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An Inventory Model having Polynomial Demand with Time Dependent Deterioration and Holding Cost

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Abstract:- In this paper, an inventory model is developed for deteriorating goods with shortages which are fully reserved. Demand rate is taken as polynomial function of time whereas deterioration rate and holding cost is taken as dependent of time.

I. INTRODUCTION

Inventory management has become the trend now for better running of an organization, as it's concerned with minimization of cost and maximization of profit. The fact that most of the goods deteriorate with time has affected inventory management. So, we are required to manage the system of inventory considering the effect caused by deterioration and generation of such type of models is required for this purpose. From the last few decades, many researchers have developed such kinds of models. Some of the work is listed below.

Bhunia and Maiti (1997) [2] created some realistic models in which rate of production depends on on-hand inventory. Wu, J.W. et al., (1999) [16] derived the EOQ Model with Weibull rate, assuming ramp type demand. Ouyanget.al., (2005) [7] considered exponential declining demand and partial backlogging in his model. Shah N.H. (2010),[8] developed policy of order for items that deteriorates with time when demand is exponentially decreasing. Mishra, V.K. et al., [6] developed model with time dependent demand and partial backlogging. Sharma (2013) et al., [9] developed model by taking Weibull Distributed Deterioration. Kumar, V., etal., [4] created inventory model by taking demand that depends on selling price and under trade credit holding cost is taken as time dependent. Ibe et al., (2016) [3] developed a Model that follows constant deterioration with time and time varying holding cost. Maragatham (2017) [5] et. al., presented Model for Deteriorating Items in single ware house and consider lead time as constant, Shortages are allowed in lead time and completely backlogged. Aliyu (2020) etal., [1], considered generalised exponential decreasing demand in his model.

Shelly and Kumar, R., (2021) [10] [11] [12] developed models by taking polynomial demand with deterioration as time dependent in one model & constant deterioration in other model and developed one model with time dependent demand and deterioration. Soni and Kumar, R., (2021) [13] [14] [15] developed models by taking bi-quadratic polynomial demand with static rate of deterioration in one model & variable rate of deterioration in other model and one model with demand as time dependent with static rate of deterioration. The working of the current work is based on the above cited works and specially on paper by **Shelly and Kumar, R**. [10] by using holding cost as a linear function of time.

II. ASSUMPTIONS AND NOTATIONS

A. Notations: -

The following are the notations used here: -

- h(t) = Inventory Holding Cost per unit per unit time.
- C_2 = Shortage cost per unit per unit time.
- C_3 = Deterioration cost per unit per unit time.
- T = Length of each cycle.
- I(t) = Inventory at any time t.
- C(t) = Average total cost.
- D(t) = Demand Rate Function
- 8.$\theta(t)$ = Deterioration Rate Function
- S = Initial Inventory

B. Assumptions: -

The following are the assumptions used here: -

- Demand Rate D(t) is assumed as polynomial function of time, given by $D(t) = t + 2t^2 + 3t^3 + \ldots + nt^n$.
- The deterioration rate function, $\theta(t)$ is assumed in the form $\theta(t) = \theta_0 t$; $0 < \theta_0 << 1$; $t > 0$.
- The holding cost is assumed in the form h(t) = h + at, where h>0, a>0.
- Replenishment size is constant and the replenishment rate is infinite.
- The Lead time is zero.
- Shortages are considered and are totally reserved.
- During the period T, neither is replacement nor repair of deteriorated units.

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III. ANALYSIS OF MODEL

Let Inventory level at any time t be I(t). Inventory level slowly decreases during time interval (0, t_1),t_1<Tand becomes exactly zero at t = t_1. Shortages takes place in the interval (0, t_1), which are totally reserved.Differential equations which govern this inventory system during the interval 0 ≤t ≤T using demand anddeterioration rate are

$$\frac{dI(t)}{dt} + \theta_0 t\, I(t) = -(t + 2t^2 + 3t^3 + \cdots + nt^n) \quad (1)$$

