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Alsultani Rules to Find the Particular Solution of Ricatti's Equation

Abdulhussein Kadhum Alsultani
Retired Mechanical Engineer
Baghdad , Iraq

Abstract:- Returning to the general Ricatti equation, we see that we can construct the general solution if a particular solution is known .Unfotunately , there is no strict algorithm to find the particular solution , which depends on the types of the functions A(x) , B(x), and C(x) , When I had studied Ricatti's equation I found that several attempts were made to find the particular soltion of some types of equations but there is no guides to solve all the types of ricatti equation .

Keyword:- *Ricatti Equation ,Particular Solution ,Alsultani Rules to find Rules to find the Particular Solution of Ricatti's Equation*

I. INTRODUCTION

Generally Ricatti equation is:

$y' = A(x)y^2 + B(x)y + C(x)$. , and ($y_1$) is a given particular solution . [1] Eq. (1)

where A(x) , B(x) and C(x) are functions of (x) .

$\frac{dy}{dx} + (2Ay_1 + B)y = -A$ Alsultani D.E. 2 [4] Eq. (2)

From Eq. (2) we see that the function C(x) didn't inter in the differential equation but when it equals zero then the equation may be Bernoulli's equation or any other type which consists of three terms , so solved directly without needing to the particular solution so for this reason I say that there is a relationship must be between (y_1) and C(x) only .

From Ricatti equation we see that ;

$y' = A(x)y^2 + B(x)y + C(x)$. , and ($y_1$) is a given particular solution

now $if \ \ y' = o$ this means that the integration of it is $\int y'dx =$ y= c (constant)

and this will affect on the right side of the equation and makes it as a function
and not a derivative so $y' \neq 0$

$y' = A(x)y^2 + B(x)y + C(x)$ Ricatti equation .

After I studied many Ricatti's equations I find that the particular solution of any one of them can be solved by one of the seven Rules which I put .

Below the disprections of seven Rules to find the Particular Solution (y_1) which produced from Eq.(1) easily with their applications ;

If Ricatti equation is in it's general form (consists of four terms) then we can arrange it by transforming B(x)y from the right side to the left as follow ;

$y' = A(x)y^2 + B(x)y + C(x)$ by arranging it ;

$y' - B(x)y = A(x)y^2 + C(x)$ (here we can consider this arrangement as a trick and we can do the solution directly without it)

So let C(x)= cK(x) where (c is constant and K(x) is a an absolute function of x) .

Then $y' - B(x)y = A(x)y^2 + c\,K(x)$

Since Ricatti's equation is quadratic then the right side of it is quadratic also because of the existence of (y^2) so we can apply our Rules as follow ;

The right side is $y^2 + cK(x)$

- *Rule 1*

if (y^2) has an opposite sign of cK(x) and $c = \mp 1$ or ∓ 4 (their square roots are integer numbers)
Then the equation represents the difference between two squares .

so at the point (y_1) and where A(x)=1 then

$y_1{}^2 = cK(x)$ so $y_1 = \pm\sqrt{|cK(x)|} = \pm\sqrt{|C(x)|}$ directly

The above two results $(\mp)$ are equal in the magnitude and opposite in the sign

- *Example 1*

$y' = -x + \frac{1}{2x}y + y^2$

So C(x)= cK(x) $= -x$ (opposite sign of y^2 , $c = -1$) then $y_1 = \sqrt{|C(x)|} = \pm\sqrt{|-x|} = \mp\sqrt{x}$

$y_1 = \sqrt{x}$ and $y_1{}' = \frac{1}{2\sqrt{x}}$

then to check

$y' = -x + \frac{1}{2x}y + y^2$

$\frac{1}{2\sqrt{x}} = -x + \frac{1}{2x}(\sqrt{x}) + (\sqrt{x})^2 = -x + \frac{1}{2\sqrt{x}} + x = \frac{1}{2\sqrt{x}}$ ok

