ISSN No:-2456-2165

Triple Connected Roman Domination Number on Some Particular Graphs

[1]R. Sivakumar ,[2]M.S. Paulraj
[1]Assistant Professor, [2]Associate Professor,
Department of Mathematics A. M. Jain College,
Chennai – 114

Abstract:- On a graph G, we can define a Roman domination function $f: V \rightarrow \{0,1,2\}$ satisfying the condition that every vertex $u \in V$ for which $f(u) = 0$ is adjacent to at least one vertex $v \in V$ for which $f(v) = 2$. The weight of a Roman dominating function is the value $f(v) = \sum_{v \in V} f(v)$. The Roman domination number $\gamma_R(G)$ is the minimum weight of a Roman dominating function on G. In a Roman dominating function if the induced subgraph $\langle V_1 \cup V_2 \rangle$ or $\langle V_2 \rangle$ is triple connected, then the Roman dominating function is called triple connected Roman dominating function (TCRDF) In this paper, we determine triple connected Roman domination number for triangle snake graph T_n, double triangle snake graph $D(T_n)$, Cartesian product $C_n \square P_n$, and Moser spindle graph.

Keywords:- *Connected graphs, Triple connected graphs, Domination number, Roman domination number and Connected Roman domination number.*

Subject Classification number: 05C40, 05C69, 05C70

I. INTRODUCTION

A subset S of $V(G)$ in G is called a dominating set, if and only if $N[S] = V(G)$. The domination number $\gamma(G)$ is the minimum cardinality of a dominating set of G. If S is a subset of $V(G)$, then we denote by $\langle S \rangle$, the subgraph induced by S. For notation and graph theory terminology in general, we follow [2]. The triple connected Roman domination number $\gamma_{TRC}(G)$ is the minimum weight of a triple connected Roman dominating function(TCRDF) on G, also we have $\gamma_{TRC}(G) = |V_1| + 2|V_2|$

II. RESULT

In this Section we found the triple connected Roman domination number for triangle snake graph T_n, double triangle snake graph $D(T_n)$, Cartesian product $C_n \square P_n$, and Moser spindle graph.

A. *Definition: Triangular snake Graph*

The triangular snake graph T_n is obtained from the path P_n by replacing each edge of the path by a triangle C_3.

Example:

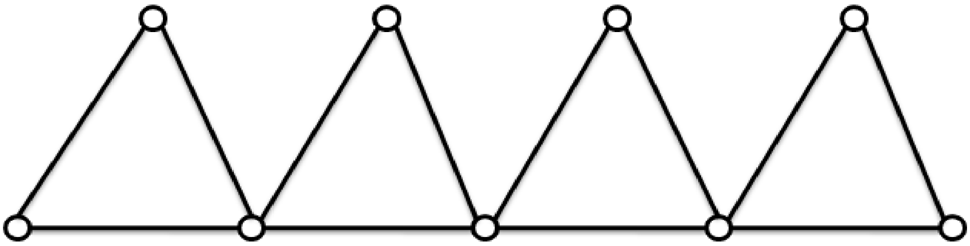

Fig. 1: Triangular snake graph T_5

B. *Definition:DoubleTriangular snake Graph*

The double triangular snake graph $D(T_n)$ consist of two triangular snakes that have a common path.

Example:

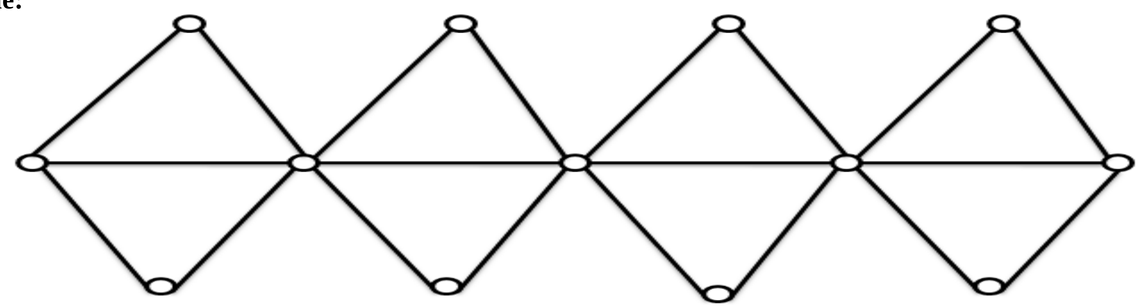

Fig. 2: Double triangular snake graph $D(T_4)$

- **Theorem**
  The triple connected Roman domination number for any triangular snake graph T_n

is $\gamma_{TRC}(T_n) = \begin{cases} \frac{3n-5}{2}, if\ n\ is\ odd, n \geq 5 \\ \frac{3n-4}{2}, if\ n\ is\ even, n \geq 6 \end{cases}$

