Comparative Study of Model Order Reduction Techniques

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Abstract:- The higher order system consumes more space because to higher order matrix. The higher order interconnected power systems stability analysis is time consuming and large system performance of the system cannot be understanding easily. The analysis of the higher system is drudging and inordinate. The analysis and control of such system presents a great challenge for system engineer. The lower order system drives a comfort exploratory and optimization which results in favorable similarity to the system. This paper's objectives are to examine the reduced order model of large scale LTI system using balanced truncation method. In this technique the reduction of nominator and denominator polynomial using balanced truncation (BT) method, which results more accurate result. This approach preserves the original system's stability and steady state value in the lower order model.

Keywords:- *Reduction Techniques, Stability Equation Method (SEM), Differentiation Equation Method (DEM), Balanced Truncation Method, Reduced Model.*

I. INTRODUCTION

Modelling is the process of brief explanation of a system using mathematical equations and matrix. The mathematical models have a capability of better understanding of a system and how it can be control[1-4]. The mathematical modelling is used in different fields like, power system, control theory and sociology and physiology etc. Study of a mathematical model which has many states variables behaviour [5]. The construction of a system or model which is nearly similar to real phenomenon. A large-scale order system is therefore challenging to analyse and simulate in terms of synthesis and control. This also affects the computational time due to large number of system variables [6].

The method of getting an approximation reduced order of a big system with the same properties as the real original system is called "model order reduction". The development of optimal model order reduction and its generalisation to discrete time non-linear systems [7]. There are various model order reduction techniques are available which are published in earlier years some of them are Hankel Norm Approximation, Stability Equation Method, Singular Value Decomposition, Pade Approximation Technique, Routh Stability Method, Pade Via Lanczos [8].

- *The Problem Associated with Model Order Reduction as follow:*

  The Error of Approximation should be Minimum.

- *The original system's characteristics must be kept protected.*
- *The computation of the reduction of system should be well-organized or sequential.*
- *The reducing procedure need to be automated[5-7, 9-15].*

For linear time invariant (LTI) dynamic system various reduction methods are proposed for model order reduction of higher order system. The technique which are more frequently used for matching the time moments of original and reduced system is pade approximation. But this technique has a drawback that it has potential to give unstable lower order reduced order model for stable higher order system. Routh stability reduction technique is commonly used for the reduction of the complexity and matching the transient response of higher order system with lower order system. Unfortunately, it also has limitation to defend the dominant poles in the reduced order model for a non-minimum phase system it lags.

In this paper, for linear time-invariant system, a mixed method approach is proposed for model order reduction. In this paper the Stability Equation Method, Differential Equation method and Balanced Truncation Method is used to reduce a higher order LTI system to a reduced order system. Now to obtain desired reduced order model again reduce the intermediate lower system.

II. STATEMENT OF PROBLEM

- *Considering a higher order (n^{th}) system and a reduced order (r^{th}) system may be represented as following:*

$$G(s)=\frac{\sum_{i=0}^{n-1} m_i s^i}{\sum_{i=0}^{r-1} ls^i} \quad (1)$$

$$R(s)=\frac{\sum_{i=0}^{r-1} q_i s^i}{\sum_{i=0}^{r} p_j s^j} \quad (2)$$

Where, m_i, q_i are scalar constants for numerator and $p_{j,}l_j$, are scalar constants for denominator of higher order and reduced order system, respectively. The objective is to find a reduced r^{th} order system model R(s) such that it retains the important properties of the original higher system model G(s) for the same type of inputs. The analysis is done by two different reduction method viz stability equation method and differentiation method. The graphical and analytical analysis of reduced order system is to be carried out.

III. METHODOLOGY

For model order reduction various techniques are proposed, but here we are using these two methodologies which are**:**

A. Stability Equation Method

In this technique the reduced order transfer function is gained straight from the pole zero arrangement of stability equations of the original higher order transfer function. Hence the order of the higher order transfer function of stability equation can be reduced [14].