Also if $y_1 = -\sqrt{x}$ then $y_1{}' = \frac{-1}{2\sqrt{x}}$

$y' = -x + \frac{1}{2x}y + \frac{1}{2\sqrt{x}}$

$\frac{-1}{2\sqrt{x}} = -x + \frac{1}{2x}(-\sqrt{x}) + (-\sqrt{x})^2 = -x - \frac{1}{2\sqrt{x}} + x = \frac{-1}{2\sqrt{x}}$ ok

- Note

Here also and from the left side

$y' - \frac{1}{2x}y = -x + y^2$ we can find the integrating factor $= e^{\int\frac{-dx}{2x}} = e^{\frac{-1}{2}\ln|x|} = \sqrt{x}$

- *Example 2*

$$\frac{dy}{dx} = \frac{-4}{x^2} - \frac{1}{x}y + y^2$$

✓ *Solution*

$C(x) = \frac{-4}{x^2} = y^2$ *i.e.* $c = -4$ (opposite sign of y^2)

and at point (y_1) we get that $y_1{}^2 = \frac{-4}{x^2}$ then $y_1 = \pm\sqrt{|C(x)|} = \pm\sqrt{|\frac{-4}{x^2}|} = \mp\frac{2}{x}$

$\frac{dy}{dx} + \frac{1}{x}y = y^2 - \frac{4}{x^2}$ so at point $y_1 = \frac{2}{x}$ and $y_1{}' = \frac{-2}{x^2}$

So to check the result

$\frac{dy}{dx} = \frac{-4}{x^2} - \frac{1}{x}y + y^2$

$\frac{-2}{x^2} = \frac{-4}{x^2} - \frac{1}{x}\left(\frac{2}{x}\right) + \left(\frac{2}{x}\right)^2 = \frac{-4}{x^2} - \frac{2}{x^2} + \frac{4}{x^2} = \frac{-2}{x^2}$ *ok*

Now if $y_1 = \frac{-2}{x}$ *so* $y_1{}' = \frac{2}{x^2}$

$\frac{dy}{dx} = \frac{-4}{x^2} - \frac{1}{x}y + y^2$

$\frac{2}{x^2} = \frac{-4}{x^2} - \frac{1}{x}\left(\frac{-2}{x}\right) + \left(\frac{-2}{x}\right)^2 = \frac{-4}{x^2} + \frac{2}{x^2} + \frac{4}{x^2} = \frac{2}{x^2}$ *ok*

- *Rule 2*

if C(x) = c K(x) and $c = \pm 2$ *or* ± 3 and they have opposite signs of (y^2) then the equation can be solved as follow ;

at the point (y_1) we find that the particular solution (y_1)= $c\sqrt{K(x)}$

and there is another solution $y_1 = \sqrt{K(x)}$ but with opposite sign of that of (c) ,

so we have two resulsts but they are unequal in their magnitudes and have opposite signs .

- *Example 3*

$$x^2y' = x^2y^2 + xy - 3$$

- *Solution*

Divide by (x^2)

$$y' = y^2 + \frac{1}{x}y - \frac{3}{x^2} \text{(also c has opposite sign of } y^2)$$

Here $C(x) = \frac{-3}{x^2} = c\,K(x)$ i.e. $c = -3$ and $K(x) = \frac{1}{x^2}$

so either $y = -3\sqrt{\frac{1}{x^2}} = \frac{-3}{x}$ *or* $y = +\sqrt{K(x)} = +\sqrt{K(x)} = \sqrt{\frac{1}{x^2}} = \frac{1}{x}$

Then at point (y_1) *we get* $y_1{}^2 = \frac{-3}{x^2}$ *so* $y_1 = c\sqrt{\frac{1}{x^2}} = \frac{-3}{x}$