and

$$\frac{dI(t)}{dt} = -(t + 2t^2 + 3t^3 + \cdots + nt^n) \quad (2)$$

Solution of differential equation (1) is

$$I(t)e^{\frac{\theta_0}{2}t^2} = -\int (t + 2t^2 + 3t^3 + \cdots + nt^n)e^{\frac{\theta_0}{2}t^2} dt + C$$
$$= -\int (t + 2t^2 + 3t^3 + \cdots + nt^n)\left(1 + \frac{\theta_0}{2}t^2\right) dt + C$$
$$= -\int \left[(t + 2t^2 + 3t^3 + \cdots + nt^n) + \frac{\theta_0}{2}(t^3 + 2t^4 + \cdots + nt^{n+2})\right] dt + C$$
$$= -\left[\left(\frac{1}{2}t^2 + \frac{2}{3}t^3 + \cdots + \frac{n}{n+1}t^{n+1}\right) + \frac{\theta_0}{2}\left(\frac{1}{4}t^4 + \frac{2}{5}t^5 + \cdots + \frac{n}{n+3}t^{n+3}\right)\right] + C$$

Putting t = 0, I(0) = C. But I(0)=S. Therefore C = S. Thus

$$I(t)e^{\frac{\theta_0}{2}t^2} = S - \left[\left(\frac{1}{2}t^2 + \frac{2}{3}t^3 + \cdots + \frac{n}{n+1}t^{n+1}\right) + \frac{\theta_0}{2}\left(\frac{1}{4}t^4 + \frac{2}{5}t^5 + \cdots + \frac{n}{n+3}t^{n+3}\right)\right]; \quad 0 \le t \le T \quad (3)$$

Again from (3), I(t_1) =0. So

$$0 = S - [\left(\frac{1}{2}t_1^2 + \frac{2}{3}t_1^3 + \cdots + \frac{n}{n+1}t_1^{n+1}\right) + \frac{\theta_0}{2}\,(\frac{1}{4}t_1^4 + \frac{2}{5}t_1^5 + \cdots + \frac{n}{n+3}t_1^{n+3})]$$

Thus

$$S = \left[\left(\frac{1}{2}t_1^2 + \frac{2}{3}t_1^3 + \cdots + \frac{n}{n+1}t_1^{n+1}\right) + \frac{\theta_0}{2}\left(\frac{1}{4}t_1^4 + \frac{2}{5}t_1^5 + \cdots + \frac{n}{n+3}t_1^{n+3}\right)\right] \quad (4)$$

Putting the value of S in (3), we get

$$I(t)e^{\frac{\theta_0}{2}t^2} = \frac{1}{2}(t_1^2 - t^2) + \frac{2}{3}(t_1^3 - t^3) + \cdots + \frac{n}{n+1}(t_1^{n+1} - t^{n+1}) + \frac{\theta_0}{2}\,(\frac{1}{4}(t_1^4 - t^4) + \frac{2}{5}(t_1^5 - t^5) + \cdots + \frac{n}{n+3}(t_1^{n+3} - t^{n+3}))$$

Hence

$$I(t) = \frac{1}{2}(t_1^2 - t^2) + \frac{2}{3}(t_1^3 - t^3) + \cdots + \frac{n}{n+1}(t_1^{n+1} - t^{n+1}) + \frac{\theta_0}{2}\left[\frac{1}{4}(t^2 - t_1^2)^2 + \cdots + \frac{n}{(n+1)(n+3)}[(n+1)t_1^{n+3} - (n+3)t^2t_1^{n+1} + 2t^{n+3}]\right] \quad (5)$$

$$I(t) = \sum_{1}^{n}\left[\frac{m}{m+1}(t_1^{m+1} - t^{m+1}) + \frac{\theta_0}{2}\frac{m}{(m+1)(m+3)}[(m+1)t_1^{m+3} - (m+3)t^2\, t_1^{m+1} + 2t^{m+3}]\right] \quad (6)$$

Solution of differential equation (2) is

$$I(t) = -\left(\frac{1}{2}t^2 + \frac{2}{3}t^3 + \cdots + \frac{n}{n+1}t^{n+1}\right) + A \quad (7)$$

Since I(t_1) = 0, we have

$$0 = -\left(\frac{1}{2}t_1^2 + \frac{2}{3}t_1^3 + \cdots + \frac{n}{n+1}t_1^{n+1}\right) + A$$

