ISSN No:-2456-2165

Solutions of Growth Decay Problems by "Emad-Falih Transform"

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Abstract:- In this paper, we find the solution of order ordinary differential equation of problems on growth and decay by using Emad-Falih integral transform. We also solve some problems on growth and decay. For that we use Emad-Falih transform. This proves the capability and efficiency of the transform to find the exact solutions of such problems.

Keywords: *Emad-Falih integral transform, growth problems and decay problems.*

I. INTRODUCTION

Recently, Integral transforms are one of the most useful and simple mathematical technique for obtaining the solutions of advance problems occurred in many fields like science, Engineering, technology, commerce and economics. isto provide exact solution of problem without lengthy calculations is the important feature of integral transforms.

Due to this important feature of the integral transforms many researchers are attracted to this field and are engaged in introducing various integral transforms. Recently, Kushare and Patil [1] introduced new integral transform called as Kushare transform for solving differential equations in time domain. Further, Savita Khakale and Dinkar Patil [2] introduced Soham transform. As researchers are interested in introducing the new integral transforms at the same time they are also interested in applying the transforms to various fields, various equations in different domain. In January 2022, Sanap and Patil [3] used Kushare transform for obtaining the solution of the problems on Newton's law of Cooling.

In April 2022 D. P. Patil, et al [4] solved the problems on growth and decay by using Kushare transform. D.P. Patil [5] also used Sawi transform in Bessel functions. Further, Patil [6] evaluate improper integrals by usingSawi transform of error functions. Laplace transforms and Shehu transforms are used in chemical sciences by Patil [7]. DinkarPatil [8] solved wave equation by using Sawi transform and its convolution theorem. Using Mahgoub transform, parabolic boundary value problems are solved by D .P. Patil [9].

D .P. Patil used double Laplace and double Sumudu transforms to obtain the solution of wave equation [10]. Dr. Patil [11] also obtained dualities between double integral transforms. Sachin Kushare and Patil [12] compared the Laplace, Elzaki and Mahgoub transforms and used it to solve the system of first order and first differential equations . D.P.Patil [13] solved boundary value problems of the system of ordinary differential equations by usingAboodh and Mahgoub transforms. Double Mahgoub transformed is used by Patil [14] to solve parabolic boundary value problems.

Laplac, Sumudu ,Aboodh , Elazki and Mahagoub transform and used it for solving boundary value problems by Patil et al [15]. D. P. Patil et al [16] solved Volterra Integral equations of first kind by usingEmad-Sara transform.Futher Patil with Tile and Shinde [17] usedAnuj transform and solvedVolterra integral equations for first kind. Rathi sisters and D. P. Patil [18] solved system of differential equations by using Soham transform. Vispute, Jadhav and Patil [19] used Emad Sara transform for solving telegraph equation.

Kandalkar, Zankar and Patil [20]evaluate the improper integrals by usinggeneral integral transform of error function. Dinkar Patil, Prerana Thakare and Prajakta Patil[21] obtained the solution of parabolic boundary value problems by using double general integral transform. Dinkar Patil used Emad- Falih transform for solving problems based on Newton's law of cooling [22]. D. P. Patil et al [23] used Soham transform to obtain the solution of Newton's law of cooling. Dinkar Patil et al [24] used HY integral transform for handling growth and Decay problems, D. P. Patil et al used HY transform for Newton's law of cooling [25]. D. P. Patil et al [26] used Emad-Falih transform for general solution of telegraph equation.Dinkar Patil et al [28] introduced double kushare transform. Recently, D. P. Patil et al [29] solved population growth and decay problems by using Emad Sara transform.Alenzi transform is used in population growth and decay problems by patil et al [30].Raundal and Patil D. P.[31] used double general integral transform for solving boundary value problems. Further Patil et al [32] used Soham thransform to solve problems in Chemical Sciences.

In this paper, we use Emad-Falih transform, introduced by Emad Kuffi and Sara Falih Maktoof [19] to solve growth decay problems. This paper is organised as follows, Second section is for preliminaries. In third section EmadFalih transform is used for growth and decay problems. Fourth section is reserved for applications and conclusion is drawn in last fifth section.

ISSN No:-2456-2165

II. PRELIMINARY

A. Definition of Emad-Falih Integral Transform:[27]

The Emad Falih integral transform is defined for an exponential order function such that

B = {f(t):∃K, m1, m2 > 0, |f(t)|< K$e^{m^2 j|t|}$ if t ϵ (-1) ^{j} X [0, ∞)} (1)

where: f(t) is a function in the set B , K is a finite constant number, m_1 and m_2 may be finite or infinite.

The Emad-Falih integral transform is denoted by (EF) and it is defined by the integral equation:

EF {f(t)} = T(ϕ) = $\frac{1}{\varphi}\int_0^\infty e^{-\varphi^2 t}$ f(t) dt (2)

Here t ≥ 0, m1 ≤ϕ≤ m2 and ϕ is a variable. It is used as a factor to the variable t in the function f.