If we take $y_1 = \frac{-3}{x}$ *then* $y_1{}' = \frac{3}{x^2}$

To check

$y' = y^2 + \frac{1}{x}y - \frac{3}{x^2}$

$\frac{3}{x^2} = \left(\frac{-3}{x}\right)^2 + \frac{1}{x}\left(\frac{-3}{x}\right) - \frac{3}{x^2} = \frac{9}{x^2} - \frac{3}{x^2} - \frac{3}{x^2} = \frac{3}{x^2}$ *ok*

Now if we take $\frac{1}{x} = y_1$ *then* $y_1{}' = \frac{-1}{x^2}$

$\frac{-1}{x^2} = (\frac{1}{x})^2 + \frac{1}{x}\left(\frac{1}{x}\right) - \frac{3}{x^2} = \frac{1}{x^2} + \frac{1}{x^2} - \frac{3}{x^2} = \frac{-1}{x^2}$ ok

- *Rule 3*

If y^2 has the same sign of cK(x) then the equation $y^2 + cK(x)$ represents the summation of two squares and it's unique solution is $y_1 = +\sqrt{|K(x)|}$

- *Example 4*

$$y' - \frac{1}{x}y + y^2 = -\frac{1}{x^2}$$

✓ *Solution*

$y' - \frac{1}{x}y = -\frac{1}{x^2} - y^2$ so they have the same signs (-ve) .

Then K=($\frac{1}{x^2}$) so $y_1 = +\sqrt{K(x)} = \sqrt{\frac{1}{x^2}}$ *and* $y_1' = \frac{-1}{x^2}$

Then to check

$y' - \frac{1}{x}y + y^2 = -\frac{1}{x^2}$

$-\frac{1}{x^2} - \frac{1}{x}\left(\frac{1}{x}\right) + \left(\frac{1}{x}\right)^2 = \frac{-2}{x^2} + \frac{1}{x^2} = -\frac{1}{x^2}$ ok

If we check $y_1 = -\frac{1}{x}$ *so it will false*

➢ *Rule 4*

If y' is multiplied (x^3) or $x(x^3 \mp \cdots)$ then the solution will be

$y_1 = \sqrt{K(x)} = +\sqrt{x^4} = cx^2$ or $y_1 = +\sqrt{K(x)} = +\sqrt{x^4} = +x^2$

- *Example 5*

$$x(1 - x^3)y' = x^2 + y - 2xy^2$$

✓ *Solution*

Here there is exception because *y' is multiplied by x^4*

$$\text{so } y_1 = \sqrt{x^4} = x^2 \text{ and } y_1' = 2x$$

to check

$x(1 - x^3)y' = x^2 + y - 2xy^2$

$x(1-x^3)(2x) = x^2 + x^2 - 2x(x^2)^2$

$2x^2 - 2x^5 = 2x^2 - 2x^5$ ok

- *Example 6*

$$y' + \frac{1}{x}y - \frac{1}{x^3}y^2 = 2x$$

✓ *Solution*

Multiply the equation by (x^3) *and arrange it*

$x^3y' = 2x^4 - x^2y + y^2$ then $C(x) = 2x^4$ i.e. $c = 2$ and $K(x) = x^4$

So $y_1 = \sqrt{x^4} = x^2$ (only + ve sign because c and y^2 have the same signs)

then $y_1' = 2x$

To check it

$y' + \frac{1}{x}y - \frac{1}{x^3}y^2 = 2x$

$2x + \frac{1}{x}(x^2) - \frac{1}{x^3}(x^2)^2 = 2x + x - x = 2x$ ok

Or $y_1 = cK(x) = 2x^2$ so $y_1' = 4x$

To check

$y' + \frac{1}{x}y - \frac{1}{x^3}y^2 = 2x$

$4x + \frac{1}{x}(2x^2) - \frac{1}{x^3}(2x^2)^2 = 2x + 4x - 4x = 2x$

The old solution

We convert this equation into into the standard form ;

