Queuing Theory Diagnostic with Fuzzy Approach

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Abstract:- Queuing theory is a classical mathematical method to analyze the congestions and delays of waiting in line. It deals with number of customers waiting in line and many other aspects of queues. Our objective of the study has to develop a new fuzzy queuing model. The study intends to examine the long-run behavior and determination of the steadiness of queuing system. For this purpose the Buckley method has been applied. In this study Buckley method has extended using the trapezoidal shaped fuzzy numbers which are introduced to find the explicit formula for membership function of the fuzzy estimation to develop a new fuzzy queuing model. Fuzzy approach has been applied for measuring different linguistic factors in queuing systems. The results concluded that fuzzy queueing model is stable and bounds obtained by Buckley estimates are at least good as compare to bounds obtained by membership function.

Keywords:- *Fuzzy Queuing, Steady State, Trapezoidal Technique.*

I. INTRODUCTION

Queueing theory is a probabilistic approach to deal with the queuing system. Queuing theory was first introduced by Calls and Erlang [8] by keeping the congestion problem in telephone exchange and introduced the basis for exponential and Poisson distribution in queuing theory.

Queue is actually a row where people wait for their turn to get the services. Queuing theory deals with one of the most unpleasant experiences of life i.e. waiting. Queues problem is commonly found in banks, airport hubs, blood bank, telecommunications and emergency medical services. People face cost of long queues in term of money and resources. Due to congestion it is difficult to facilitate each and every person to fulfill their heavy demands. It deals with number of customers waiting in line and many other aspects of queue. Queueing theory is helpful to develop efficient queuing systems which at one hand reduce customer wait times and in contrast the number of customers will increase that can be served. A queuing system is therefore, characterized by three components which are Arrival process, Service mechanism and Queue discipline. Queues are classified by crisp queues and fuzzy queues. Crisp queue deals with probabilistic approach for performance measures. The "service time" and "interarrival time" follow Poisson distribution. In real world cases both the service and arrival rates described subjectively. In fuzzy approach service and arrival rates are described in possibilistic way because most statistical data is gathered subjectively.

According to Zadeh [16] crisp queues are not more realistic as compare to fuzzy queues. These queueing models are most suitable in case of multiple servers when crisp queues are extended into fuzzy queues. Queueing theory is used to develop the mathematical models for customer services. The main uniqueness of system can characterize by customers and servers. Queue is basically a line where people wait and served. Arrival rate follows Poisson distribution at a given time, whereas classical queueing models follow discrete distributions with continuous distribution of service time.There is some time needed to wait and served. So, time between service and arrival rates depends on probability distribution.

Fuzzy queueing method is a more realistic approach than classical queueing theory methods if service time explained in possibilistic way. Fuzzy approach is quite different from crisp set approach because its boundary is defined with a limited membership degree. In steady-state behavior of the model, its parameters are not time-varying. In case of steady state $\rho < 1$ means that arrival rate is lower than the service rate and steady state occurs and queue size is small. On contrary, when $\rho > 1$ steady state solution does not exist for queue size and service rate is slower than arrival rate. The queue will become longer and longer with time and system will explode. When $\rho = 1$ steady state condition do not hold because of random nature of queue. In addition, queue will infinitely increase. The state of statistical equilibrium is reached from any starting state. The process remains in statistical equilibrium once it has reached. In real world situations queueing behavior is finite. There is strong effect of more than one customer on future arrival distribution when size of customers is small. When size of population is finite then ending time of one service and time of next call service follow exponential distribution. Likewise, service time follows exponential distribution. Theoretically system reaches to steady state as time changes. Steady-state performance refers to the performance of a system with time-stationary parameters that has been in operation for a sufficiently long time which have no longer affects the distributions of number in system, number in different queues, waiting times, and total delay. In contrast, transient queues arise when either system parameters are not time-stationary or the queueing system does not remain in operation long enough to reach a steady state. The steady state performance of queue is referred the negligible effect of starting position of server where no one is in the queue or plenty of customers reaching before the server is started. The steady state condition can be achieved in different ways for different situations. For example, car is needed four vehicles to reach at steady state. On the other hand, momentary behavior will take place in absence of steady state. This short period of state includes both start up time and also time of rush hours. The performances measures of queueing model are "Ls" (expected number of customer in organization), "L_q" (expected number of customer in the waiting line), "Ws" (expected number of waiting time in the organization) and "W_q" (expected number of waiting time in the waiting line).