This implies

$$A = \frac{1}{2}t_1^2 + \frac{2}{3}t_1^3 + \cdots + \frac{n}{n+1}t_1^{n+1}$$

Hence

$$I(t) = \frac{1}{2}(t_1^2 - t^2) + \frac{2}{3}(t_1^3 - t^3) + \cdots + \frac{n}{n+1}(t_1^{n+1} - t^{n+1}); t_1 \le t \le T \quad (8)$$

Thus, the entire amount of deteriorated units = I(0) – stock loss due to demand

$$= S - \int_0^{t_1}(t + 2t^2 + \cdots . + nt^n)dt$$
$$= S - \left(\frac{1}{2}t_1^2 + \frac{2}{3}t_1^3 + \cdots + \frac{n}{n+1}t_1^{n+1}\right)$$

$$= \left(\frac{1}{2}t_1^2 + \frac{2}{3}t_1^3 + \cdots + \frac{n}{n+1}t_1^{n+1}\right) + \frac{\theta_0}{2}\left(\frac{1}{4}t_1^4 + \frac{2}{5}t_1^5 + \cdots + \frac{n}{n+3}t_1^{n+3}\right) - \left(\frac{1}{2}t_1^2 + \frac{2}{3}t_1^3 + \cdots + \frac{n}{n+1}t_1^{n+1}\right)$$

$$= \frac{\theta_0}{2}\left(\frac{1}{4}t_1^4 + \frac{2}{5}t_1^5 + \cdots + \frac{n}{n+3}t_1^{n+3}\right) \quad (9)$$

Inventory Holding Cost is given by

$$=\int_0^{t_1} h(t)I(t)dt$$

$$= \int_0^{t_1}(h+at)I(t)dt$$

$$= h\int_0^{t_1} I(t)dt + a\int_0^{t_1} I(t)dt$$

$$= h\int_0^{t_1}\left[\frac{1}{2}(t_1^2 - t^2) + \frac{2}{3}(t_1^3 - t^3) + \cdots + \frac{n}{n+1}(t_1^{n+1} - t^{n+1})\right]dt$$

$$+ h\frac{\theta_0}{2}\int_0^{t_1}\left[\frac{1}{4}((t^2 - t_1^2)^2) + \cdots + \frac{n}{(n+1)(n+3)}[(n+1)t_1^{n+3} - (n+3)t^2 t_1^{n+1} + 2t^{n+3}\right]dt$$

$$+ a\int_0^{t_1}\left[\frac{1}{2}(t_1^2 t - t^3) + \frac{2}{3}(t_1^3 t - t^4) + \cdots + \frac{n}{n+1}(t_1^{n+1}t - t^{n+2})\right]dt$$

$$+ a\frac{\theta_0}{2}\int_0^{t_1}\left[\frac{1}{4}(t(t^2 - t_1^2)^2) + \cdots + \frac{n}{(n+1)(n+3)}t[(n+1)t_1^{n+3} - (n+3)t^2 t_1^{n+1} + 2t^{n+3}\right]dt$$

$$= h\left[\left(\frac{1}{3}t_1^3 + \frac{1}{2}t_1^4 + \cdots + \frac{n}{n+2}t_1^{n+2}\right) + \frac{\theta_0}{2}\left(\frac{2}{15}t_1^4 + \cdots + \frac{2n}{3(n+4)}t_1^{n+4}\right)\right]$$

$$+ a\left[\left(\frac{1}{8}t_1^4 + \frac{1}{5}t_1^5 + \cdots + \frac{n}{2(n+3)}t_1^{n+3}\right) + \frac{\theta_0}{2}\left(\frac{1}{24}t_1^6 + \cdots + \frac{n}{4(n+5)}t_1^{n+5}\right)\right] \quad (10)$$

Deterioration Cost = C_3 *the entire amount of deteriorated units

$$= C_3\left[\frac{\theta_0}{2}\left(\frac{1}{4}t_1^4 + \frac{2}{5}t_1^5 + \cdots + \frac{n}{n+3}t_1^{n+3}\right)\right] \quad (11)$$