Let us Assume that the Transfer Function of the Higher Order System is:

$$G(s) = \frac{b_m s^m + b_{m-1} s^{m-1} + \dots\dots\dots\; b_1 s + b_0}{a_n s^n + a_{n-1} s^{n-1} + \dots\dots\dots a_1 s + + a_0} \quad (3)$$

$N(s)$ and $D(s)$ are the Numerator and Denominator of considered transfer function $G(s)$ respectively. By separating the numerator $N(s)$ and denominator $D(s)$ into their even and odd parts we get

$$G(s) = \frac{N_e(s) + N_o(s)}{D_e(s) + D_o(s)} \quad (4)$$

Where,

$$N_e(s) = \sum_{i=2,2,4}^{m} b_i s^i \quad (5)$$

$$N_o(s) = \sum_{i=1,3,5}^{m} b_i s^i \quad (6)$$

$$D_e(s) = \sum_{i=0,2,4,}^{n} a_i s^i \quad (7)$$

$$D_o(s) = \sum_{i=1,3,5}^{n} a_i s^i \quad (8)$$

The roots of the even part of numerator $N_e(s)$ and denominator $D_e(s)$ are called zeros $z_i(s)$ while the odd part of the numerator $N_o(s)$ and denominator $D_o(s)$ are called poles $p_i(s)$. In this method, we reduce a polynomial's less significant factor by individual eliminating. Let us illustrate the method by reducing the numerator and the denominator is reduced as well. The reduced order model is attained by the ratio of reduced numerator and denominator.

The Numerator is Separated as:

$$N(s) = N_e(s) + N_o(s) \quad (9)$$

Were

$$N_e(s) = b_0 + b_2 s^2 + b_4 s^4 + \dots \quad (10)$$

$$N_o(s) = b_1 + b_3 s^3 + b_5 s^5 + \dots\dots\dots \quad (11)$$

The Equation May be written as

$$N_e(s) = b_0 \sum_{i=1}^{k_1} (1 + s^2/z_i^2) \quad (12)$$

$$N_o(s) = b_1 s \sum_{i=1}^{k_2} (1 + s^2/p_i^2) \quad (13)$$

Where k_1 and k_2 are integer parts of n/2 and (n-2)/2 respectively.

Here $z_1^2 < p_1^2 < z_2^2 < p_2^2 \dots\dots\dots$ by ignoring the factor with higher magnitude of z_i & p_i, we get thee desired reduced order 'r' of stability equation and it can be written as

$$N_{er}(s) = b_0 \sum_{i=1}^{r_1} (1 + s^2/z_i^2) \quad (14)$$

$$N_o(s) = b_1 \sum_{i=1}^{r_2} (1 + s^2/p_i^2) \quad (15)$$

Where r_1 and$r_2$are the integer parts of r/2 and (r-2)/2 respectively. The reduced order numerator formed as:

$$N_r(s) = N_{er}(s) + N_{or}(s) \quad (16)$$

Similarly, the Reduced Order Denominator is Obtained:

$$D_r(s) = D_{er}(s) + D_{or}(s) \quad (17)$$

The Reduced Order Transfer Function R(S) is Written as:

$$R(s) = \frac{N_{er}(s) + N_{or}(s)}{D_{er}(s) + D_{or}(s)} \quad (18)$$

As the result of the stability equation method, we get stable reduced order models. It is observed that poles and zeros of lower models have a greater degree of dominance than those of higher magnitude. In this method, the poles or zeros with higher magnitudes ore disregarded.

B. Differentiation Equation Method-

This method was introduced by Gutman et al. [14]. This method based on polynomial's differentiation. The differentiation approach of Gutman et al. may also be considered a multipoint pade method; it is another stability preserving method. Any reduction technique in the frequency domain that obtains the lower order models, as a result of this novel connection between the pade and stability preserving methods.