There are some researchers Li and Lee [12] and Negi and Lee [13] have employed Zadeh's extension principle for fuzzy queueing models.

In this study, membership function is constructed using trapezoidal fuzzy numbers i.e. service rate and arrival rate and alpha-cut approach is useful to find the higher and lesser bounds of function. Robert (2010) has used α-cut method used to explain the performance and fuzzy priority discipline queuing system. Likewise, for fuzzy queue system Negi and Lee [13] have employed alpha-cut and two-variable simulation. The main drawback in their study is that their approach provides an only crisp solution and cannot explain the queueing system. In addition, Li and Lee [12] have used general approach of Zadeh principle for fuzzy model. Another study has done by Jau-Chuan et al [2006] in which membership function for crisp queues have been constructed in non linear parametric models.

The alpha-cuts method has used by many researchers which is the most used on fuzzy queueing theory. Unfortunately, this method is manually difficult and inappropriate because it will require for each station, to solve two mathematical programs called PNLP (Parametric Non Linear Programming). To solve this problem our study is used to compute the steady state behavior which allows us to compute these performance measures separately.

There are few studies done on the determination of steadiness of queuing system.

Aïssani et al (2011) have proposed a discrete event simulation and t-test to find the stability performance of queueing theory. This approach has efficient results to by solving problem of queueing sytem.

Lukata et al (2015) have introduced fuzzy set theory in classical queueing networks theory to evaluate performance measures of a fuzzy product form queueing network. This scholar has used L-R method and used steady state behavior to solve the problem of alpha-cuts because it is inappropriate. Triangular fuzzy numbers are used in this study. The results of Left-Right method show that the two bounds of its support show the interval where the customer sojourn time can be found at steady state.

Buckley (2005) has used triangular fuzzy numbers to capture the problem of uncertainty and membership functions are not explicitly defined.

In this study we have extended the Buckley method using trapezoidal shaped fuzzy numbers.

In this study we have introduced a method to find the explicit formula for membership function of the fuzzy estimation by using Buckley's approach. The values of unknown fuzzy parameters are estimated by explicit and unique membership function. We assumed that population drawn from normal distribution. Finally we have examined the long-run behavior of the queuing system whether they are steady state or transient.

II. THE PROPOSED PROCEDURE

The primary objective of research is to analyze the fuzzy approach on multiple server queuing models. Study area is major hospital of Islamabad where the service quality of services is evaluated and also user satisfaction of patients in a queue and the evolution of the queuing system. If the arrival and service rate is inexact the performance measures will also be inexact.

We are intended here to develop a new fuzzy queuing model. We have considered the fuzzy queuing model in different systems.

➢ *Multiple Servers Fuzzy Queuing Models*

The multiple servers multiple queues system is basic queuing model observed in counter when there is two or more than two servers to dealing with customers and this model is notified as FM/FM/***c***, where FM/FM represents the fuzzy arrivals and departures of customers and c denote number of servers in the system. Queuing models would have wider appliance when the crisp queueing system with many attendants are expanded to many attendants of fuzzy queueing system.

