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A Study on Fuzzy Inventory Model with Fuzzy Demand with No Shortages Allowed using Pentagonal Fuzzy Numbers

ONYENIKE K. and OJARIKRE, H.I
[1]Department of Computer Science/Mathematics, Novena University Ogume, Nigeria
[2]Department of Mathematics, Delta State University, Abraka

Abstract:- This work considers inventory systems that models uncertainties in demand and various fuzzy inventory cost parameters. However, in many cases where there is little or no historical data available to decision makers due to recent changes in the supply chain environment, probability distribution may simply not be available, or may not easily or accurately be estimated. In this paper we consider fuzzy inventory model with three fuzzy parameters and shortages not permitted. Fuzziness in this model occurs in the demand of product, holding costs and ordering costs. The inventory lead time is assumed to be zero. To obtain the total fuzzy costs, the three fuzzy parameters have been represented using Pentagonal fuzzy numbers (PFN). The economic ordered quantity (EOQ) and total inventory costs has also been computed by defuzzification of the fuzzy economic ordered quantity and total fuzzy costs using the graded mean integration approach. A numerical example is provided to compare our model with the four fuzzy set parameters with those with crisp parameters.

Keywords:- *Demand, Defuzzification, Inventory cost, pentagonal fuzzy numbers, Graded mean integration representation method.*

I. INTRODUCTION

Inventory is an essential component of supply chain management. Inventory is defined as the list of items held in stock for sale, production or distribution. Stock consists of goods and materials used for production and distribution processes such as raw materials, work-in-progress and finished products. Uncertainties are the main reason for a marketer to keep inventory. Uncertainty is the probability that unexpected or risky events may occur in all business. Uncertainty increases the complexity of business decision such as inventory quantity determination. The major sources of uncertainties in an inventory are demand fluctuation and supply disruptions. The fuzzy set theory in inventory modeling is the closest possible approach to reality, as reality is not exact and can only be calculated to some extent. Same way, fuzzy theory helps one to incorporate uncertainties in the design of the model, thus bringing it closure to real life situation. Recent research has employed fuzzy logic in modeling uncertainties in inventory management system. Dutta and Pavan (2012) considered inventory model in a fuzzy environment with constant demand and shortages not allowed using two fuzzy parameters, the holding cost and setup cost which they represented using trapezoidal fuzzy numbers. The defuzzification of the total fuzzy cost is done using the signed distance method. Saranya and Varajan (2018) proposed a fuzzy inventory model with acceptable shortages which was completely backlogged with three fuzzy parameters, the carrying cost, holding cost and backorder cost using the triangular and trapezoidal fuzzy numbers to obtain the fuzzy total cost. The total fuzzy cost defuzzified using the graded mean integration value method. Sharmilla and Uthaya kumar (2015) presented fuzzy inventory model for deteriorating items for power demand under fully backlogged condition were holding cost, shortage cost, purchasing cost and deterioration cost all assumed to be fuzzy with exponential demand. The defuzzification of the total fuzzy cost is carried out using the signed distance method. Palami and Maragatham (2017) developed deterministic fuzzy inventory model for time dependent deteriorating items with lead time and stock dependent demand rate. The ordering cost, deterioration cost, holding cost and shortage cost are assumed as triangular fuzzy number. In this model shortages are allowed during the lead time and completely backlogged. The graded mean approach is used to find the total cost. Indarsih (2019) proposed an inventory model with fuzzy parameters and back ordering is allowed. The annual demand and lead time is considered fuzzy with crisp cost parameters. The optimal quantity and reorder point which minimizes the fuzzy total cost is calculated using the graded mean integration and a non linear optimization method by MatLab was employed to find the order quantity. De and Rawat (2011), In their work considered an inventory model in a fuzzy environment without shortage by considering real-life data from the LPG store of Banasthali University. Triangular fuzzy numberswas used to consider the ordering and holding costs. For defuzzification, signed-distance method was used to compute the optimum order quantity. Rajalaskhmi and Michael Rosarioin their paper, investigates a fuzzy inventory model with allowable shortage which is completely backlogged. The carrying costs, backorder cost and ordering cost were treated as fuzzy using triangular, trapezoidal, pentagonal fuzzy numbers to obtain the fuzzy total cost. Signed distance method is used for defuzzification to estimate the total cost.