The coefficient of the reduced order transfer function is produced by differentiating the reciprocal of the numerator and denominator polynomial of the higher transfer functions repeatedly. The differentiated reduced order is reciprocated again and normalized reduced polynomials are used. Because of the problem that zeros with big modules tend to be better approximation than those with a small module, the straightforward differentiation is abandoned.

➢ *Algorithm of Differentiation-*

- *The Reciprocal of the Transfer Function is Taken.*
- *To Get the Desired Reduced Order the Reciprocated Transfer Function is Differentiated.*
- *The Reduced Transfer Function is Again Reciprocated*
- *The Decreased Order System is then Subjected to the Steady State Correction.*

C. *Balance Truncation Method*

➢ *Review on BTM-*

This approach transforms the controllability Gramian and the observability Gramian into a diagonal matrix for the altered realisation [11]. Balance realisation is the name given to this process. Balancing of a particular reality is the first stage in the balance truncation procedure.

Let us consider a LTI system in a state space form as: -

$$X(t) = Ax(t) + Bu(t) \quad (19)$$

$$y(t) = Cx(t) + Du(t) \quad (20)$$

Where for each 't',

$u(t)€R^q$ input vector,

$x(t)€R^n$ state vector,

$y(t)€R^p$ output vector respectively.

The transfer function of original state space model is obtained as: -

$$G(s) = C(sI - A)^{-1}B + D \quad (21)$$

Here A, B, C and D are the matrix of order $n * n, n * q$ and $p * q$ in n-dimensional space R.

The objective is the higher order model to be transformed into lower order model. The reduced order state space equation is written as: -

$$x_r'(t) = A_r x_r(t) + B_r u_r(t) \quad (22)$$

$$y_r(t) = C_r x_r(t) + D_r u_r(t) \quad (23)$$

The state space matrix for reduced order into transfer function is written as: -

$$G_r(s) = C_r(sI - A)^{-1}B_r + D_r \quad (24)$$

The resulted error of output of both original and reduced model is kept low promising for input *u(t).*

The level of observability and control Gramians can be used to measure observability and controllability in particular state space directions. In this the transfer function equation of higher order is transformed into equivalent state space model then the matrix is reduced into the lower order matrix and the lower order state space matrix is again transformed into the transfer function equation.

The input and output state space models are inversely correlated with the controllability and observability Gramians. The controllability of the system is given by: -

$$P = \int_0^\infty e^{At} B B^T e^{A^T t}\, dt \quad (25)$$

Similarly, the observability of the system is given by:

$$Q = \int_0^\infty e^{A^T t} C^T C e^{A^T t}\, dt \quad (26)$$

The two Lyapunov equations can be used to determine:

$$AP + PA^T + BB^T = 0 \quad (27)$$

$$A^T Q + QA + C^T C = 0 \quad (28)$$

➢ *Diminution by Balanced Truncation Technique: -*

There is a transformation procedure that, for every stable dynamic system, renders controllability and observability equal and in diagonal form. The idea of system observability and controllability provides the basis for the combination of singular value decomposition, principal component analysis, and the balanced truncation approach.

- *Minimal Phase System: -*

Consider that (A, B) is observable, (A, C) is controllable, and A, is stable. A matrix can be changed into if it is symmetric and positive definite into the lower order triangular matrix by Cholesky factorization method. The Cholesky factor of lower triangular matrix are $L_c and\ L_o$, of the P and Q.

$$P = L_c L_c^T \quad (29)$$

$$Q = L_o L_o^T \quad (30)$$

The matrix's singular value decomposition $L_o L_c^T$,

$$L_o L_c^T = W \Sigma V^H \quad (31)$$

Here, W, V = orthogonal matrix, V^H =Hermitian transpose

The column of matrix u is referred to as a left single vector and column of matric V is named as a single right vector. The strength of controllability and observability for each specific state is obtained by Hankel singular values.