$x^3y' = 2x^4 - x^2y + y^2$

As you can see ,we have a Ricatti equation . Tryto find a particular solutionin the form $y_1 = x^2$. Substituting this into Ricatti equation , we can determine the coefficient cc

$$(cx^2)' + \frac{cx^2}{x} - \frac{(cx^2)^2}{x^3} = 2x \rightarrow 2cx + cx - c^2x = 2x, \rightarrow c^2 - 3c + 2 = 0$$

Solving this quadratic equation, we obtain the value of c ;

$D = 9 - 4 \times 2 = 1. \rightarrow c_{1,2} = \frac{3 \mp \sqrt{1}}{2} = 1, 2$

Thus, there are even two particular solutions . However , we needonly one of them. So we take , for example , $y_1 = x^2$

- *Rule 5*

$y' = A(x)y^2 + B(x)y + C(x)$ and $C(x)$ consists of many terms

so $y_1 = \pm\sqrt{x \text{ (here x has the largest even exponent)}}$ only when C(x) and y^2

have opposite signs otherwise only (+ve)

- *Example 7*

$$y' = y^2 - (2x - 1)y + x^2 - x + 1$$

✓ *Solution*

$C(x) = x^2 - x + 1$ so $y_1 = \sqrt{x^2} = x$ so $y_1' = 1$ (only one solution because y^2 and x^2 have the same signs)

To check the result

$y' = y^2 - (2x - 1)y + x^2 - x + 1$

$1 = x^2 - (2x - 1)x + x^2 - x + 1 = x^2 - 2x^2 + x + x^2 - x + 1 = 1$ ok

- *Example 8*

$$\frac{dy}{dx} = y^2 - 2e^xy + e^{2x} + e^x$$

✓ *Solution*

C(x)= $e^{2x} + e^x$ *so* $y_1 = \sqrt{e^{2x}} = e^x$ *and* $y_1{}' = e^x$ (one solution)

To check

$\frac{dy}{dx} = y^2 - 2e^x y + e^{2x} + e^x$

$e^x = (e^x)^2 - 2e^x(e^x) + e^x = e^{2x} - 2e^{2x} + e^{2x} + e^x = e^x$ ok

➢ *Rule 6*

$y' \neq 0$ but after the substitution by (y_1) the right side becomes = 0 then we must find another magnitude of y_1 .

- *Example 9*

$$y' = (y - x)^2$$

✓ *Solution*

$y' = (y - x)^2 = y^2 - 2xy + x^2$

Here C(x)= x^2 then $y_1 = \sqrt{x^2} = x$ so $y_1{}' = 1$

To check the result

$y' = (y - x)^2$

1=$(x - x)^2 = 0 \neq 1$ so to make the result of the right side equal (1)

we choose y_1 = x+1 then $y_1{}' = 1$(one solution)

to check the result

$y' = (y - x)^2$

$1 = (x + 1 - x)^2 = 1$ ok

➢ *Rule 7*

$$y' = A(x)y^2 + B(x)y + C(x)$$

But if B(x)= 0

Then $y' = A(x)y^2 + C(x)$ so $C(x) = cK(x)$ and $y_1 = c\sqrt{|K(x)|}$

and the other solution is y_1=$\sqrt{K(x)}$ with the opposite sign of (c)$\sqrt{K(x)}$

- *Example 10*

$$y' + y^2 = \frac{2}{x^2}$$

✓ *Solution*

$y' + y^2 = \frac{2}{x^2}$ so C(x) = $\frac{2}{x^2} = c\,K(x)$ *then* $c = 2$ *and* $K(x) = \frac{1}{x^2}$

So $y_1 = c\sqrt{|K(x)|} = 2\sqrt{\frac{1}{x^2}} = \frac{2}{x}$ *and* $y_1{}' = \frac{-2}{x^2}$

To check

$y' + y^2 = \frac{2}{x^2}$

$\frac{-2}{x^2} + \left(\frac{2}{x}\right)^2 = \frac{-2}{x^2} + \frac{4}{x^2} = \frac{2}{x^2}$ *ok*