Shortage units Quantity = $\int_{t_1}^{T} -I(t)dt$

$$= -\int_{t_1}^{T}\left[\frac{1}{2}(t_1^2 - t^2) + \frac{2}{3}(t_1^3 - t^3) + \cdots + \frac{n}{n+1}(t_1^{n+1} - t^{n+1})\right]dt$$

$$= T\left[\frac{1}{2}\left(\frac{1}{3}T^2 - t_1^2\right) + \frac{2}{3}\left(\frac{1}{4}T^3 - t_1^3\right) + \cdots + \frac{n}{n+1}\left(\frac{1}{n+2}T^{n+1} - t_1^{n+1}\right)\right] + \left[\frac{1}{3}t_1^3 + \frac{1}{2}t_1^4 + \cdots + \frac{n}{n+2}t_1^{n+2}\right] \quad (12)$$

Shortage Cost = C_2* shortage units quantity

$$= C_2 T\left[\frac{1}{2}\left(\frac{1}{3}T^2 - t_1^2\right) + \frac{2}{3}\left(\frac{1}{4}T^3 - t_1^3\right) + \cdots + \frac{n}{n+1}\left(\frac{1}{n+2}T^{n+1} - t_1^{n+1}\right)\right] +$$

$$C_2\left[\frac{1}{3}t_1^3 + \frac{1}{2}t_1^4 + \cdots + \frac{n}{n+2}t_1^{n+2}\right] \quad (13)$$

The Total Cost per unit time

= Inventory Holding Cost + Deterioration Cost + Shortage Cost

$$= h\left[\left(\frac{1}{3}t_1^3 + \frac{1}{2}t_1^4 + \cdots + \frac{n}{n+2}t_1^{n+2}\right) + \frac{\theta_0}{2}\left(\frac{2}{15}t_1^4 + \cdots + \frac{2n}{3(n+4)}t_1^{n+4}\right)\right]$$

$$+ a\left[\left(\frac{1}{8}t_1^4 + \frac{1}{5}t_1^5 + \cdots + \frac{n}{2(n+3)}t_1^{n+3}\right) + \frac{\theta_0}{2}\left(\frac{1}{24}t_1^6 + \cdots + \frac{n}{4(n+5)}t_1^{n+5}\right)\right]$$

$$+ C_3\left[\frac{\theta_0}{2}\left(\frac{1}{4}t_1^4 + \frac{2}{5}t_1^5 + \cdots + \frac{n}{n+3}t_1^{n+3}\right)\right] + C_2\left[\frac{1}{3}t_1^3 + \frac{1}{2}t_1^4 + \cdots + \frac{n}{n+2}t_1^{n+2}\right]$$

$$+ C_2 T\left[\frac{1}{2}\left(\frac{1}{3}T^2 - t_1^2\right) + \frac{2}{3}\left(\frac{1}{4}T^3 - t_1^3\right) + \cdots + \frac{n}{n+1}\left(\frac{1}{n+2}T^{n+1} - t_1^{n+1}\right)\right]$$

The Average Total Cost per unit time,

$$C(t_1) = \frac{1}{T}\left[\,Total\ Cost\ per\ unit\ time\,\right]$$

$$C(t_1) = \frac{h}{T}\left[\left(\frac{1}{3}t_1^3 + \frac{1}{2}t_1^4 + \cdots + \frac{n}{n+2}t_1^{n+2}\right) + \frac{\theta_0}{2}\left(\frac{2}{15}t_1^4 + \cdots + \frac{2n}{3(n+4)}t_1^{n+4}\right)\right]$$

$$+ \frac{a}{T}\left[\left(\frac{1}{8}t_1^4 + \frac{1}{5}t_1^5 + \cdots + \frac{n}{2(n+3)}t_1^{n+3}\right) + \frac{\theta_0}{2}\left(\frac{1}{24}t_1^6 + \cdots + \frac{n}{4(n+5)}t_1^{n+5}\right)\right] + \frac{C_3}{T}\left[\frac{\theta_0}{2}\left(\frac{1}{4}t_1^4 + \frac{2}{5}t_1^5 + \cdots + \frac{n}{n+3}t_1^{n+3}\right)\right] +$$

$$C_2\left[\frac{1}{2}\left(\frac{1}{3}T^2 - t_1^2\right) + \frac{2}{3}\left(\frac{1}{4}T^3 - t_1^3\right) + \cdots + \frac{n}{n+1}\left(\frac{1}{n+2}T^{n+1} - t_1^{n+1}\right)\right]$$

$$+ \frac{C_2}{T}\left[\frac{1}{3}t_1^3 + \frac{1}{2}t_1^4 + \cdots . + \frac{n}{n+2}t_1^{n+2}\right]$$

For minimum average total cost, the necessary and sufficient conditions are $\frac{dC(t_1)}{dt_1} = 0\ and\ \frac{d^2C(t_1)}{dt_1^2} > 0$.