➢ *General Procedure*

The 'membership function' of different 'performance measures' are,

- *Membership Function for 'W_S'*

It is important for the construction of membership function of 'μ_{ws}' to find the lower bounds 'W_S^L' and upper bounds 'W_S^U' of the 'α' cuts which are given below.

$$W_{s\alpha}^{\ L} = \min_{v,v\in R+}\left[\frac{v^{k+1}-(k+1)u^k v+ku^{k+1}}{(v-u)(v^{k+1}-u^{k+1})}\right] \quad (1)$$

Where in (1) '$W_{S\alpha}^{\ L}$' represents the waiting time of lower bound of the system.

So that $U_\alpha^L \le U \le U_\alpha^U$ and $V_\alpha^L \le V \le V_\alpha^U$

$$W_{s\alpha}^{\ U} = \max_{v,v\in R+}\left[\frac{v^{k+1}-(k+1)u^k v+ku^{k+1}}{(v-u)(v^{k+1}-u^{k+1})}\right] \quad (2)$$

Where in (2) '$W_{S_\alpha}^{\ L}$' represents the waiting time of upper bound of the system.

So that $U_\alpha^L \le U \le U_\alpha^U$ and $V_\alpha^L \le V \le V_\alpha^U$

The optional solution of mathematical program changes in non linear parametric programming when $U_{\alpha}^{U}, U_{\alpha}^{L}, V_{\alpha}^{L}, V_{\alpha}^{L}$ changes over the interval $\alpha \in [0,1]$.

There is difficulty arises in solving pair of mathematical program since objective function become complex as k increases. Left shape function $L(z) = \left(W_{S\alpha}^{L}\right)^{-1}$ and a right shape function $R(z) = \left(W_{S\alpha}^{U}\right)^{-1}$ can be attained if 'W_S^L' and 'W_S^U' are invariable with respect to 'α' and help to make membership function 'μ_{ws}'.

$$\mu_{ws}(z) = \begin{cases} \text{L(z)}, & z_1 \le z \le z_2 \\ 1, & z_2 \le z \le z_3 \\ \text{R(z)}, & z_3 \le z \le z_4 \end{cases}$$

- *Membership Function for 'L_S'*

The performance measure in which we are interested is L_s i.e $P(u,v) = L_s$. $\rho = \dfrac{u}{v}$

$$L_s = \frac{\rho\left[1-(k+1)\rho^{k} + k\rho^{k+1}\right]}{\left[\left(1-\rho^{k+1}\right)(1-\rho)\right]} \tag{3}$$

Following the membership function for 'L_S' is written as

$$\mu_{LS}(z) = \sup_{X\in x.Y\in y} \min \left\{ \begin{array}{c} \mu_{\tilde{\lambda}}(u), \mu_{\tilde{\mu}}(v) \\ z = \dfrac{\rho\left[1-(k+1)\rho^{k} + k\rho^{k+1}\right]}{\left(1-\rho^{k+1}\right)(1-\rho)} \end{array} \right\} \tag{4}$$

The lower bounds 'L_S^L' and upper bounds 'L_S^U' of the α-cuts of μ_{LS} are given below

$$L_{s\alpha}^{L} = \min_{u,v\in R+} \left[\frac{\rho\left[1-(k+1)\rho^{k} + k\rho^{k+1}\right]}{\left(1-\rho^{k+1}\right)(1-\rho)} \right] \tag{5}$$

Where in (5) '$L_{s\alpha}^{L}$' represents the average mean length of lower bound of the system.

$$L_{s\alpha}^{U} = \max_{u,v\in R+} \left[\frac{\rho\left[1-(k+1)\rho^{k} + k\rho^{k+1}\right]}{\left(1-\rho^{k+1}\right)(1-\rho)} \right] \tag{6}$$

Where in (6) '$L_{S\alpha}^{U}$' represents the average mean length of upper bound of the system. Left shape function $L(z) = \left(L_{s\alpha}^{L}\right)^{-1}$ and a right shape function $L(z) = \left(L_{s\alpha}^{U}\right)^{-1}$ can be attained If '$L_{S\alpha}^{L}$' and '$L_{S\alpha}^{U}$' are invariable with respect to 'α' and help to make membership function 'μ_{LS}'.