- **Proof**:
  Let G be a triangular snake graph T_n

  The triangular snake consists of $u_1, u_2, \dots, u_{n-1}$ vertices in the top of triangle and $v_1, v_2, \dots, v_n$ vertices at the bottom of the triangle. So, $V(G) = \{u_i, v_j : 1 \leq i \leq n-1, 1 \leq j \leq n\}$and $E(G) = \{v_i u_i, u_i v_{i+1}, v_i v_{i+1} : 1 \leq i \leq n-1\}$.

  Now $V_2 = \{v_2, v_4, \dots, v_{n-1}\}$, $V_1 = \{v_1, v_3, \dots, v_{n-2}\}$ and $V_0 = \{v_1, v_2, u_1, u_2, \dots, u_{n-1}\}$

  Clearly, V_2 is a dominating set of V_0 and $\langle V_1 \cup V_2 \rangle$ is triple connected.

  Hence the function $f = (V_0, V_1, V_2)$ is a triple connected Roman dominating function on G.

  Case(i): If n is odd, $n \geq 5$, then we have$\gamma_{TRC}(T_n) = |V_1| + 2|V_2|$
  Here$|V_1| = \frac{n-1}{2} - 1$, $|V_2| = \frac{n-1}{2}$

  $$\gamma_{TRC}(T_n) = \frac{n-1}{2} - 1 + 2\left(\frac{n-1}{2}\right) \Rightarrow \gamma_{TRC}(T_n) = \frac{3n-5}{5}$$

  Case (ii): If n is even $n \geq 6$, then
  Here$|V_1| = \frac{n}{2} - 2$, $|V_2| = \frac{n}{2}$

  $$\gamma_{TRC}(T_n) = \frac{n}{2} + 2\left(\frac{n}{2}\right)$$

  $$\gamma_{TRC}(T_n) = \frac{3n-4}{5}$$

- **Theorem**
  The triple connected Roman domination number for a double triangular snake graph $D(T_n)$

is $\gamma_{TRC}D(T_n) = \begin{cases} \frac{3n-5}{2}, if\ n\ is\ odd, n \geq 5 \\ \frac{3n-4}{2}, if\ n\ is\ even, n \geq 6 \end{cases}$

- **Proof:**
  Let G be a double triangular snake graph $D(T_n)$

  The graph$D(T_n)$ consists of $u_1, u_2, \dots, u_{n-1}$ vertices in the top triangle, $w_1, w_2, \dots, w_{n-1}$ are vertices in the bottom triangle and $v_1, v_2, \dots, v_n$ vertices at the common path of the double triangle. So, $V(G) = \{u_i, v_j, w_i : 1 \leq i \leq n-1, 1 \leq j \leq n\}$and

  $E(G) = \{v_i u_i, u_i v_{i+1}, v_i v_{i+1}, v_i w_i, w_i v_{i+1} : 1 \leq i \leq n-1\}$.

  Now $V_2 = \{v_2, v_4, \dots, v_{n-1}\}$, $V_1 = \{v_1, v_3, \dots, v_{n-2}\}$ and $V_0 = \{v_1, v_2, u_1, u_2, \dots, u_{n-1}, w_1, w_2, \dots, w_{n-1}\}$

  Clearly, V_2 is a dominating set of V_0 and $\langle V_1 \cup V_2 \rangle$ is triple connected.

  Hence the function $f = (V_0, V_1, V_2)$ is a triple connected Roman dominating function on G.

