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Stochastic Fuzzy Transportation Problem in Deliveries – A Case Study

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Abstract:- Industries growth drives transportation development, leading to diverse methods. This expansion brings challenges, notably the stochastic fuzzy transportation problem (SFTP), a probabilistic chance-constrained programming (CCP) issue. SFTP handles fuzzy objectives amid supply-demand randomness. The transportation problem's (TP) core aim is efficient product movement between customers and producers to meet demand at a lower cost. TP's parameters encompass cost, supply, and demand. Uncertainties in reality include randomness and fuzziness. Randomness reflects potential outcomes and is quantified using random variables (RVs). Real-world scenarios involve multiple objectives, e.g., cost and time minimization. This study addresses the multi-objective transportation problem within a stochastic-fuzzy context, using Weibull distribution. The goal is to optimize transportation quantities considering real-world uncertainties.

Keywords:- *SFTP; CCP; TP.*

I. INTRODUCTION

In Decision-making is a fundamental process that spans diverse domains such as Economics, Psychology, Philosophy, Mathematics, and Statistics. It's essential to recognize the pivotal role of transportation within distribution networks. The transportation problem (TP) aims to optimize the movement of goods between producers and customers, minimizing costs and meeting demands. This involves considering parameters like cost, supply, and demand. In some cases, different transportation modes can be used to economize expenses or meet deadlines. When supply and demand capacity change significantly, the traditional transportation problem transforms into a mixed-constraint unequal scenario (TPMC).

Real-world situations are marked by uncertainties, primarily characterized by randomness and fuzziness. Randomness refers to the inherent variability of outcomes, often represented using random variables. Fuzziness, on the other hand, arises from the imprecision of human knowledge and manifests in various contexts such as data processing, uncertain parameter limits, and the absence of precise information. To address these uncertainties, models like stochastic transportation problems (STP) and fuzzy transportation problems (FTP) are formulated. It's worth noting that these uncertainties greatly influence decision-making in practical scenarios, including weather prediction, stock market analysis, and economic studies. Traditional approaches like fuzzy programming and stochastic programming alone aren't sufficient to handle these complex situations, leading to the integration of both randomness and fuzziness in the form of fuzzy stochastic programming.

The concept of fuzzy decision-making was introduced by Zadeh and has found applications in fields like financial engineering and risk management. By utilizing fuzzy set theory, decision-makers can effectively handle unknown or vague elements. This proves particularly useful when dealing with imperfect knowledge of asset returns and uncertainties in capital market behavior. Fuzzy numbers, representing fuzzy subsets of real numbers, provide a powerful tool for representing imprecise numerical data.

Stochastic programming (SP) comes into play when dealing with mathematical programming problems involving probabilistic elements. It addresses decision-making under uncertain conditions by incorporating random variables to represent these uncertainties. The chance-constrained approach is an effective strategy to optimize problems involving multiple uncertainties. It ensures that certain constraints are met with a high level of confidence, thus refining the feasible solution region. However, this method can be complex to implement. In our research, we delve into the combined utilization of probability theory and fuzzy set theory to tackle uncertainty in decision-making scenarios.

In transportation problems, encountering mixed constraints is a common real-world occurrence. The mixed constraint paradox emerges when it's possible to ship more goods at a lower total cost, while adhering to quantity requirements. However, uncertainties stemming from factors beyond control can lead to variations in cost coefficients, availability, and demand quantities. These uncertainties can generally be categorized into fuzzy transportation problems (FTP), where cost, supply, and demand possess fuzzy characteristics, and stochastic transportation problems (STP), where parameters are described by random variables with known distributions. The benefits of integrating fuzziness and randomness in decision-making lie in their ability to handle imprecise input data, accounting for subjective evaluations and emotional factors that conventional approaches may overlook.

A. Transportation Scenario in Indian Industries

In India, how things are transported is crucial for businesses. Whether it's making goods or providing services, getting stuff from one place to another matters a lot. India has roads, trains, planes, and ports to move things, but there are issues like bad roads and traffic jams that slow everything down. Confusing rules and pollution from transportation also pose challenges.

The government is working to improve this. They're making better roads, using smart technology to manage traffic, and simplifying rules for businesses. They're also focusing on ways to transport things in an eco-friendly manner. As businesses grow, a good transportation system not only helps them but also boosts the entire economy.

II. LITERATURE REVIEW

The transportation problem is a widely recognized operational research challenge due to its practical relevance. It falls within the realm of network optimization problems, focusing on efficiently moving goods from sources to destinations, considering supply and demand, with the goal of minimizing transportation costs. Hitchcock introduced the fundamental transportation problem in 1941 [1]. Dantzig enhanced solution methods in 1951 [2], followed by contributions by Charnes, Cooper, and Henderson in 1953 [3].