Rexlin Jeyakumanietal (2020). Studied EOQ model under uncertainty with no reasonable deficiencies for Pentagonal Fuzzy Numbers. Here the parameters like purchasing costs, storage cost and yearly interest are thought to be Pentagonal Fuzzy Numbers. The PFN was defuzzify by utilizing the Graded Mean Integration Representation technique to get the advancement in the least difficult

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manner.Dutta and Pavan (2013) formulated and solved a fuzzy inventory model for deteriorating items with shortages under fully backlogged condition. Deterioration rate and demand were assumed to be constant. Shortages are allowed and assumed to be fully backlogged. Fuzziness is introduced by allowing the cost components (holding cost, shortage cost, etc.), demand rate and the deterioration. In fuzzy environment, all related inventory parameters were assumed to be trapezoidal fuzzy numbers.Sahidul Islam and WasimAkram Mandal (2017) formulated and solved Inventory model with unit production cost, time depended holding cost, without shortages. They considered a single objective inventory model. The nearest interval approximation method was used to convert a triangular fuzzy number to an interval number then transform this interval number to a parametric interval-valued functional form and solve the parametric problem by geometric programming technique. Lathasri and Varadharajan (2020) studied a fuzzy inventory model with shortages where the setup costs dependence on the size in step as the lot size strengthened. The fuzzy costs in this model include setup costs, shortage costs, holding costs and transportation costs and they are represented using fuzzy triangular numbers which was defuzzify using Graded mean integration value.

In this paper, we study a fuzzy inventory model with fuzzy demand and no shortages allowed. Fuzziness in this model occurs in the demand of product, holding costs and ordering costs. The inventory lead time is assumed to be zero. To obtain the total fuzzy costs, the three fuzzy parameters have been represented using PFN. EOQ and total inventory costs has also been computed by defuzzification of the fuzzy economic ordered quantity and total fuzzy costs using the graded mean integration approach. A numerical example is provided to compare our model with the three fuzzy set parameters with those with crisp parameters. Hence, the work entails the definitions of the basic concepts in section 2, while the assumptions and model description have been considered in section 3,section 4 presents the inventory model in the fuzzy & crisp form with model analysis in section 5.

II. DEFINITION AND PRELIMINARIES

A. FUZZY SETS

Fuzzy set is a mathematical model of vague qualitative or quantitative data, frequently generated by means of the natural language. The model is based on the generalization of the classical concepts of set and its characteristic function. Let X be a collection of objects and x a member of X, then a fuzzy sets $\tilde{A}$ in X is a pair of ordered set $\tilde{A} = \{(x, \mu_{\tilde{A}}(x))\}$, where $\mu_{\tilde{A}}(x)$ is the membership function(also called the degree of compatibility or degree of truth).

B. FUZZY NUMBER:

A fuzzy number A is a subset of real line R, with the membership function $\mu_{\tilde{A}}(x)$satisfying the following properties:

- $\mu_{\tilde{A}}(x)$ is piecewise continuous in its domain.
- A is normal, i.e., there exist a $x_0 \in A$ such that $\mu_{\tilde{A}}(x_0) = 1$
- A is convex, i.e. $\mu_{\tilde{A}}(\gamma x_1 + (1-\gamma)x_2 > min\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)) \forall x_1, x_2 \in X.$