Now $\frac{dC(t_1)}{dt_1} = 0\ gives$

$$(t_1+2t_1^2+3t_1^3+\cdots+nt_1^n)\left[\frac{a\theta_0}{8}t_1^4+\frac{h\theta_0}{3T}t_1^3+\left(\frac{C_3\theta_0}{2T}+\frac{a}{2T}\right)t_1^2+\frac{(h+C_2)}{T}t_1-C_2\right]=0$$

Which further implies

$$\frac{a\theta_0}{8}t_1^4+\frac{h\theta_0}{3T}t_1^3+\left(\frac{C_3\theta_0}{2T}+\frac{a}{2T}\right)t_1^2+\frac{(h+C_2)}{T}t_1-C_2=0 \quad (14)$$

Since (14) is a bi-quadratic equation in t_1 having only one change of sign then this equation has atmost one positive root by applying Descartes's rule of signs. Also $\frac{d^2C(t_1)}{dt_1^2}>0$. Let t_1^* be the positive root of (14). So optimum value of t_1 is t_1^*. Substituting it in (4), the optimized value of S is

$$S^*=\left[\left(\frac{1}{2}t_1^{*2}+\frac{2}{3}t_1^{*3}+\cdots+\frac{n}{n+1}t_1^{*(n+1)}\right)+\frac{\theta_0}{2}\left(\frac{1}{4}t_1^{*4}+\frac{2}{5}t_1^{*5}+\cdots+\frac{n}{n+3}t_1^{*(n+3)}\right)\right]$$

(15)

Minimum value of $C(t_1)$ is

$$C(t_1^*)=\frac{h}{T}\left[\left(\frac{1}{3}t_1^{*3}+\frac{1}{2}t_1^{*4}+\cdots+\frac{n}{n+2}t_1^{*(n+2)}\right)+\frac{\theta_0}{2}\left(\frac{2}{15}t_1^{*5}+\cdots+\frac{2n}{3(n+4)}t_1^{*(n+4)}\right)\right]$$
$$+\frac{a}{T}\left[\left(\frac{1}{8}t_1^{*4}+\frac{1}{5}t_1^{*5}+\cdots+\frac{n}{2(n+3)}t_1^{*(n+3)}\right)+\frac{\theta_0}{2}\left(\frac{1}{24}t_1^{*6}+\cdots+\frac{n}{4(n+5)}t_1^{*(n+5)}\right)\right]+\frac{C_3}{T}\left[\frac{\theta_0}{2}\left(\frac{1}{4}t_1^{*4}+\frac{2}{5}t_1^{*5}+\cdots+\right.\right.$$
$$\left.\left.\frac{n}{n+3}t_1^{*(n+3)}\right)\right]+\frac{C_2}{T}\left[\frac{1}{3}t_1^{*3}+\frac{1}{2}t_1^{*4}+\cdots.+\frac{n}{n+2}t_1^{*(n+2)}\right]$$
$$+C_2\left[\frac{1}{2}\left(\frac{1}{3}T^2-t_1^{*2}\right)+\frac{2}{3}\left(\frac{1}{4}T^3-t_1^{*3}\right)+\cdots+\frac{n}{n+1}\left(\frac{1}{n+2}T^{n+1}-t_1^{*(n+1)}\right)\right]$$

(16)

Thus equation (16) gives optimal value of total average cost per unit time. These equations can be further solved for different values of variables used here, using software like Matlab and/or Mathematica.

IV. CONCLUSION

In this paper, an inventory model is generated for goods that deteriorate with time by considering demand as a polynomial function of time with time-dependent deterioration and holding cost as a linear function of time, and then by using mathematical analysis the optimized value of cost is calculated.

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