A. The Solution Process for the Transportation Problem Entails Three Phases:

Formulation of the Mathematical Problem.
Determining an Initial Feasible Solution.
Optimizing the Initial Solution.

Notable researchers like Abdur Rashid et al. [4], Aminur Rahman Khan et al. [5]-[8], and Kasana & Kumar [10] have extensively explored this problem. This section presents a literature review of the transportation problem (TP) and the fixed transportation problem (FTP) within the realm of supply chain planning (SP).

Charnes and Copper [11] established the TP, while Kataoka [12] discussed the shortest path approach for it. Stochastic variations were addressed by Spoerl and Wood [13], who examined the stochastic generalized assignment problem. Williams [14] introduced stochastic variables for demand using cumulative distribution functions, studying stochastic TP with both supply and demand uncertainties. Agrawal and Ganesh [15] proposed a solution incorporating stochastic demand and nonlinear costs.

Powell and Topaloglu [16] explored SP in transportation and logistics. Anholcer [17] introduced a stochastic generalized TP with discrete demand distributions. Ojha et al. [18] designed a stochastic discounted multi-objective STP, converting stochastic variables to deterministic using expected value criteria. Holmberg and Tuy [19] developed a branch and bound method for the TP when demand and production costs are stochastic. Real-world scenarios typically involve uncertainty in both supply and demand.

Mahapatra et al. [20] focused on inequality constraints in a multi-objective STP, where supply and demand are log-normal random variables (RVs). Agrawal [21] employed artificial intelligence for solving the STP in a stochastic setting. Several studies have addressed both fuzziness and randomness. Aruna Chalam [22] presented a fuzzy goal programming method for STPs with resource restrictions. Giri et al. [23] introduced a fuzzy stochastic solid TP using fuzzy goal programming. Acharya et al. [24] computed multi-objective fuzzy STPs.

Gessesse [25] proposed a genetic algorithm-based fuzzy programming approach for multi-objective linear fractional stochastic TP. Maity et al. [26] explored optimal intervention in transportation networks using fuzzy stochastic multimodal systems. The Weibull distribution (WD) was used for characterizing stochastic variables, originally introduced by Weibull [27] in the 1950s for reliability studies. It's mathematically defined by a probability density function involving location, shape, and scale parameters. The WD is useful for describing lifetimes and failure rates, with applications in various fields.

While the Weibull distribution is suitable for many cases, it may have limitations in certain practical situations. Klakattawi [29] and Mahapatra [30] investigated features and applications of the Weibull-gamma distribution.acteristics and applications of the Weibull gamma distribution.

B. Gap in the Literature

The occurrence of combined limitations within Transportation Problems (TPs) is a widely observed phenomenon in real-world scenarios. The conundrum of mixed constraints within a TP emerges when the potential to transport a greater overall volume of goods exists, all while incurring an equal or lesser total cost. This situation involves transporting at least the same quantities, if not more, from each point of origin to each destination, while upholding the principle of nonnegative shipment costs. However, instances may arise where the uncertainty stemming from uncontrollable variables affects the cost coefficients, availability, and demand quantities. This uncertainty can be predominantly categorized into two distinct types: fuzzy TP and stochastic TP.

The term "fuzzy TP" denotes a transportation scenario wherein the elements of transportation cost, supply levels, and demand quantities are expressed as fuzzy quantities. The fundamental aim of a Fuzzy Transportation Problem (FTP) is to determine an optimal shipping schedule that minimizes the cumulative fuzzy transportation expenses, all while adhering to the limits of fuzzy supply and fuzzy demand. Conversely, when the parameters possess imprecision in a stochastic sense and are described by random variables defined through known probability distributions, the scenario is labeled a stochastic TP. The principal advantages associated with approaches rooted in fuzziness and randomness lie in their capacity to function without reliance on anticipatable regular patterns. Additionally, they can effectively handle input information characterized by imprecision, taking into account sentiments and evaluations of emotions quantified according to the subjective judgment of the Decision Maker (DM).

Recent attention has been directed towards problems related to Stochastic Transportation Problems (STPs) and Stochastic Fuzzy Transportation Problems with equality constraints (SFTP). These problems have garnered notable interest, leading to the development of various models and solution methodologies tailored to stochastic environments. An investigation of the existing literature (as depicted in Table 1) has uncovered a lack of structured models designed to address Stochastic Fuzzy Transportation Problems with mixed constraints (SFTPMC). In actuality, mixed constraints involving uncertain supply levels at origins might materialize due to potential disruptions or delays resulting from diverse factors. Additionally, uncertainties in demand requirements often stem from inaccuracies in forecasting or the inherent volatility of requirements.

It is against this backdrop that we are inspired to formulate a model designed to tackle the complexities of an FTP with mixed constraints within a stochastic environment.