$$\mu_{LS}(z) = \begin{cases} \text{L(z)}, & z_1 \le z \le z_2 \\ 1, & z_2 \le z \le z_4 \\ \text{R(z)}, & z_3 \le z \le z_4 \end{cases}$$

III. NUMERICAL COMPUTATION

- *The Parametric Non Linear Program*

In FM/FM/1 queue, both the arrival and service rate are fuzzy numbers and is represented by λ = [4, 5, 6, 7] and μ = [35, 37, 40, 42], per hour assumed the arrivals are served as FCFS basis in the system. At possibility level α the confidence of interval as [$4+\alpha, 7-\alpha$] and [2α + 35, 42 – 2α].

The Parametric Non Linear Program for the Membership Function of 'W$_s$'

In multiple queuing model both incoming and utility rates fuzzy numbers are shown by
λ = [4, 5, 6, 7] and μ = [35, 37, 40, 42]. At possibility level the confidence interval are shown below

$$\begin{aligned} [u_{\alpha}^{L}, u_{\alpha}^{U}] &= [\min\ \mu_{\lambda}^{-1}(\alpha), \max\ \mu_{\lambda}^{-1}(\alpha)] = [4+\alpha, 7-\alpha] \\ [v_{\alpha}^{L}, v_{\alpha}^{U}] &= [\min\ \mu_{\mu}^{-1}(\alpha), \max\ \mu_{\mu}^{-1}(\alpha)] = [2\alpha+35, 42-2\alpha] \end{aligned} \tag{7}$$

$$W_S = \frac{\left(v^2 - 2uv + u^2\right)}{(v-u)(v^2-u^2)}$$

As a result the smallest 'W_s' occurs when 'u' is at its lower bound $(4+\alpha)$ and 'v' is its upper bound $(42-2\alpha)$, therefore by using equation (1) we obtained

$$W_{S\alpha}^{L} = \frac{(38-3\alpha)}{(3\alpha^2 - 176\alpha + 1748)} \tag{8}$$

By solving these equations we get the limit values of z

$$z = 0, z = \frac{1}{46} = .021 \text{ and } z = 0, z = \frac{1}{45} = .022$$

In contrast, for maximization, the largest values of 'u' and smallest values of 'v' are required, therefore by using equation (2) we obtained,

$$W_{S\alpha}^{U} = \frac{(3\alpha+28)}{(3\alpha^2 + 154\alpha + 1176)} \tag{9}$$

By solving these values we get the limit values of z

$$z = 0, z = \frac{1}{42} = .023 \text{ and } z = 0, z = \frac{1}{41} = .024$$

Therefore membership function of 'μ_{ws}' is defined by using (8) and (9)

$$\mu_{ws}(z) = \begin{cases} \frac{(176z-3) \pm \sqrt{(176z-3z)^2 - 12z(1748z-38)}}{2(3z)} & 0.021 \le z \le 0.022 \\ 1 & 0.022 \le z \le 0.023 \\ \frac{-(154z-3) \pm \sqrt{(154z-3)^2 - 4(3z)(1176z-28)}}{2(3z)} & 0.023 \le z \le 0.024 \end{cases}$$

α	$l_a(\alpha)$	$u_a(\alpha)$	$l_b(\alpha)$	$u_b(\alpha)$	$l_{ws}(\alpha)$	$u_{ws}(\alpha)$
0.0	4	7	35.0	42	0.0217	0.0238
0.1	4.1	6.9	35.2	41.8	0.0218	0.0237
0.2	4.2	6.8	35.4	41.6	0.0219	0.0236
0.3	4.3	6.7	35.6	41.4	0.0220	0.0235
0.4	4.4	6.6	35.8	41.2	0.0221	0.0234
0.5	4.5	6.5	36.0	41.0	0.0222	0.0233
0.6	4.6	6.4	36.2	40.8	0.0223	0.0232
0.7	4.7	6.3	36.4	40.6	0.0224	0.0231
0.8	4.8	6.2	36.6	40.4	0.0225	0.0230
0.9	4.9	6.2	36.8	40.2	0.0226	0.0229
1	5	6	37.0	40.0	0.0227	0.0228