C. PENTAGONAL FUZZY NUMBER

A Pentagonal fuzzy number set $\varphi(\widetilde{A})$ is defined as

$\tilde{A} = (a_1, a_2, a_3, a_4, a_5)$ where a_1, a_2, a_3, a_4, a_5 are real numbers and it has a membership function which is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, \ for x \le a_1 \\ \dfrac{(x-a_1)}{(a_2-a_1)}, \ for a_1 \le x \le a_2 \\ \dfrac{(x-a_2)}{(x-a_3)}, \ for a_2 \le x \le a_3 \\ 1, \ for x = a_3 \\ \dfrac{(a_4-x)}{(a_4-a_3)}, \ for a_3 \le x \le a_4 \\ \dfrac{(a_5-x)}{(a_5-a_4)}, \ for a_4 \le x \le a_5 \\ 0, \ for x \succ a_5 \end{cases}$$

D. Operations on Pentagonal fuzzy numbers

Suppose $\tilde{A} = (a_1, a_2, a_3, a_4, a_5)$and $\tilde{B} = (b_1, b_2, b_3, b_4, b_5)$ are two pentagonal fuzzy numbers and $\alpha \in R$. Then,

- $\tilde{A} \oplus \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4, a_5 + b_5)$
- $\tilde{A} \otimes \tilde{B} = (a_1 b_1, a_2 b_2, a_3 b_3, a_4 b_4, a_5 b_5)$
- $-\tilde{A} = (-a_1, -a_2, -a_3, -a_4, -a_5)$
- $\frac{1}{\tilde{A}} = (\frac{1}{a_1}, \frac{1}{a_2}, \frac{1}{a_3}, \frac{1}{a_4}, \frac{1}{a_5})$
- If $\alpha \ge 0$, then $\alpha \otimes \tilde{A} = (\alpha a_1, \alpha a_2, \alpha a_3, \alpha a_4, \alpha a_5)$
- If $\alpha < 0$, then $\alpha \otimes \tilde{A} = (\alpha a_5, \alpha a_4, \alpha a_3, \alpha a_2, \alpha a_1)$

Where $a_1, a_2, a_3, a_4, a_5, b_1, b_2, b_3, b_4, b_5$ are all non positive real numbers.

E. Defuzzification of Fuzzy Numbers

Defuzzification is defined as mapping from fuzzy set $\tilde{A}$ in $U \in R$ to a crisp set $B \in U$

F. Graded mean integration representation

This is a method of Defuzzifying a number into a crisp value. Let $\tilde{A} = (a_1, a_2, a_3, a_4, a_5)$ be a Pentagonal fuzzy number, then the Graded mean integration representation method is given as:

$$\beta(\widetilde{A}) = \frac{(a_1 + 2a_2 + 2a_3 + 2a_4 + a_5)}{8}$$

III. MODEL DESCRIPTION

A. NOTATIONS

The following notations were used to develop the proposed model.

D	Demand in Crisp value
$\widetilde{D}$	Fuzzy demand
C_o	Inventory ordering cost in crisp value
$\tilde{C}_o$	Fuzzy inventory ordering cost
C_h	Inventory holding cost in crisp value
$\tilde{C}_h$	Fuzzy inventory holding cost
TC	Total inventory cost in crisp valued
$\widetilde{TC}$	Fuzzy total inventory cost
Q	Total quantity ordered
Q^0	Economic ordered quantity in crisp value
$\tilde{Q}^0$	Fuzzy economic ordered quantity

B. ASSUMPTIONS

The following assumptions were used to develop the proposed model

- Demands are unknown and fuzzy
- Lead time zero
- No shortages allowed
- Holding cost, ordering cost, are all fuzzy.