B. Emad-Falih Transform of fundamental functions:[27]

Sr. No.	Function	Emad-Fallih Transform
1	k	$\frac{k}{\varphi^2}$
2	t^n	$\frac{n!}{\varphi^{2n+3}}$
3	e^{at}	$\frac{1}{\varphi(\varphi^2 - a)}$
4	$\sin at$	$\frac{1}{\varphi(\varphi^4 + a^2)}$
5	$\cos at$	$\frac{\varphi}{(\varphi^4 + a^2)}$
6	$\sinh at$	$\frac{a}{\varphi(\varphi^4 - a^2)}$
7	$\cosh at$	$\frac{\varphi}{(\varphi^4 - a^2)}$

Table 1

C. Inverse Emad-Falilh Transform:

Sr. No.	Inverse Emad-Falih Transform	Function
1	$\frac{k}{\varphi^2}$	K
2	$\frac{n!}{\varphi^{2n+3}}$	t^n
3	$\frac{1}{\varphi(\varphi^2 - a)}$	e^{at}
4	$\frac{1}{\varphi(\varphi^4 + a^2)}$	$\sin at$
5	$\frac{\varphi}{(\varphi^4 + a^2)}$	$\cos at$
6	$\frac{a}{\varphi(\varphi^4 - a^2)}$	$\sinh at$
7	$\frac{\varphi}{(\varphi^4 - a^2)}$	$\cosh at$

Table 2

D. Emad-Falih Transform of derivative of the function f(t):[27]

Let T(ϕ) is the Emad-Falilh integral transform of a function $f(t)$, $EF\{f(t)\} = T(\phi)$, then:

Then New Integral Transform "Emad-Falih" is $EF\{f'(t)\} = \frac{-f(0)}{\phi} + \phi^2 T(\phi)$

III. APPLICATION OF EMAD-FALIH TRANSFORM IN POPULATION GROWTH AND DECAY PROBLEMS

A. *Emad-Falih Transform of Growth Problems:*

Now, we use **Emad-Falih** Transform to obtain solution of population growth problems. The population growth is governed by the linearordinary differential equation of first order,

$$\frac{dN}{dt} = PN \quad (1)$$

With the initial condition,

$$N(t_0) = N_0 \quad (2)$$

Where P > 0 is a real number, N is the population at time t and N_0 is the initial population at time $t = t_0$.

We apply **Emad-Falih** transform on equation (1)

$$EF\left\{\frac{dN}{dt}\right\} = EF\{PN(t)\}$$

$$EF\left\{\frac{dN}{dt}\right\} = P\ EF\{N(t)\} \quad (3)$$

Now we apply **Emad-Falih** Transform of derivatives of function property to this equation and obtain

$$\frac{-N(0)}{\varphi} + \phi^2\, T(\phi) = P\, T(\phi)$$

Where EF {N(t)} = T(ϕ).

$$T(\phi)\,(\phi^2 - P) = \frac{N(0)}{\varphi}$$

Since, $t_0 = 0$ then $N(0) = N_0$

$$T(\phi)\,(\phi^2 - P) = \frac{N^0}{\varphi}$$

$$\Rightarrow T(\phi) = \frac{N_0}{\varphi(\varphi^2 - P)} \quad (4)$$

We apply **Inverse Emad-Falih** transform to equation (4), we have

$$EF^{-1}[T(\phi)] = EF^{-1}\left[\frac{N^0}{\varphi(\varphi^2 - P)}\right]$$

$$\Rightarrow N(t) = N_0 e^{Pt}$$

It is the population at any time t.

B. *Emad-Falih Transform for Decay Problems:*

Now, we use **Emad-Falih** Transform to obtain the solution of decay problems.

The decay problems of the substance is defined as

$$\frac{dN}{dt} = -PN \quad (4)$$

With initial condition as,

$$N(t_0) = N_0 \quad (5)$$

Where N is the amount of substance at time t, P is a positive real number and N_0 is the initial amount of the substance at time t_0.

As mass of the substance is decreasing with respect to time we take negative sign in the right hand side of the equation. Thus the derivative $\frac{dN}{dt}$ must be negative.

We apply**Emad-Falih** transform on the both sides of (4),

$$EF\left\{\frac{dN}{dt}\right\} = EF\{-PN(t)\}$$

$$EF\left\{\frac{dN}{dt}\right\} = -P\ EF\{N(t)\} \quad (6)$$

Now we apply **Emad-Falih** Transform of derivatives of function property to this equation and obtain

$$\frac{-N(0)}{\varphi} + \phi^2\, T(\phi) = -P\, T(\phi)$$

Since, $t_0 = 0$ then $N(0) = N_0$

$$T(\phi)\,(\phi^2 + P) = \frac{N^0}{\varphi}$$

$$\Rightarrow T(\phi) = \frac{N^0}{\varphi(\varphi^2 + P)}$$

Apply **InverseEmad-Falih** transform on above equation, we have

$$EF^{-1}\{T(\phi)\} = EF^{-1}\left\{\frac{N^0}{\varphi(\varphi^2 + P)}\right\}$$

$$\Rightarrow N(t) = N_0 e^{-Pt}$$

It is required amount of the substance at time t.