C. *Objective of this Study*

The aim of this study is to propose a mathematical model to solve transportation problems, that deals with fuzziness and randomness under one roof and to present a simplified computation conversion of probabilistic constraints to their equivalent deterministic constraint.

III. METHODOLOGY

A. *Some Preliminaries*

Definition 1

(Fuzzy set, Zadeh., [38]). Let R be a collection of sets and $\mu_R(x)$ be a membership function from R to [0,1]. A fuzzy set $\bar{R}$ with the membership function $\mu_R(x)$ is defined by

$$\bar{R} = \{(x, \mu_R(x))/x \in \text{R and } \mu_R(x) \in [0, 1]\}.$$

Definition 2

(Fuzzy set, Zadeh., [38]). A real fuzzy number $\bar{r} = (r_1, r_2, r_3)$ is a fuzzy subset of the real line R with the membership function $\mu_{\bar{r}}(r)$ satisfying the following conditions:

- $\mu_{\bar{r}}(r)$: R → [0, 1] is continuous
- $\mu_{\bar{r}}(r)$ = 0 for all $(-\infty, r_1] \cup [r_1, \infty)$
- $\mu_{\bar{r}}(r)$ is strictly increasing on $[r_1, r_2]$ and strictly decreasing on $[r_2, r_3]$
- $\mu_{\bar{r}}(r)$ = 1 for all $r \in r_2$ where $r_1 \le r_2 \le r_3$

Definition 3

(Triangular fuzzy number, Zadeh., [38]). A fuzzy number $\bar{R}$ is denoted as a triangular fuzzy number by (r_1, r_2, r_3) where a_1, a_2 and a_3 are real numbers, and its membership function $\mu_{\bar{A}}(x)$ is given as follows:

$$\mu_{\bar{R}}(x) = \begin{cases} \dfrac{x - r_1}{r_2 - r_1} & r_1 \le x \le r_2 \\ \dfrac{x - r_3}{r_2 - r_3} & r_2 \le x \le r_3 \\ 0 & otherwise \end{cases}$$

Definition 4

(Alpha Cut, Zadeh., [38]). The alpha cut of a fuzzy number $R(x)$ is defined as

$$R(\alpha) = \{((x) \,/\, \mu(x)) \ge \alpha, \;\; \alpha \in [0, 1]\}$$

Definition 5

(Linear membership function, [10]). A linear membership function can be defined as

$$\mu_R(x) = \begin{cases} 0 & if\; x_{ij} < \underline{x}_{ij} \\ \dfrac{\bar{x}_{ij} - x_{ij}}{\bar{x}_{ij} - \underline{x}_{ij}} & if\; \underline{x}_{ij} < x_{ij} < \bar{x}_{ij} \\ 1 & if\; x_{ij} > \bar{x}_{ij} \end{cases}$$

In order to transform the fuzzy system to a deterministic set, the alpha cut representation using linear membership function is

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$((\bar{x}_{ij} - x_{ij})/(\bar{x}_{ij} - \underline{x}_{ij})) = \alpha$ such that $x_{ij} = (1 - \alpha)\bar{x}_{ij} + \alpha\underline{x}_{ij}$ for all $\alpha \in [0, 1]$.

Definition 6

(Feasible Solution). Any set of { $x_{ij} \geq 0$, i = 1, 2, …, m; j = 1, 2, …, n} that satisfies all the constraints is called a feasible solution to the problem.

Definition 7

(Optimal Solution). A feasible solution to the problem which minimizes the total shipping cost is called an optimal solution to the problem.

The probability density function (pdf) and cumulative distribution function (cdf) of random variable t following a three-parameter Weibull distribution are given, respectively, as

$$f(t) = \frac{\beta}{\eta}\left(\frac{t-\gamma}{\eta}\right)^{\beta-1} e^{-\left(\frac{t-\gamma}{\eta}\right)^{\beta}} \tag{1}$$

And

$$F(t) = 1 - e^{-\left(\frac{t-\gamma}{\eta}\right)^{\beta}}$$

Where $f(t) \geq 0, t \geq 0$ or $\gamma, \beta > 0, \eta > 0$, $-\infty < \gamma < \infty$. Note that β, η and γ are the shape parameter (also known as the Weibull slope), the scale parameter and the location parameter respectively.

The Weibull distribution consists of the failure rate function, which defines the frequency at which an engineered system or component fails. The Weibull distribution is often suitable when the conditions for the strict randomness of the exponential distribution are not satisfied, where the shape parameter β depends on the fundamental nature of the problem. The location parameter γ shifts sample median and mode and the value of scale parameter η. However, it does not change β or the shape of the distribution. More importantly, it does not change the goodness of fit of data to the distribution function despite the commonly accepted idea that this additional parameter should improve the fit. This property is helpful to define the probabilistic constraints in the proposed mathematical model since the constraints are estimated using previously and partially known information on uncertain variables. For these reasons, the Weibull distribution has been adopted to describe the uncertainty in this study.