IV. INVENTORY MODEL IN CRISP FORM

Here we present inventory model without in crisp form, in this model, we assumed that demand over time is known and uniform with zero lead time. In this model, the total inventory cost (TC) and the economic ordered quantity (Q) is calculated using the following model equations:

The total inventory cost TC= holding cost + ordering cost4.1

We calculate separate components per cycle as follows:

- Unit ordering cost $= \frac{C_o D}{Q}$4.2
- Holding cost $= \frac{C_h Q}{2}$ 4.3

Equation 3.1 becomes,

Total cost $TC = \frac{C_o D}{Q} + \frac{C_h Q}{2}$ 4.4

TC is minimum, if its first derivative with respect to Q is zero and the second derivative positive. i.e

$$\frac{d(TC)}{dQ} = 0, \text{ and } \frac{d^2(TC)}{(dQ)^2} > 0$$

Differentiating TC in equation (4.4) and setting it equal to zero, we have

$$\frac{d(TC)}{dQ} = -\frac{C_o D}{Q^2} + \frac{C_h}{2} = 0$$

$$-\frac{C_o D}{Q^2} + \frac{C_h}{2} = 0$$

$$\frac{C_o D}{Q^2} = \frac{C_h}{2}$$

$$Q^2 = \frac{2C_o D}{C_h}$$

Taking the square root of both sides we have that,

$$Q = \sqrt{\frac{2C_o D}{C_h}} \qquad 4.5$$

Taking the second derivative of TC, we have,

$\frac{d^2(TC)}{(dQ)^2} = \frac{C_o D}{Q^3}$ which is obviously greater than since parameters C_o, D and Q are

not equal to zero and hence equation (4.6) gives us the optimal quantity.

Thus $Q^0 = \sqrt{\frac{2C_o D}{C_h}}$ …………………………………………………………………… 4.6

is the economic ordered quantity which minimizes the total inventory cost.

Also substituting for Q in equation (3.4) gives the minimum inventory cost as

$$(TC)^* = \sqrt{2C_o D C_h} \qquad 4.7$$

C. INVENTORY MODEL IN FUZZY FORM

In this model, we consider the demand rate, ordering cost and holding cost as fuzzy and are been represented as pentagonal fuzzy numbers.

Lets denote the above mentioned costs respectively using LR-form as:

$\widetilde{D} = (d_1, d_2, d_3, d_4, d_5)$, $\widetilde{C_0} = (c_{01}, c_{02}, c_{03}, c_{04}, c_{05})$, $\widetilde{C_h} = (c_{h1}, c_{h2}, c_{h3}, c_{h4}, c_{h5})$

$$\widetilde{TC} = \frac{\widetilde{C_0}(\widetilde{D})}{Q} + \frac{\widetilde{C_h} Q}{2} \quad 4.8$$

$$\widetilde{TC} = \frac{(c_{01}, c_{02}, c_{03}, c_{04}, c_{05}) \otimes (d_1, d_2, d_3, d_4, d_5)}{Q} + \frac{(c_{h1}, c_{h2}, c_{h3}, c_{h4}, c_{h5}) \otimes Q}{2}$$

$$\widetilde{TC} = \frac{c_{01}d_1, c_{02}d_2, c_{03}d_3, c_{04}d_4, c_{05}d_5}{Q} + \frac{c_{h1}Q, c_{h2}Q, c_{h3}Q, c_{h4}Q, c_{h5}Q}{2} \ldots\ldots 4.9$$

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$$\widetilde{TC} = \frac{c_{01}d_1}{Q} + \frac{c_{h1}Q}{2}, \frac{c_{02}d_2}{Q} + \frac{c_{h2}Q}{2}, \frac{c_{03}d_3}{Q} + \frac{c_{h3}Q}{2}, \frac{c_{04}d_4}{Q} + \frac{c_{h4}Q}{2}, \frac{c_{05}d_5}{Q} + \frac{c_{h5}Q}{2} \quad \ldots\ldots 4.10$$

Next we defuzzify using the graded mean integration representation formula for Pentagonal fuzzynumbers.