Or $y_1 = \sqrt{K(x)}$ with the opposite sign of (c) $\sqrt{K(x)}$ $i.e. = -\sqrt{\frac{1}{x^2}} = -\frac{1}{x}$ so $y_1' = \frac{1}{x^2}$

Then to check the result

$y' + y^2 = \frac{2}{x^2}$

$\frac{1}{x^2} + (\frac{-1}{x})^2 = \frac{1}{x^2} + \frac{1}{x^2} = \frac{2}{x^2}$ ok

- Note

We can solve the last example as follow ;

Suppose that $y = \frac{a}{x}$ so $y' = \frac{-a}{x^2}$ where a = constant

So

$y' + y^2 = \frac{2}{x^2}$

$\frac{-a}{x^2} + (\frac{a}{x})^2 = \frac{2}{x^2}$ then $a^2 - a - 2 = 0$

So $(a-2)(a+1) = 0$ then either a=2 or a=-1

II. MAIN RESULTS

Find the particular soltion of the following Ricatti equations and solve them .

1)

$$x(x^2 - 1)y' + x^2 - (x^2 - 1)y - y^2 = 0$$

- ✓ *Solution*

Using Rule 5

C(x)= x^2 *then* $y_1 = \sqrt{x^2} = +x$ so $y_1' = 1$

Now to check

$x(x^2 - 1)y' + x^2 - (x^2 - 1)y - y^2 = 0$

$x(x^2 - 1)(1) + x^2 - (x^2 - 1)(x) - (x)^2 = 0$

$x^3 - x + x^2 - x^3 + x - x^2 = 0$ ok

Dividing the problem by $x(x^2 - 1)$ *we get*

$$y' = \frac{1}{x(x^2 - 1)}y^2 + \frac{x^2 - 1}{x(x^2 - 1)}y - \frac{x^2}{x(x^2 - 1)}$$

Then

A(x)= $\frac{1}{x(x^2-1)}$, B(x)=$\frac{1}{x}$ *and* $C(x) = \frac{-x}{x^2-1}$

$\frac{dy}{dx} + (2Ay_1 + B)y = -A$ (Alsultani D.E. 2) [4]

$$\frac{dy}{dx}+\left[\frac{2\times 1\times x}{x(x^2-1)}+\frac{1}{x}\right]y=\frac{-1}{x(x^2-1)}$$

$\frac{dy}{dx}+(\frac{2}{x^2-1})y=-\frac{1}{x(x^2-1)}$

So $p(x)=\frac{2}{(x^2-1)}+\frac{1}{x}$ so by partial decompsition ;

Then $\frac{2}{x^2-1}=\frac{C}{x+1}+\frac{D}{x-1}$

So $C(x-1)+D(x+1)=2$ then $Cx-C+Dx+D=2$

$D-C=2$ and $C+D=0$ so $D=-C$

$C=-1$ and $D=1$

I.f.$=e^{\int(\frac{-1}{x+1}+\frac{1}{x-1}+\frac{1}{x})dx}=e^{\ln x(\frac{x-1}{x+1})}=\frac{x(x-1)}{x+1}$

$\frac{x(x-1)y}{x+1}=-\int\frac{x(x-1)}{x+1}\times\frac{1}{x(x^2-1)}dx=\int\frac{-dx}{(x+1)^2}$

Let $x+1=u$ then $du=dx$

$\int\frac{-dx}{(x+1)^2}=\int\frac{-du}{u^2}=\frac{1}{u}+c=\frac{1}{x+1}+c=\frac{c(x+1)+1}{x+1}$