  Case (i): If n is odd, $n \geq 5$, then we have $\gamma_{TRC}D(T_n) = |V_1| + 2|V_2|$

  Here$|V_1| = \frac{n-1}{2} - 1$, $|V_2| = \frac{n-1}{2}$

  $$\gamma_{TRC}D(T_n) = \frac{n-1}{2} - 1 + 2\left(\frac{n-1}{2}\right)$$

  $$\gamma_{TRC}D(T_n) = \frac{3n-5}{5}$$

  Case (ii): If n is even $n \geq 6$, then
  Here$|V_1| = \frac{n}{2} - 2$, $|V_2| = \frac{n}{2}$

  $$\gamma_{TRC}D(T_n) = \frac{n}{2} + 2\left(\frac{n}{2}\right)$$

  $$\gamma_{TRC}D(T_n) = \frac{3n-4}{5}$$

C. Definition: Cartesian product of graphs

The **Cartesian product** *G*□*H* of graphs *G* and *H* is a graph such that, the vertex set of *G*□*H* is the Cartesian product *V*(*G*) × *V*(*H*); andtwo vertices (*u*,*u'*) and (*v*,*v'*) are adjacent in *G*□*H* if and only if either*u* = *v* and *u'* is adjacent to *v'* in *H*, **or***u'* = *v'* and *u* is adjacent to *v* in *G*.

- **Theorem**
  The triple connected Roman domination number for Cartesian product $C_n \square P_n$

is $\gamma_{TRC}(G) = \begin{cases} 6, if\ n = 3 \\ (2i+1)(3i+1) + i, if\ n = 3i+1, i \geq 1 \\ 2i(3i-1) + i - 1, if\ n = 3i-1, i \geq 2 \\ 2i(3i) + i - 1, if\ n = 3i, i \geq 2 \end{cases}$

- **Proof**:
  Let G be a Cartesian product $C_n \square P_n$ graph

  The graph $C_n \square P_n$consists of $V(G)$as a vertex set and E(G) as a edge set

  So,$V(G) = \{u_i,\ u'_i,\ v_i,\ v'_i : 1 \leq i \leq n\}$and

  $E(G) =$
  $\begin{cases} \text{either } u_i = v_i \text{ and } u'_i \text{ is adjacent to } v'_i \text{ in } P_n, \\ \textbf{or } u'_i = v'_i \text{ and } u_i \text{ is adjacent to } v_i \text{ in } , 1 \leq i \leq n, 1 \leq j \leq \end{cases}$
  .

  Case (i): If $n = 3$, then we have $\gamma_{TRC} = |V_1| + 2|V_2|$

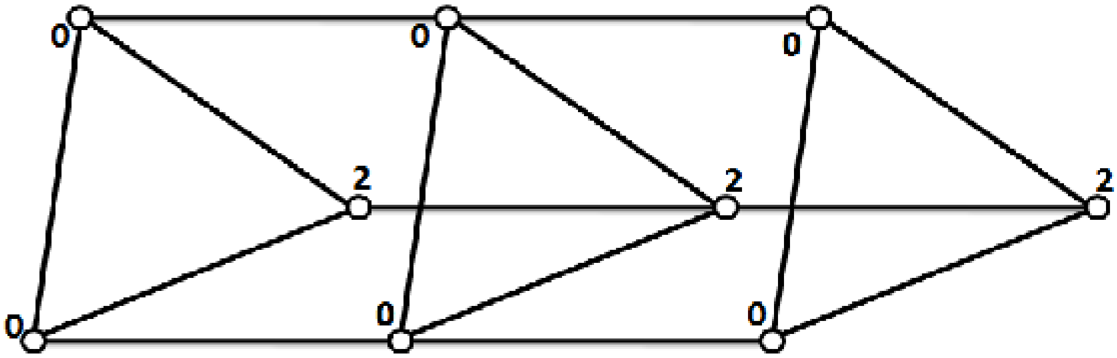

Fig. 3: Cartesian product $C_3□P_3$ graph

Clearly, V_2 is a dominating set of V_0 and $\langle V_2 \rangle$ is triple connected.
Hence the function $f = (V_0, V_1, V_2)$ is a triple connected Roman dominating function on G.