Now we will solve some problems on population growth and decay.

IV. APPLICATIONS

In this section we solve some problems of growth and decay.

Application (1): The population of the city grows at the rate proportional to the number of people presently living in the city. If after two years, the population has doubled and after three years the population is 20,000. Estimate the number of people initially in the city.

Solution: We can write this problem in the form of differential equation as:

$$\frac{dN}{dt} = PN \quad (7)$$

Here N denotes the number of people living in the city at any time t and P is the constant of proportionality.

Consider N_0 be the number of people initially living in the city at t = 0.

We apply **Emad-Falih** transform on both sides of equation (7)

ISSN No:-2456-2165

$$\text{EF}\left\{\frac{dN}{dt}\right\} = \text{P EF}\{\text{N(t)}\}$$

Now we apply **Emad-Falih** Transform of derivatives of function property to this equation and obtain

$$\frac{-N(0)}{\varphi} + \phi^2\, T(\phi) = P\, T(\phi)$$

Since, t =0 then N(0) = N_0,

$$T(\phi)\,(\phi^2 - P) = \frac{N^0}{\varphi}$$

$$\Rightarrow T(\phi) = \frac{N^0}{\varphi(\varphi^2 - P)}$$

Apply **inverse Emad-Falih** transform on above equation,

$$EF^{-1}\{T(\phi)\} = EF^{-1}\left\{\frac{N^0}{\varphi(\varphi^2 - P)}\right\}$$

$$\Rightarrow N(t) = N_0 e^{Pt} \qquad (8)$$

Now at t = 2, N= $2N_0$

$\therefore 2N_0 = N_0\, e^{2P}$

$\therefore 2 = e^{2P}$

$\therefore 2P = \log_e (2)$

$\Rightarrow P = \frac{1}{2} \log_e (2)$

$\Rightarrow$ P = 0.3465

Now t = 3, N = 20000

Put this value in (8), we get

$\therefore 20000 = N_0 e^{3\times 0.3465}$

$\therefore 20000 = N_0 \times 2.8278$

$\therefore N_0 = 7072.6359 \cong 7072$

Therefore number of people living in the city initially is 7072.

Application (2): A radioactive substance is known to decay at a rate proportional to the amount present. Suppose that initially there is 100 milligrams of the radioactive substance present and after two hours it is observed that the radioactive substance has lost 10 percent of its original mass. Find the half-life of the radioactive substance.

Solution: We can write this decay problem as follows,

$$\frac{dN}{dt} = -PN \qquad (9)$$

Where N denotes the amount of radioactive substance at time t and P is the constant of proportionality. Suppose at time $t = 0$ the initial amount of the radioactive substance is N_0.

We apply **Emad-Falih** transform to the equation (9)

$$\text{EF}\left\{\frac{dN}{dt}\right\} = -\text{P EF}\{\text{N(t)}\}$$

Now we apply **Emad-Falih** Transform of derivatives of function property to this equation and obtain

$$\frac{-N(0)}{\varphi} + \phi^2\, T(\phi) = -P\, T(\phi)$$

Since t = 0 then N = N_0 = 100

$$T(\phi)\,(\phi^2 + P) = \frac{100}{\varphi}$$

$$\Rightarrow T(\phi) = \frac{100}{\varphi(\varphi^2 + P)}$$

Apply **inverse Emad-Falih** transform on above equation

$$EF^{-1}\{T(\phi)\} = EF^{-1}\left\{\frac{100}{\varphi(\varphi^2 + P)}\right\}$$

$$\Rightarrow N(t) = 100\, e^{-Pt} \qquad (10)$$

Now at t = 2, the radioactive substance has lost 10 percent of its original mass 100 mg, so N = 100 – 10 = 90

$\therefore 90 = 100\, e^{-2P}$

$\therefore e^{-2P} = 0.9$

$\therefore -2P = \log_e (0.9)$

$\therefore P = 0.0526$

We required half time of radioactive substance (t)

When $N = \frac{N_0}{2} = \frac{100}{2} = 50$

Substitute this value in equation (10)

$\therefore 50 = 100\, e^{-0.0526t}$

$\therefore e^{-0.0526t} = 0.5$

$\therefore -0.0526t = \log_e (0.5)$

$\therefore -0.0526t = -0.6931$

$\therefore$ t = 13.1768 hours

Hence required half life period of the radioactive substance is 13.1768 hours.

V. CONCLUSION

We have successfully used “Emad-Falih Transform“ to solve the problems on population Growth and Decay.

ISSN No:-2456-2165

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