Table 1:- The α-cut of '$\mu_{ws}(x)$' at 11 distinct α value k=1

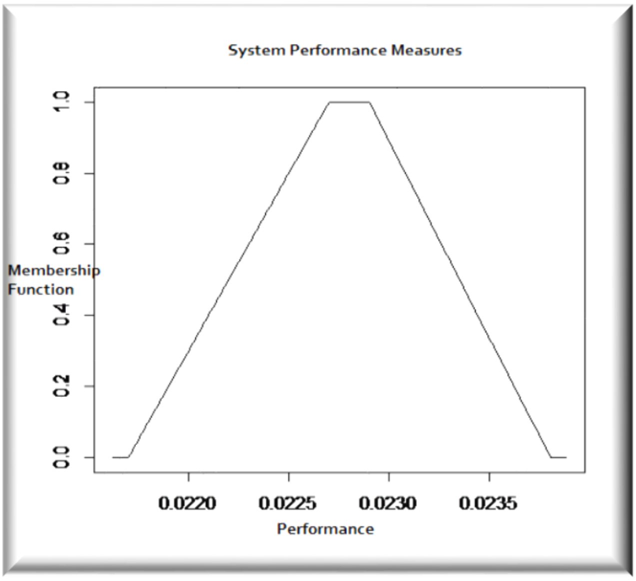

Fig 1:- Membership function for 'W_s'

The α-cut shows that the values of measures of performance lie between 0 and 1.Defining the alpha-cut approach on 11 different values, series of performance measure can be calculated at different possibility levels. The most likely mean value of waiting time of client in the organization α = 1, we suggested that expected waiting time of the 2% of the patients in the system are treated by the doctors falls between 1.36 to 1.37 min approximately. In addition, the series of queuing length in the organization at alpha = 0 according to Table 1 using the same relation showing that expected waiting time of the 3% of the patients are treated by the doctors is approximately between 1.3 to 1.4 min which shows that patient does not exceed 1.4 min in hospital or leaves before 1.3 min. The above information will be definitely useful for scheming queuing system. Number of the patients increased and decreased within α=1 and they become stable at one point at α=0 and this point is called a steady state.The number of the patients in the system are limited and hence there occurs steady state when the system is stable.

The Parametric Non Linear Program for the Membership Function of 'L_s'

In multiple queuing model both the incoming and utility rate are fuzzy numbers are shown by

In multiple queuing model both the incoming and utility rate are fuzzy numbers are shown by λ = [4, 5, 6, 7] and μ = [35, 37, 40, 42]. At possibility level the confidence interval are shown below,

$$[u_\alpha^L, u_\alpha^U] = [\min \mu_\lambda^{-1}(\alpha), \max \mu_\lambda^{-1}(\alpha)] = [4+\alpha, 7-\alpha]$$
$$[v_\alpha^L, v_\alpha^U] = [\min \mu_\mu^{-1}(\alpha), \max \mu_\mu^{-1}(\alpha)] = [2\alpha+35, 42-2\alpha] \quad (10)$$

Thus the parametric nonlinear programs for deriving the membership function of L_s are:

$$L_S = \frac{\left(\frac{u}{v}\right)\left[1 - 2\left(\frac{u}{v}\right) + \left(\frac{u}{v}\right)^2\right]}{\left(1 - \left(\frac{u}{v}\right)^2\right)\left(1 - \frac{u}{v}\right)}$$