Here $|V_1| = 0$, $|V_2| = 3$

$\gamma_{TRC} = 0 + 2(3) \Rightarrow \gamma_{TRC} = 6$

Case (ii): If n = 4, then we have $|V_1| = 5$, $|V_2| = 4$,
$\gamma_{TRC} = 5 + 2(4) = 3(4) + 1 = 13$
$\Rightarrow \gamma_{TRC} = 13$

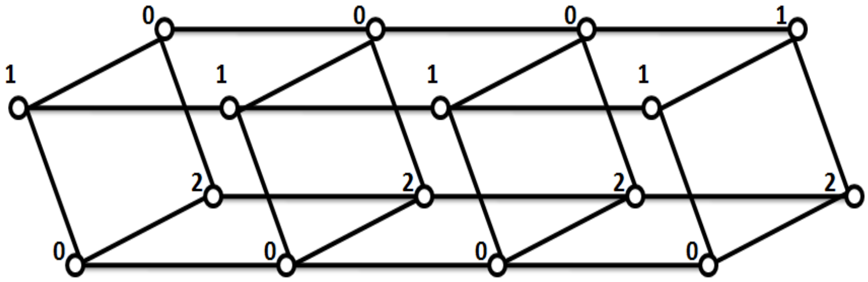

Fig. 4: Cartesian product $C_4□P_4$ graph

Similarly, if n = 7,(n will be increased by 3 in each case)
We get $|V_1| = 9$, $|V_2| = 14$, $\gamma_{TRC} = 9 + 2(14) = 5(7) + 2 = 37$

For n = 10, we have $|V_1| = 15$, $|V_2| = 29$, $\gamma_{TRC} = 15 + 2(29) = 7(10) + 3 = 73$
The following table gives the triple Roman domination number for the cartesian product upto n values.

S.No	The value of n	TCRDN
1	4	$\gamma_{TRC} = 3(4) + 1$
2	7	$\gamma_{TRC} = 5(7) + 2$
3	10	$\gamma_{TRC} = 7(10) + 3$
4	13	$\gamma_{TRC} = 9(13) + 4$
5	16	$\gamma_{TRC} = 11(16) + 5$
6	19	$\gamma_{TRC} = 13(19) + 6$
7	22	$\gamma_{TRC} = 15(22) + 7$
8	25	$\gamma_{TRC} = 17(25) + 8$
9	28	$\gamma_{TRC} = 19(28) + 9$
10	31	$\gamma_{TRC} = 21(31) + 10$
n	$n = 3i + 1, i \geq 1$	$\gamma_{TRC=} (2i + 1)(3i + 1) + i$

Table 1: Triple connected Roman domination numbers

In each case V_2 is a dominating set of V_0 and $\langle V_1 \cup V_2 \rangle$ is triple connected.

$If\ n = 3i + 1, i \geq 1$, then we have the triple connected Roman domination number as

$\gamma_{TRC} = (2i + 1)(3i + 1) + i = 6i^2 + 6i + 1$

Case (iii): If n = 5, then we have $|V_1| = 1$, $|V_2| = 10$, $\gamma_{TRC} = 1 + 2(10) = 4(5) + 1 = 21$

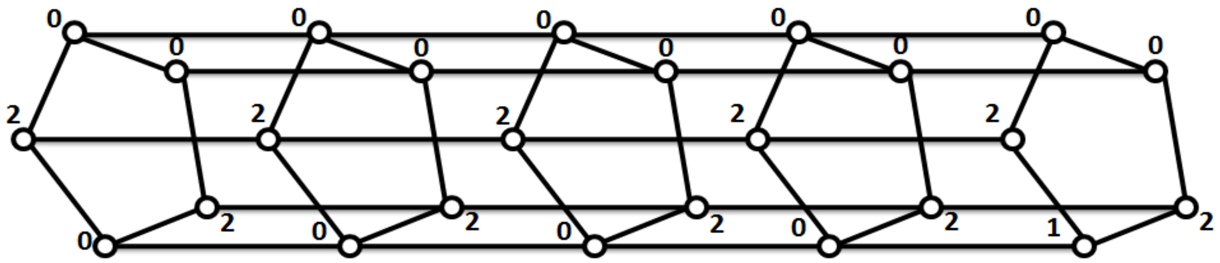

Fig. 5: Cartesian product $C_5 \square P_5$ graph

$\Longrightarrow \gamma_{TRC} = 21$

Similarly, if n = 8,(n will be increased by 3 in each case)

We get $|V_1| = 4$, $|V_2| = 23$, $\gamma_{TRC} = 4 + 2(23) = 6(8) + 2 = 50$

For n = 11, we have $|V_1| = 7$, $|V_2| = 42$, $\gamma_{TRC} = 7 + 2(42) = 8(11) + 3 = 91$