B. Assumptions and Notations

The direct value is the cost of transportation per unit amount. The mixed constraint occurs for the transportation activity between a source and a destination. The following notations are introduced in order to develop the mathematical model.

a_i:	The amount of homogeneous product availability at the source i
b_j:	The amount of homogeneous product demand at the destination j
co_{ijk}:	Fuzzy time for carrying of goods from source i to destination j by using vehicle k
$\bar{c}_{ijk}$:	Fuzzy transportation cost per unit for carrying a unit of goods from source i to destination j by using vehicle k
x_{ij}:	The amount shipped from source i to destination j
P_{a_i}:	Probabilities for a_i
P_{b_j}:	Probabilities for b_j
χ_{a_i}:	Shape parameter for a_i
χ_{b_j}:	Shape parameter for b_j
λ_{a_i}:	Scale parameter for a_i
λ_{b_j}:	Scale parameter for b_j
ξ_{a_i}:	Location parameter for a_i
ξ_{b_j}:	Location parameter for b_j

C. Problem Formulation

The objective value and constraints play an important role for transportation model. Minimizing the total transportation costs is our main objective. In different real-life situations, parameters (cost, supply and demand) become uncertain in nature; then, decision maker (DM) faces difficulty to make the optimal decision. This situation can be handled with fuzzy and random variables. In this model, we consider cost as triangular fuzzy variables and constraints as random variables. The uncertainty in supply or demand constraints may or may not occur, depending on the situation of decision maker. Therefore, formulated models based on the uncertainty in constraints. The model explained below shows the formulation of model with m sources and n destinations.

Chance-Constrained Programming Model

To obtain the quantiles of a probability distribution function in a closed form, it is necessary to apply the constraints in the suggested model to the deterministic constraints.

(P) Minimize $\bar{z}_q = \sum_{k=1}^{k}\sum_{i=1}^{m}\sum_{j=1}^{n}\bar{c}_{q_{ijk}}x_{ijk}$ (2)

Subject to

$P\left(\sum_{k=1}^{k}\sum_{j=1}^{n}x_{ijk} \geq a_i\right) \geq P_{a_i},\quad i = 1, 2, \dots m$ (3)
$P\left(\sum_{k=1}^{k}\sum_{i=1}^{m}x_{ijk} \geq b_j\right) \geq P_{b_j},\quad j = 1, 2, \dots n$ (4)
$P\left(\sum_{j=1}^{n}\sum_{i=1}^{m}x_{ijk} \leq e_j\right) \geq P_{e_j},\quad k = 1, 2, \dots k$ (5)
$x_{ijk} \geq 0$ (6)

Where P_{a_i} and P_{b_j} are probabilities given. It is observed that random variables a_i, b_j, e_k represent supply, demand and conveyance of vehicle, following the Weibull distribution, respectively. The Weibull distribution has a_i has three parameters ξ_{a_i}, λ_{a_i} and χ_{a_i} which serve as location, shape and scale parameters. Similarly, the parameters for b_j are defined as ξ_{b_j}, λ_{b_j} and χ_{b_j}. Similarly for vehicle conveyance Constraints (3) are the probabilistic constraint for the quantity of supply at origin i, which ensures with a given probability P_{a_i}, the sum of quantities of shipments from supply source i must distribute exactly a_i units of supply or it must distribute at least a_i units of supply which {q_1, q_2, and q_3} are partitions of i = {1, 2, 3…, m}. In a similar manner, constraints (4) can be construed for the demand at destination j, must receive exactly b_j units or it receives at most b_j units of demand which {r_1, r_2, and r_3} are partitions of j = {1, 2, 3…, n}. $\bar{c}_{1_{ij}}, \bar{c}_{2_{ij}}$ is the fuzzy time and transportation cost and (1) per unit for carrying one unit of goods from sources i to destination j. The aim is to describe the quantity x_{ij} transported from origin i to destination j while minimizing the total transport cost when satisfying mixed type supply and demand constraints. For the general cases, in which a single random variable among a_i , b_j and e_k is uncertain, the transformation of the probabilistic parameter into a deterministic parameter.