$\frac{x(x-1)y}{x+1}=\frac{c(x+1)+1}{x+1}$

$y=\frac{c(x+1)+1}{x(x-1)}$ then $\frac{1}{y}=\frac{x(x-1)}{c(x+1)+1}$

$y_2=y_1+\frac{1}{y}=x+\frac{x(x-1)}{c(x+1)+1}$ End

2)

$$y'+7x^{-1}y-3y^2=3x^{-2}$$

- ✓ *Solution*

$$y'=\frac{3}{x^2}-\frac{7}{x}y+3y^2$$

- ➢ *Rule 5*

$\frac{3}{x^2}$ and $3y^2$ have the same signs (+ve) then $y_1=\sqrt{K(x)}=\sqrt{\frac{1}{x^2}}=\frac{1}{x}$ only

$y_1'=\frac{-1}{x^2}$

To check

$y'=\frac{3}{x^2}-\frac{7}{x}y+3y^2$

$\frac{-1}{x^2}=\frac{3}{x^2}-\frac{7}{x}\left(\frac{1}{x}\right)+3\left(\frac{1}{x}\right)^2=\frac{3}{x^2}-\frac{7}{x^2}+\frac{3}{x^2}=\frac{-1}{x^2}$ *ok*

A=3 , B= $\frac{-7}{x}$ *and* $C=\frac{3}{x^2}$

$\frac{dy}{dx}+(3Ay_1+B)y=-A$

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$\frac{dy}{dx} + \left(\frac{2\times1\times3}{x} - \frac{7}{x}\right)y = -3$ so $\frac{dy}{dx} - \frac{1}{x}y = -3$

Then I.f.= $e^{\int\frac{-dx}{x}} = \frac{1}{x}$

So $\frac{y}{x} = -3\int\frac{dx}{x} = -3\ln|x| + c$ then $y = x(c - 3\ln|x|)$

$\frac{1}{y} = \frac{1}{x(c-3\ln x)}$

$y_2 = y_1 + \frac{1}{y} = \frac{1}{x} + \frac{1}{x(c-3\ln x)}$ End

3)

$$y' = x^3 + \frac{2}{x}y - \frac{1}{x}y^2$$

Multiply by (x) *we get*

$$xy' = x^4 + 2y - y^2$$

So $C(x) = x^4$ then $y_1 = \mp\sqrt{|C(x)|} = \mp\sqrt{x^4} = \mp x^2$ (becaus $C(x)$and y^2 have opposite signs)

When $y_1 = x^2$ then $y_1' = 2x$

To check

$y' = x^3 + \frac{2}{x}y - \frac{1}{x}y^2$

$2x = x^3 + \frac{2}{x}(x^2) - \frac{1}{x}(x^2)^2 = x^3 + 2x - x^3 = 2x$ ok

Now $A = \frac{-1}{x}$, $B = \frac{2}{x}$ and $y_1 = x^2$

$\frac{dy}{dx} + (2Ay_1 + B)y = -A$

$\frac{dy}{dx} + \left(\frac{-2}{x}\times x^2 + \frac{2}{x}\right)y = \frac{1}{x}$

$$\frac{dy}{dx} + \left(-2x + \frac{2}{x}\right)y = \frac{1}{x}$$

I.f. $= e^{\int(-2x+\frac{2}{x})dx} = x^2e^{-x^2}$

Let $e^{-x^2} = u$ then $-2xe^{-x^2}dx = du$

$yx^2e^{-x^2} = \int\frac{1}{x}\left(x^2e^{-x^2}\right)dx = \int xe^{-x^2}dx = -\frac{e^{-x^2}}{2} + c = \frac{2c-e^{-x^2}}{2}$

Then $y = \frac{2c-e^{-x^2}}{2}\times\frac{1}{x^2e^{-x^2}} = \frac{2c-e^{-x^2}}{2x^2e^{-x^2}}$ *so* $\frac{1}{y} = \frac{2x^2e^{-x^2}}{2c-e^{-x^2}}$