S.No	The value of n	TCRDN
1	5	$\gamma_{TRC} = 4(5) + 1$
2	8	$\gamma_{TRC} = 6(8) + 2$
3	11	$\gamma_{TRC} = 8(11) + 3$
4	14	$\gamma_{TRC} = 10(14) + 4$
5	17	$\gamma_{TRC} = 12(17) + 5$
6	20	$\gamma_{TRC} = 14(20) + 6$
7	23	$\gamma_{TRC} = 16(23) + 7$
8	26	$\gamma_{TRC} = 18(26) + 8$
9	29	$\gamma_{TRC} = 20(29) + 9$
10	32	$\gamma_{TRC} = 22(32) + 10$
n	$n = 3i - 1, i \geq 2$	$\gamma_{TRC=} 2i(3i - 1) + i - 1$

Table 2: Triple connected Roman domination numbers

In each case V_2 is a dominating set of V_0 and $\langle V_1 \cup V_2 \rangle$ is triple connected.

$If\ n = 3i - 1, i \geq 2$, then the triple connected Roman domination number founded as

$\gamma_{TRC} = 2i(3i - 1) + i - 1 = 6i^2 - i - 1$

Case (iv):If n = 6, then we have $|V_1| = 2$, $|V_2| = 12$, $\gamma_{TRC} = 2 + 2(12) = 4(6) + 2 = 26$

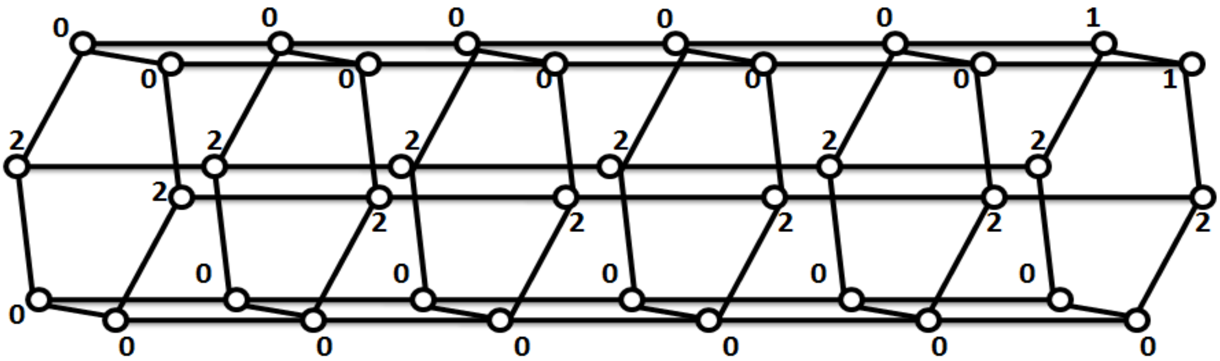

Fig. 6: Cartesian product $C_6 \square P_6$ graph

$\Longrightarrow \gamma_{TRC} = 26$

Similarly, if n = 9,(n will be increased by 3 in each case)

We get $|V_1| = 5$, $|V_2| = 18$, $\gamma_{TRC} = 5 + 2(27) = 6(9) + 3 = 57$

For n = 12, we have $|V_1| = 8$, $|V_2| = 48$, $\gamma_{TRC} = 15 + 2(48) = 8(12) + 4 = 100$

S.No	The value of n	TCRDN
1	6	$\gamma_{TRC} = 4(6) + 2$
2	9	$\gamma_{TRC} = 6(9) + 3$
3	12	$\gamma_{TRC} = 8(12) + 4$
4	15	$\gamma_{TRC} = 10(15) + 5$
5	18	$\gamma_{TRC} = 12(18) + 6$
6	21	$\gamma_{TRC} = 14(21) + 7$
7	24	$\gamma_{TRC} = 16(24) + 8$
8	27	$\gamma_{TRC} = 18(27) + 9$
9	30	$\gamma_{TRC} = 20(30) + 10$
10	33	$\gamma_{TRC} = 22(33) + 11$
n	$n = 3i, i \geq 2$	$\gamma_{TRC=} 2i(3i) + i - 1$

Table 3: Triple connected Roman domination numbers

In each case V_2 is a dominating set of V_0 and $\langle V_1 \cup V_2 \rangle$ is triple connected.