Case (I) Only a_i is Uncertain

For $P\left(\sum_{k=1}^{k}\sum_{j=1}^{n}x_{ijk} \geq a_i\right) \geq P_{a_i},\quad i \in q_1$, the Proof is Shown below

It is considered that a_i ($i \in q_1 = 1, 2, \dots, m_1$) are independent random variables with Weibull distribution and three known parameters ξ_{a_i}, λ_{a_i} and χ_{a_i}. Equation (3) can therefore be transformed as follows:

$P\left(a_i \leq \sum_{k=1}^{k}\sum_{j=1}^{n}x_{ijk}\right) \geq P_{a_i},\quad i \in q_1$ (7)
Let us consider $\sum_{k=1}^{k}\sum_{j=1}^{n}x_{ijk} = \delta_{a_i}$
$P\left(a_i \leq \delta_{a_i}\right) \geq P_{a_i},\quad i \in q_1$, by using WD for the probability function

$$\int_{-\infty}^{\delta_{a_i}} \frac{\chi_{a_i}}{\lambda_{a_i}}\left(\frac{a_i-\xi_{a_i}}{\lambda_{a_i}}\right)^{\chi_{a_i}-1} e^{-\left(\left(a_i-\xi_{a_i}\right)/\lambda a_i\right)^{\chi_{a_i}}} da_i \geq P_{a_i},\quad i \in q_1 \tag{8}$$

Simultaneously, $a_i \geq \xi_i,\ i \in q_1$, and the integration of equation (8) gives the following form:

$$\int_{\xi_{a_i}}^{\delta_{a_i}} \frac{\chi_{a_i}}{\lambda_{a_i}}\left(\frac{a_i-\xi_{a_i}}{\lambda_{a_i}}\right)^{\chi_{a_i}-1} e^{-\left(\left(a_i-\xi_{a_i}\right)/\lambda a_i\right)^{\chi_{a_i}}} da_i \geq P_{a_i} \tag{9}$$

After integrating equation (9), we get

$$1 - e^{-\left(\delta_{a_i} - a_i/\lambda a_i\right)^{\chi_{a_i}}} \geq P_{a_i} \tag{10}$$

Equation (10) can be further simplified by applying the logarithm as,
$\delta_{a_i} \geq \xi_{a_i} + \lambda_{a_i}\left(-ln\left(1-p_{a_i}\right)\right)^{1/\chi_{a_i}},\ i \in q_1$. Lastly, this can be stated as a deterministic constraint in the equivalent terms: $\sum_{k=1}^{k}\sum_{j=1}^{n}x_{ijk} \geq \xi_{a_i} + \lambda_{a_i}\left(-ln\left(1-p_{a_i}\right)\right)^{1/\chi_{a_i}},\ i \in q_1$.

i. For $P\left(\sum_{k=1}^{k}\sum_{j=1}^{n}x_{ijk} \leq a_i\right) \geq P_{a_i},\quad i \in q_3,\ (i = m_2+1, m_2+2, \dots, m)$ the proof of converting the probabilistic values into deterministic values by using WD is obtained.

All parameters are considered stochastic in this project. Based on the above formulation for supply similar way for demand and vehicle capacity considered.

(D_1) Minimize

$$z_q = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{q_{ij}}((1-\alpha)\bar{x}_{ijk} + \alpha\underline{x}_{ijk}) \text{ for each q} \quad (11)$$

Subject to

$$\sum_{k=1}^{k}\sum_{j=1}^{n} x_{ijk} \geq \xi_{a_i} + \lambda_{a_i}\left(-ln\left(1-p_{a_i}\right)\right)^{1/\chi_{a_i}}, \quad (12)$$

$i = 1, 2, \dots$ m

$$\sum_{k=1}^{k}\sum_{j=1}^{n} x_{ijk} = \xi_{b_j} + \lambda_{b_j}\left(-ln\left(1-p_{b_j}\right)\right)^{1/\chi_{b_j}}, \quad (13)$$

$i = 1, 2, \dots$ m

$$\sum_{j=1}^{j}\sum_{j=1}^{n} x_{ijk} \leq \xi_{e_i} + \lambda_{e_i}\left(-ln\left(1-p_{e_i}\right)\right)^{1/\chi_{e_i}}, \quad (14)$$

$i = 1, 2, \dots$ k

$$x_{ijk} \geq 0 \quad (15)$$

Fuzzy Goal Programming (FGP)

The fuzzy goal programming model can be framed as;

Find X $(x_1, x_2, x_3, \dots, x_n)$

To satisfy

$$H_k(X) \begin{pmatrix} \gtrsim \\ \cong \\ \lesssim \end{pmatrix} P_k \quad (16)$$

subject to

$$CX \begin{pmatrix} \gtrsim \\ \cong \\ \lesssim \end{pmatrix} b, X \geq 0 \quad (17)$$

Where $H_k(X)$ is the kth goal of fuzzy and P_k is the objective level related to $H_k(X)$. The syn $\lesssim, \gtrsim$ and $\cong$ refers to the fuzziness of aspiration level (i.e., approximately greater than or equal to, approximately equal to, and approximately less than or equal to) and CX ($\lesssim, \gtrsim, \cong$) b reflects of constraints in vector notation.