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As a result the shortest 'L_s' occurs when 'u' is at its lower bound $(4+\alpha)$ and 'v' is its upper bound $(42-2\alpha)$, therefore by using equation (5) we obtained

$$L_{S\alpha}^{L} = \frac{-(\alpha+4)}{(\alpha-46)}, \quad (11)$$

By solving these equations we get the limit values of z

$$z \geq \frac{1}{9} \text{ and } z \leq \frac{4}{46}$$

In contrast, for maximization the largest values of 'u' and smallest values of 'v' are required, therefore by using equation (6) we obtained,

$$L_{S\alpha}^{U} = \frac{-(\alpha-7)}{(\alpha+42)} \quad (12)$$

By solving these equations we get the limit values of z

$$z \leq \frac{7}{42} \text{ and } z \geq \frac{6}{43}$$

Therefore membership function of 'μ_{Ls}' is defined by using (11) and (12)

$$\mu_{L\tilde{s}} = \begin{cases} \frac{46z-4}{z+1} & .0869 \leq z \leq .1111 \\ 1 & .1111 \leq z \leq .1395 \\ \frac{7-42z}{z+1} & .1395 \leq z \leq .1667 \end{cases}$$

α	$l_a(\alpha)$	$u_a(\alpha)$	$l_b(\alpha)$	$u_b(\alpha)$	$l_{Ls}(\alpha)$	$u_{Ls}(\alpha)$
0.0	4.0	7.0	35	42	0.0869	0.1667
0.1	4.1	6.9	35.2	41.8	0.0893	0.1639
0.2	4.2	6.8	35.4	41.6	0.0917	0.1611
0.3	4.3	6.7	35.6	41.4	0.0941	0.1584
0.4	4.4	6.6	35.8	41.2	0.0965	0.1557
0.5	4.5	6.5	36.0	41.0	0.0989	0.1529
0.6	4.6	6.4	36.2	40.8	0.1013	0.1502
0.7	4.7	6.3	36.4	40.6	0.1038	0.1475
0.8	4.8	6.2	36.6	40.4	0.1062	0.1449
0.9	4.9	6.2	36.8	40.2	0.1086	0.1422
1.0	5.0	6.0	37.0	40.0	0.1111	0.1395

Table 2:- The α-cut of '$\mu_{Ls}(x)$' at 11 distinct α value

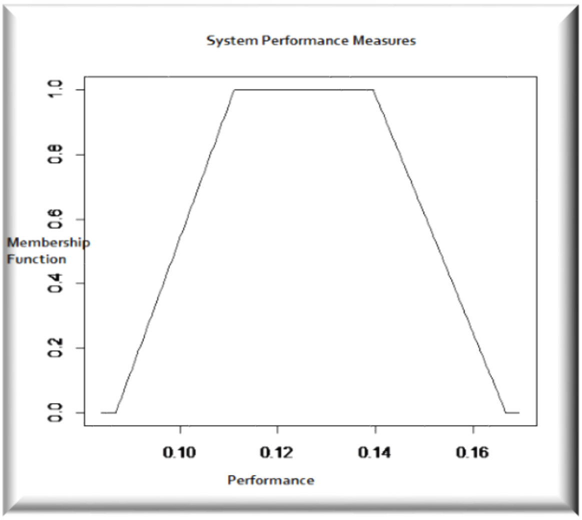

Fig 2:- Membership function for 'L_s'

The α-cut shows that the values of measures of performance lie between 0 and 1.Defining the alpha-cut approach on 11 different values, the range of a performance measure at different possibility levels has been derived. The mean number of customers waiting in the organization at L_s =.1026 customer per one server. The length of the customers at α=0 lies in the interval [.0869,.1667].This means that the number of the patients in hospital that will never exceed 16% or never fall below 8%. Moreover, the number of patients at α=1 lies between the interval [.1111, .1395].This means that the minimam length of the patients is 11% and maximam length of the patients is 14%. The above information will be very useful for designing the queuing system. Number of the patients increased and decreased within α=1 and they become stable at one point at α=0 and this point is called a steady state.The number of patients in the system are limited and hence there occurs steady state when the system is stable.