$If\ n = 3i, i \geq 2$, then the triple connected Roman domination num0062er founded as

$\gamma_{TRC} = 2i(3i) + i - 1 = 6i^2 + i - 1$

- **Theorem**

For a Moser spindle graph, the triple connected Roman domination number is

$\gamma_{TRC} D(T_n) = 5.$

- **Proof**:

Moser spindle graph contains 7 vertices and 11 edges.

Here $|V_1| = 1, |V_2| = 2$, V_2 is a dominating set of V_0 and $\langle V_1 \cup V_2 \rangle$ is triple connected.

Hence the function $f = (V_0, V_1, V_2)$ is a triple connected Roman dominating function on G.

So, we have $\gamma_{TRC} = |V_1| + 2|V_2|$

Hence $\gamma_{TRC} = 1 + 2(4) \Rightarrow \gamma_{TRC} = 5$

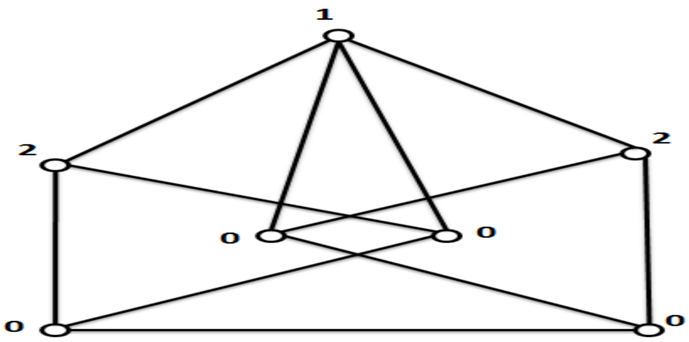

Fig. 7: Moser spindle graph

III. CONCLUSION

In this paper, we obtained the triple connected Roman domination number for triangular snake graphs, double triangular snake graphs, Cartesian product $C_n \square P_n$ graph and Moser spindle graph. We will find triple connected Roman domination number for some special graphs such as alternate triangular snake graphs, double alternate triangular snake graphs, quadrilateral snake graph, etcin our future work

REFERENCES

[1] International Journal of pure and applied mathematics, Volume 116, no 1, 2017, 105 - 113 Connected domination path decomposition of triangle snake graph, E. Ebin Raja Merly, D. JeyaJothi

[2] Text book Bondy and Murthy

[3] International Journal of Current Research and Modern Education Impact Factor 6.725, Special Issue, July – 2017, Page Number 10-12, Triple Connected Domination Number For Some Special Graphs A. Josephine Lissie& S. Jaya

[4] Roman Domination In Cartesian Product Graphs And Strong Product Graphs, Ismael González Yero and Juan Alberto Rodríguez-VelázquezApplicable Analysis and

Discrete Mathematics, Vol. 7, No. 2 (October 2013), pp. 262-274 (13 pages)
[5] Roman Domination Number of the Cartesian Products of Paths and CyclesJanuary 2013 Kragujevac Journal of Mathematics 37(2) DOI:10.37236/2595PolonaRepoluskInstitute of Mathematics, Physics and Mechanics Janez Zerovnik University of Ljubljana
[6] International J.Math. Combin. Vol.3(2012), 93-104, Triple Connected Domination Number of a Graph, G.Mahadevan∗ , SelvamAvadayappan† , J.Paulraj Joseph‡ and T.Subramanian∗
[7] I. Jadav, G. V. Ghodasara, Snakes related stronglygraphs, International Journal of Advanced Engineering Research and Science (IJAERS)[Vol-3, Issue-9, Sep- 2016
[8] The Graceful chromatic number for some particular classes of graphs, Alexandrupopa, Radu minchConference 2019, 21st SYNASC
[9] Labeling of double Triangular Snake, Dushyant Tanna, July 2013, Research Gate The graceful chromatic number for some particular classes of graphs The graceful chromatic number for some particular classes of graphs International Journal of Pure and Applied Mathematics Volume 116No.12017, 105-113
[10] In CONNECTED DOMINATION PATH DECOMPOSITION OF TRIANGULAR SNAKE GRAPH.