In a fuzzy decision-making situation, the goals are defined by the membership function associated with them which are attained by the definition of tolerable variations of up and down and the type of membership function is dependent on kind of goal. The aspiration level of the goal in equation 16 expresses that the decision-maker is satisfied up to a certain tolerance if the goal is not fully achieved. A linear membership function $H_k(X)$ can be expressed according $\mu_k(X)$ can be expressed according to Zimmermann (1976, 1978).

For the limitation of this type $\gtrsim$, $\mu_k(X)$ will be framed algebraically as follows:

$$\mu_k = \begin{cases} 1 & if\ H_k(X) \geq P_k \\ \dfrac{H_k(X) - L_k}{P_k - L_k} & if\ L_k \leq H_k(X) \leq P_k \\ 0 & if\ H_k(X) \leq L_k \end{cases} \quad (18)$$

where L_k is the lower tolerance limit for the fuzzy goal $H_k(X)$.

For the limitation of this type $\lesssim$, $\mu_k(X)$ will be framed algebraically as follows:

$$\mu_k = \begin{cases} 1 & if\ H_k(X) \leq P_k \\ \dfrac{U_k - H_k(X)}{U_k - P_k} & if\ P_k \leq H_k(X) \leq U_k \\ 0 & if\ H_k(X) \geq U_k \end{cases} \quad (19)$$

Where U_k is the upper tolerance limit for the fuzzy goal $H_k(X)$.

For the limitation of this type $\cong$, $\mu_k(X)$ will be framed algebraically as follows:

$$\mu_k = \begin{cases} 1 & if\ H_k(X) = P_k \\ \dfrac{U_k - H_k(X)}{U_k - P_k} & if\ P_k \leq H_k(X) \leq U_k \\ \dfrac{H_k(X) - L_k}{P_k - L_k} & if\ L_k \leq H_k(X) \leq P_k \\ 0 & if\ H_k(X) \geq U_k;\ if\ H_k(X) \leq L_k \end{cases} \quad (20)$$

By adding the membership functions (18, 19, 20) together with the fuzzy goal programming problem (16), the additive model can be framed as:

$Z(\mu) = \sum_{k=1}^{m} f_k \mu_k$

Subject to

$\mu_k = \frac{H_k(X) - L_k}{P_k - L_k}, \quad \mu_k = \frac{U_k - H_k(X)}{U_k - P_k}$

$CX \leq b$

$\mu_k \leq 1$

$X, \; \mu_k \geq 0$, k = 1, 2, … m,

Where Z (μ) is fuzzy decision function or fuzzy achievement function and f_k represents preferential weight.

D. Steps followed in the problem (Methodology adopted)

Step-1: Collection of data
Step-2: Convert the given fuzzy time problem (P) into an equivalent deterministic cost by using alpha cut
Step-3: Deterministic equations are formulated keeping in view the demand, supply constraints for each objective function for alpha level =0.5
Step-4. Compute min cost z_{lc} and time z_{ut}, similarly min time z_{lt} and cost z_{uc}

U1 $\leq (z_{uc} - z_1) / (z_{uc} - z_{lc})$
U2 $\leq (z_{ut} - z_2) / (z_{ut} - z_{lt})$
Objective function max = U1+U2

Fuzzy goal programming values are solved for each value.

Step-5: Sensitivity analysis for different probabilities

IV. ILLUSTRATION OF PROPOSED METHOD

An illustration to demonstrate the efficacy and applicability of SFTPMC and its variants. In order to make smooth functioning of some of the 3 stocking yards, products are transported from 4 sources of mills. The material considered is spare parts. Two vehicles (vehicle 1 and vehicle 2) are used for transportation of material from sources to destinations. The transportation costs in rupees for vehicle 1 with respect to source and destination is depicted in Table I.

Table 1 Transportation Costs for Vehicle 1

	Destination 1	**Destination 2**	**Destination 3**
Source 1	1200	2300	2000
Source 2	1000	1100	1700
Source 3	2000	2200	1300
Source 4	2200	1400	1400

The Transportation Cost in Rupees for Vehicle 2 with Respect to Source and Destination is Depicted in Table 2.

Table 2 Transportation Costs for Vehicle 2

	Destination 1	**Destination 2**	**Destination 3**
Source 1	1400	300	2300
Source 2	1200	2000	1400
Source 3	1200	2300	1200
Source 4	1400	2300	2100

Time taken in Hours by Vehicles from Source to Destination is Shown in Table 3 and Table 4.