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➢ *Comparison of Membership Function Vs Buckley Estimates*

Degree of membership (alpha)	Membership Function(Trapezoidal)		Buckley Estimates		Efficiency (percentage)	
	LB	UB	LB	UB	MF	Buckley Estimates
0.0	0.0217	0.0238	.0295	.0333	91.18	88.59
1.0	0.0227	0.0228	.0321	.0322	99.56	99.69
0.0	.0452	.0507	.0270	0.0299	89.15	90.30
1.0	.0466	.0492	.0286	0.0287	94.72	99.65

Table 3:- Comparison of membership function Vs Buckley estimates

Hence the bounds obtained by Buckley estimates are at least good as compare to bounds obtained by trapezoidal membership function. The bounds are very efficient in Buckley estimates as compare to membership function .So, Buckley technique is better than trapezoidal membership function.

➢ *Correlation Between Utilization Factor'LQ and WQ*

Pearson partial correlation are calculated between different factors of queuing model such as utilization factor, number of waiting customer's in a waiting line (L_q) and waiting time of customer's in a queue (W_q) to find out the Strength of relationship between these factors.

		Utilization factor	Expected no of patients waiting in queue Wq	Expected no of patients in queue
utilization factor	Pearson Correlation	1	.909	.923
	Sig. (2-tailed)		.091	.077
expected waiting time of patients waiting in queue Wq	Pearson Correlation	.909	1	.998**
	Sig. (2-tailed)	.091		.002
expected no of patients in queue	Pearson Correlation	.923	.998**	1
	Sig. (2-tailed)	.077	.002	

Table 4:- Correlation coefficient between three different factors of performance measure.

From the above results, it is clearly observed that positive correlation exist between factors of performance measure. There exist a strong positive correlation between utilization factor and 'L_q' (customers waiting time in a queue), as compared to the correlation which occurs between utilization factor and 'W_q' (no of customers waiting in a queue) which shows that they are highly correlated. So we can say that when queue length increased utilization factor also increased rapidly, so by increasing the number of servers utilization factor inevitably reduced.

IV. CONCLUSION

The study intends to examine the long-run behavior and determination of the steadiness of queuing system. For this purpose the Buckley method has applied. In addition, fuzzy approach has used for measuring different linguistic factors in queuing systems. Buckley (2005) has used triangular fuzzy numbers to capture the problem of uncertainty and membership functions which are not explicitly defined. In this study we have extended the Buckley method using trapezoidal shaped fuzzy numbers. By developing Buckley's approach a method has introduced to find the explicit formula for membership function of the fuzzy estimation. The specific parameter of trapezoidal function (W_s) in multiple fuzzy queueing models has calculated. More specifically, Performance measure of fuzzy queuing models for multiple servers has explicitly calculated. Study area is major hospital of Islamabad where the service quality of services is evaluated and also user satisfaction of patients in a queue and the evolution of the queuing system.

In this study two approaches which are Buckley and membership function approach have been used and their efficiency has checked. The results concluded that the bounds obtained by Buckley estimates are at least good as compare to bounds obtained by membership function. The bounds are very efficient using the Buckley estimates as compare to trapezoidal membership function. Pearson partial correlation are calculated between different factors of queuing model such as utilization factor, number of waiting customer's in a queue(L_q) and waiting time of customer's in a queue (W_q) to find out the Strength of relationship between these factors.

There exist a strong positive correlation between utilization factor and 'L_q' (customers waiting time in a queue), as compared to the correlation which occurs between utilization factor and 'W_q' (no of customers waiting in a queue) which shows that they are highly correlated. It means when queue length has increased utilization factor also increased rapidly, so by increasing the number of servers' utilization factor inevitably reduced.

The results of steadiness of queueing system show that system is stable which means that arrival rate is lower than the service rate and queue size is small.

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