The graded mean integration representation value is:

$$\beta(\widetilde{A}) = \frac{(a_1 + 2a_2 + 2a_3 + 2a_4 + a_5)}{8}$$

$$\widetilde{TC} = \frac{1}{8}\left(\frac{c_{01}d_1}{Q} + \frac{c_{h1}Q}{2} + \frac{2c_{02}d_2}{Q} + \frac{2c_{h2}Q}{2} + \frac{2c_{03}d_3}{Q} + \frac{2c_{h3}Q}{2} + \frac{2c_{04}d_4}{Q} + \frac{2c_{h4}Q}{2} + \frac{c_{05}d_5}{Q} + \frac{c_{h5}Q}{2}\right)$$

$$\widetilde{TC} = \frac{1}{8}\left(\frac{(c_{01}d_1 + 2c_{02}d_2 + 2c_{03}d_3 + 2c_{04}d_4 + c_{05}d_5)}{Q} + \frac{(c_{h1} + 2c_{h2} + 2c_{h3} + 2c_{h4} + c_{h5})Q}{2}\right) \quad \ldots\ldots 4.11$$

$\widetilde{TC}$ is minimum, if its first derivative with respect to Q is zero and its second derivative positive.

Differentiating $\widetilde{TC}$in equation (4.0) and setting equal to zero and evaluating for Q, we have the optimum economic ordered quantity as,

$$\widetilde{Q^*} = \sqrt{\frac{2(c_{01}d_1 + 2c_{02}d_2 + 2\,c_{03}d_3 + 2c_{04}d_4 + c_{05}d_5}{c_{h1} + 2c_{h2} + 2c_{h3} + 2c_{h4} + c_{h5}}} \quad \ldots\ldots 4.12$$

And the optimal inventory cost as

$$\widetilde{TC}^* = \sqrt{2(d_1 + 2d_2 + 2d_3 + 2d_4 + d_5)(c_{01} + 2c_{02} + 2c_{03} + 2\,c_{04} + c_{05})(c_{h1} + 2c_{h2} + 2c_{h3} + 2c_{h4} + c_{h5})} \ldots\ldots 4.13$$

V. NUMERICAL ANALYSIS

A. CRISP MODEL

Let the demand for a particular product be given as D=1500 over a given period with the following parameters: ordering cost $C_o = 6$ and holding cost $C_h = 12$ in Naira.

Using equation (4.4) and (4.6) we have the economic ordered quantity Q^0 and optimal inventory cost $(TC)^*$as shown in table 1 below:

Demand D	C_o	C_h	Q^0	**TC**
1500	6	12	38.730	464.758

Table 1

B. PENTGONAL FUZZY MODEL

Let the three fuzzy parameters be given as:

$\widetilde{D} = (1000,\ 1200,\ 1500, 1700, 1900)$, $\widetilde{C_0} = (2, 4, 6, 8, 10\ )$, $\widetilde{C_h} = (8, 10, 12, 14,16)$ the two cost parameter $\widetilde{C_0}$ and $\widetilde{C_h}$ are in Naira.

Using equation 4.11 and 4.12 above we have the economic ordered $\widetilde{Q^*}$ and Total inventory cost $\widetilde{TC}$ as shown in table 2.

$\widetilde{D}$	$\tilde{C}_o$	$\tilde{C}_h$	$\tilde{Q}^*$	$\widetilde{TC}$
(1000,1200,1500,1700,1900)	(2,4,6,8,10)	(8,10,12,14,16)	39.739	476.865

Table 2

Analysis from table 2 shows that using Pentagonal fuzzy numbers takes the total inventory cost closer to its crisp value.

VI. CONCLUSION

In this paper, we studied Fuzzy Inventory model with fuzzy demand and no shortages allowed. For the purpose this study we considered two cost parameters, the holding cost and ordering cost which we treated as fuzzy. The three Fuzzy parameters; Demand, holding cost and ordering cost were modeled using Pentagonal fuzzy numbers. To Defuzzify the fuzzy Economic order quantity and Total inventory cost, the graded mean integration representation method was used. Comparing our Pentagonal Fuzzy model results with the crisp model shows very insignificant increase in the total inventory cost, we can therefore state that the Pentagonal fuzzy number minimizes the fuzzy optimal inventory cost.

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