Table 3 Time taken for Vehicle 1 (in hours)

	Destination 1	**Destination 2**	**Destination 3**
Source 1	(2, 3, 4)	(4, 5, 6)	(5, 6, 7)
Source 2	(2, 3, 4)	(10, 11, 12)	(5, 7, 9)
Source 3	(5, 6, 7)	(8, 10, 12)	(9, 11, 13)
Source 4	(4, 6, 8)	(5, 6, 7)	(8, 9, 10)

Table 4 Time taken for Vehicle 2 (in hours)

	Destination 1	**Destination 2**	**Destination 3**
Source 1	(5, 7, 8)	(3, 5, 6)	(3, 4, 6)
Source 2	(6, 8, 10)	(4, 6, 8)	(3, 5, 7)
Source 3	(2, 3, 4)	(3, 4, 6)	(2, 3, 4)
Source 4	(9, 10, 11)	(3, 4, 5)	(2, 3, 4)

Nominal Values of Uncertainties are Mentioned in Table 5 Along with their Probabilities.

Table 5 Arbitrary Probabilities

	Nominal Values	**Probabilities**	χ	λ	ξ
Source 1	21	0.95	2	2	20
Source 2	23	0.93	2	2	20
Source 3	22	0.92	2	2	20
Source 4	20	0.92	2	2	20
Destination 1	21	0.36	2	2	20
Destination 2	20	0.35	2	2	20
Destination 3	22	0.38	2	2	20
Vehicle 1	31	0.32	2	2	30
Vehicle 2	34	0.52	2	2	30

Conversion of Fuzzy time into Equivalent Deterministic Time for Vehicle 1 is Depicted in Table 6. α Value is taken as 0.5.

Table 6 Conversion of Time into Equivalent Deterministic Time for Vehicle 1

	Destination 1	**Destination 2**	**Destination 3**
Source 1	$(1-\alpha)4 + 2\alpha$	$(1-\alpha)6 + 4\alpha$	$(1-\alpha)7 + 5\alpha$
Source 2	$(1-\alpha)4+ 2\alpha$	$(1-\alpha)12 + 10\alpha$	$(1-\alpha)9+ 5\alpha$
Source 3	$(1-\alpha)7 + 5\alpha$	$(1-\alpha)12+ 8\alpha$	$(1-\alpha)13 + 9\alpha$
Source 4	$(1-\alpha)18 + 4\alpha$	$(1-\alpha)7 + 5\alpha$	$(1-\alpha)10 + 8\alpha$

Conversion of Fuzzy Time into Equivalent Deterministic Time for Vehicle 2 is Depicted in Table 7. α Value is taken as 0.5.

Table 7 Conversion of Time into Equivalent Deterministic Time for Vehicle 2

	Destination 1	**Destination 2**	**Destination 3**
Source 1	$(1-\alpha)8+5\alpha$	$(1-\alpha)6+ 3\alpha$	$(1-\alpha)6+ 3\alpha$
Source 2	$(1-\alpha)10 + 6\alpha$	$(1-\alpha)8 + 4\alpha$	$(1-\alpha)7 + 3\alpha$
Source 3	$(1-\alpha)4 + 2\alpha$	$(1-\alpha)6 + 3\alpha$	$(1-\alpha)4+ 2\alpha$
Source 4	$(1-\alpha)11 + 9\alpha$	$(1-\alpha)5 + 3\alpha$	$(1-\alpha)4 + 2\alpha$

Tables 8, 9 and 10 Depicts the Demand, Supply and Vehicle Capacity Values Respectively.

Table 8 Destination Capacities

	χ	λ	ξ	Probability	Demand
Destination 1	2	2	20	0.36	21.33
Destination 2	2	2	20	0.35	21.31
Destination 3	2	2	20	0.38	21.38

Table 9 Source Capacities

	χ	λ	ξ	Probability	Supply
Source 1	2	2	20	0.95	23.46

Source 2	2	2	20	0.93	23.26
Source 3	2	2	20	0.92	23.18
Source 4	2	2	20	0.92	23.18

Table 10 Vehicle Capacities

	χ	λ	ξ	Probability	Supply
Vehicle 1	2	2	30	0.34	31.289
Vehicle 2	2	2	30	0.44	31.522

Problem is solved using the Lingo and the results are as follows which are depicted in Tables XI and XII for vehicle 1 and vehicle 2 respectively.

All equations are formulated as mentioned in methodology and solved using lingo software linear programming.

Steps Carried out

Firstly: Linear programming is performed. cost minimization has been done taking into as its objective. For this cost minimization, total time taken is computed called as maximum time taken for transporting.
Secondly: Linear programming is performed. Transporting is taken for minimization as its objective. For this transporting time, cost is calculated called as maximum cost.
Thirdly: Fuzzy multi objective programming is performed considering simultaneously time and cost. maximization of satisfaction level is computed at which tradeoff between cost and transporting time is achieved.