$y_2 = y_1 + \frac{1}{y} = x^2 + \frac{2x^2e^{-x^2}}{2c-e^{-x^2}}$

also $y_1 = -x^2$ is another answer then $y_1' - 2x$ to check

$y' = x^3 + \frac{2}{x}y - \frac{1}{x}y^2$

$-2x = x^3 + \frac{2}{x}(-x^2) - \frac{1}{x}(-x^2)^2 = x^2 - 2x - x^3 = -2x$ End

4) $y' + 6y^2 = \frac{1}{x^2}$

✓ *Solution*

$y_2 = y^{-1}$ then $y_2' = -\frac{1}{y^2}$ substitute in the problem

$$\frac{dy_2}{dx} = -6y_2^2 + \frac{1}{x^2}$$

$$\frac{dy_2}{dx} + \frac{P(x)}{n}y^2 = Q(x)y^k \quad [3] \text{ i.e. } k = 2 = 1 - n \text{ so } n = -1$$

$$-\frac{1}{y_2^2}\frac{dy_2}{dx} = -6\left(\frac{1}{y_2}\right)^2 + \frac{1}{x^2} \text{ so multiply by } (-y_2^2) \text{ we get}$$

$\frac{dy_2}{dx} = -6 + \frac{y_2^2}{x^2}$ homogeneous equation then let $y_2 = vx$ so $dy = vdx + xdv$

$$\frac{dy_2}{dx} = \frac{xdv + vdx}{dx} = x\frac{dv}{dx} + v = -6 + v^2$$

$$x\frac{dv}{dx} = v^2 - v - 6 \quad \text{so} \quad \frac{dx}{x} = \frac{dv}{v^2 - v - 6}$$

and we can solve it by partial decomposition

$$\frac{1}{v^2 - v - 6} = \frac{A}{v-3} + \frac{B}{v+2}$$

$A(v+2)+B(v-3)=1$ then $Av+2A+Bv-3B = 1$ so $A + B = 0$ and $2A - 3B = 1$

Then $A=-B$ and $2A + 3A = 1$ so $A = \frac{1}{5}$ and $B = \frac{-1}{5}$

$$\frac{dx}{x} = \frac{dv}{5(v-3)} - \frac{dv}{5(v+2)} \quad \text{so } \ln|x| = \frac{1}{5}\ln\frac{v-3}{v+2} + c$$

$$cx^5 = \frac{v-3}{v+2} \quad \text{since } v = \frac{y_2}{x} \text{ then } cx^5 = \frac{\frac{y_2}{x} - 3}{\frac{y_2}{x} + 2} = \frac{y_2 + 3x}{y_2 - 2x}$$

$$\text{But } y_2 = \frac{1}{y} \text{ then } cx^5 = \frac{\frac{1}{y} - 3x}{\frac{1}{y} + 2x} = \frac{1 - 3xy}{1 + 2xy}$$

III. CONCLUSION

- After reading this research we recognize that $C(x)=cK(x)$ and finding the particular Solution of the problem directly .
- We see that the particular solution is the key of solving Ricatti's equation and so without it the work will become without advantage .
- Before this research so the procedure of finding the particular solution needs a strict algorithm but now it is easy to find it and may be with more than one result as we saw .
- Although I put seven Rules but they are simple and we can keep them in our mind quickly .
- By these Rules and my differential equation $\boxed{Alsultani\ D.E.2}$ [3] then the solutions of Ricatti's differential equations will be faster and without the complicated substitutions.

REFERENCES

[1]. Ricatti Jacope (1724) 'Animadversiones inaequaiones differentials secunde Gradus'(Observations regarding differential equations of the second order), Actorum Eruditorum, quae Lipsiaepubliccantur .Supplementa, 8: 66- y3. Translation Of the original Latin into English by Ian Bruce .
[2]. Ince. E. L.(1956) [1926] Ordinary Differential Equations, New York: Dover Publications, pp. 23-25
[3]. https;//ijisrt.com/new-method-to-solve-the-first-order-de-which-consists-of-three-terms
[4]. https://ijisrt.com/fast-way-to-solve-ricatti-equation.