases}$$

Here $tm_h(0) = tm_h(1) = tm_h(2) = tm_h(3) = r + 2t + 1$. Suppose $n \equiv 2 \pmod 4$, then we reconstruct h by $h(x) = g(x)$ for all $x \in V(G)$, $h(u) = 2$ and

$$h(u_i)=\begin{cases} 0 & \text{if } 1 \le i \le 2t+1 \\ 2 & \text{if } 2t+2 \le i \le 3t+1 \\ 3 & \text{if } 3t+2 \le i \le 4t+2 \end{cases}$$

In this case $tm_h(0) = tm_h(1) = tm_h(2) = tm_h(3) = r + 2t + 2$ and hence h is a 4-total mean cordial labeling of $G \cup K_{1,n}$.Q

Illustration 1. A 4-total mean cordial labeling of $H_8 \cup K_{1,6}$ is given in Figure 1, where H_n is a helm graph obtained from a wheel by appending n pendent vertices to the rim vertices of that wheel.

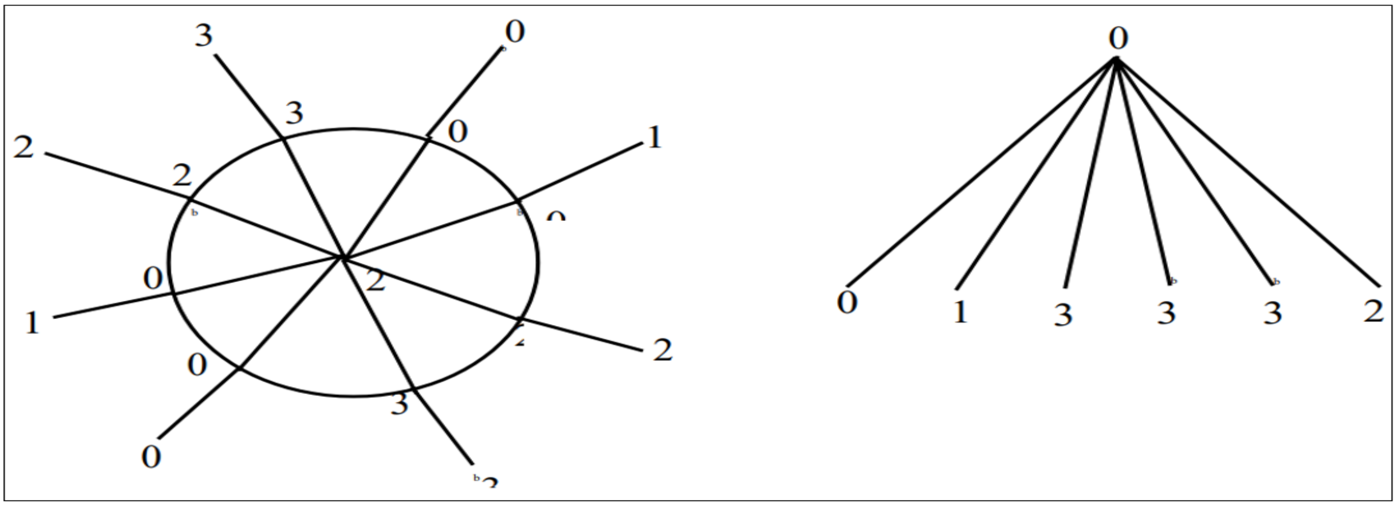

Fig 1 A 4-total mean cordial labeling of H8 ∪ K1,6

Conclusion. In this manuscript, we discussed 4-total mean cordial labeling behviour of disjoint union of star with a 4-total mean cordial graph. With these idea we can construct new 4-total mean cordial graphs from the existing graphs.

REFERENCES

[1]. I.Cahit, Cordial graphs:A weaker version of graceful and harmonious graphs,

[2]. Ars Combinatoria, 23(1987), 201-207.

[3]. J.A.Gallian, A Dynamic survey of graph labeling, The Electronic Journal of Combinatorics, 19 (2017) #Ds6.

[4]. F.Harary, Graph theory, Addision wesley, New Delhi (1969).

[5]. R.Ponraj, S.Subbulakshmi and S.Somasundaram, k-total mean cordial graphs,

[6]. J. Math. Comput. Sci, 10(5)(2020), 1697-1711.