Table 11 Result Table of Units Transported by Vehicle 1

	Destination 1	**Destination 2**	**Destination 3**
Source 1	0	0	0
Source 2	21.3	1.93	0
Source 3	0	0	1.08
Source 4	0	7.68	0

Table 12 Result Table of Units Transported by Vehicle 2

	Destination 1	**Destination 2**	**Destination 3**
Source 1	0	11.7	0
Source 2	0	0	0
Source 3	0	0	23.3
Source 4	0	0	0

By using the Lingo software, we obtain the optimal transportation cost of 63,479 rupees and optimal time taken for transportation is 256.73 hours.

Sensitivity Analysis

A sensitivity analysis of optimal solutions concerning the variations of probabilities on uncertain parameters in transportation problem is conducted in this section. We have used the same test problems for sensitivity analysis while we vary only the probability values and the results are depicted in in Table XIII.

Table 13 Sensitivity Analysis Concerning the Varying Probabilities

S. No.	**Probabilities**	**Optimal Cost**	**Optimal Time**	**Vehicle 1 Units**	**Vehicle 2 Units**
1	Destination 1 = 0.36 Destination 2 = 0.35 Destination 3 = 0.38 Source 1 = 0.95 Source 2 = 0.93 Source 3 = 0.92 Source 4 = 0.92 Vehicle 1 = 0.8 Vehicle 2 = 0.9	63917	238.39	30.91	33.11
	Destination 1 = 0.36 Destination 2 = 0.35 Destination 3 = 0.38 Source 1 = 0.8				

2	Source 2 = 0.84 Source 3 = 0.86 Source 4 = 0.81 Vehicle 1 = 0.32 Vehicle 2 = 0.52	64428.5	239.09	31.37	32.64
3	Destination 1 = 0.28 Destination 2 = 0.45 Destination 3 = 0.18 Source 1 = 0.95 Source 2 = 0.93 Source 3 = 0.92 Source 4 = 0.92 Vehicle 1 = 0.32 Vehicle 2 = 0.52	63461	237.73	30.95	32.62
4	Destination 1 = 0.36 Destination 2 = 0.35 Destination 3 = 0.38 Source 1 = 0.95 Source 2 = 0.93 Source 3 = 0.92 Source 4 = 0.92 Vehicle 1 = 0.34 Vehicle 2 = 0.44	63479	256.73	31.99	32.00

V. RESULTS AND DISCUSSION

A. Results and Discussion

Keeping the probabilities of destination and vehicle capacity same and varying the probabilities of source are brought down. It is found the cost is increased from rupees 63917 to 64428, time of transportation also increased from 238.39 hours to 239.0925 hours. The units transported in vehicle 1 are changed from 30.91 to 31.37 and units transported in vehicle 2 are changed from 33.11 to 32.64.

Similarly, keeping the probabilities of source and vehicle capacity same and varying the probabilities of destination are brought down. It is found the cost is decreased from rupees 63917 to 63461, time of transportation also decreased from 238.39 hours to 237.73 hours. The units transported in vehicle 1 are changed from 30.91 to 30.95 and units transported in vehicle 2 are changed from 33.11 to 32.62. And by keeping the probabilities of source and destination capacity same and varying the probabilities of vehicles are brought down. It is found the cost is decreased from rupees 63917 to 63479, time of transportation increased from 238.39 hours to 256.73 hours. The units transported in vehicle 1 are changed from 30.91 to 31.99 and units transported in vehicle 2 are changed from 33.11 to 32.00.

VI. CONCLUSION AND FUTURE SCOPE

A. Concluding Remarks

A model for solving a transport model that includes the cost coefficient of the objective function as a fuzzy number and probabilistic constraints accompanying the WD. The fuzzy objective value is converted into an equivalent deterministic objective function using alpha cut representation, and all stochastic constraints are converted into an equivalent deterministic constraint using WD. The analysis demonstrates the need of grasping the sensitivity of mixed constraints in the face of rising uncertainty. It assists a decision maker to choose the right level of uncertainty for uncertain parameters. The computed results clearly indicate that the designed model is robust with respect to the different parameters managerial decision-making situations such as the planning of many complex resource allocation problems in the areas of industrial production, in which demand and supply are random variables.

B. Future Scope of Study

In reality, the decision-making process handling with the complex organizational situation cannot depend exclusively on a single objective. So, it is understood the presence of numerous criteria that can improve multi criteria decision making. In view of this, in our future study, we plan to investigate scenarios by considering multi-objective with multiitem parameters. Using the approach presented in this work, supply and demand variations can be considered in the economic order quantity model. In our forthcoming research, we plan to get real-world data from proper authorities and employ statistical regularity criteria to derive its probability distribution. In this case, the WD has a broad range of applications.

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