A dynamic system can be changed into a balanced system by utilising the non-singular transformation T, which is defined as

$$T = L_c V \Sigma^{-\frac{1}{2}} \alpha \quad (32)$$

The balanced system matrix is obtained as.

$$A^{\sim} = T^{-1}AT, B^{\sim} = T^{-1}B, C^{\sim} = CT \quad (33)$$

The transmission matrix D remain unchanged during the reduction process of original system. In a balanced system the $P^{\sim}$ and $Q^{\sim}$ matrix became same and converted into diagonal matrix as

$$P^{\sim} = Q^{\sim} = \sum \alpha = \begin{bmatrix} \alpha_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \alpha_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \dots & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \alpha_r & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \alpha_{r+1} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \alpha_n \end{bmatrix} \quad (34)$$

Where $\alpha_i i = 1,2, \dots \dots, r, r+1, \dots \dots n$ are Hankel singular values, listed in decreasing order as $\alpha_1 > \alpha_2 > \alpha_3 > \cdots > \alpha_r \gg \alpha_{r+1} > \cdots > \alpha_n$.

To calculate the desired order (r) reduced model the balance system $(A^{\sim}, B^{\sim}, C^{\sim}, D^{\sim})$ is partitioned as

$$A^{\sim} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, B^{\sim} = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}, C^{\sim} = [\,C_1 \quad C_2], D^{\sim} = D \qquad (35)$$

The order of the reduced order matrix element which are used for computation are $A_{11}(r * r)$, $B_1(r * m)$ and $C_1(p * r)$. Therefore, the simplified model's transfer function is: -

$$R_r(s) = D^{\sim} + C_1(Si - A_{11})^{-1} B_1 \qquad (36)$$

- *Non-Minimal Phase System: -*

Assume that A is stable, for obtaining the balance realization of given non-minimal stable system which is continuous-time system The square root approach was put out by Tombs and Postlethwaite. The observability and controllability Gramians for non-minimal phase systems are positive semi-definite matrices. By using the Choleskey factorization method, this sort of matrix cannot be converted into a lower order triangular matrix (L). Use is made to acquire the lower order triangular matrix of the LDL^T decomposition of non-minimal phase systems. The information concerning LDL^T breakdown is provided. The lower triangular matrices L_o and L_c are obtained when the controllability and observability Gramians are obtained by equations (28) and (29)

$$P = L_c d_C L_C^T \qquad (37)$$

$$Q = L_O d_O L_O^T \qquad (38)$$

Instead of being positive definite, the lower triangular matrices are symmetric positive semi-definite. Decomposing the lower triangular matrix into singular values $L_o^T L_c$ is given as

$$L_o^T L_C = [U_1 \; U_2][\textstyle\sum_1 V_1 \quad \sum_2 V_2] \qquad (39)$$

IV. NUMERICAL EXAMPLE

Example1. Let us consider an 8th order transfer function equation:

The main purpose is to reduce the model of the eighth order to second order. Here we are using

$$T(s) = \frac{18s^7+514s^6+5982s^5+36380s^4+122664s^3+222088s^2+185760s+40320}{s^8+36s^7+546s^6+4536s^5+22449s^4+67284s^3+118124s^2+109606s+40320} \qquad (40)$$

Various steps for obtaining the reduced order model, which are as follow: -

Step 1: Separating the N(s) and D(s) of the above system into even and odd parts.

Numerator part:

Using equation (10) and (11) the numerator further divided: as

Even part:

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$$N_e(s) = 514s^6 + 36380s^4 + 222088s^2 + 40320 \quad (41)$$

This can be written as: -

$$N_{er}(s) = 36380s^4\left(\frac{514s^2}{36380} + 1\right) + 222088s^2 + 40320 \quad (42)$$

Odd part:

$$N_{or}(s) = 18s^7 + 5982s^6 + 122664s^3 + 185760s \quad (43)$$

Also, the odd part is: -

$$N_{or}(s) = 5982s^5 + \left(\frac{18s^2}{5982} + 1\right) + 122664s^3 + 185760s \quad (44)$$

As per stability equation concept, the factors which have large magnitudes can be neglected. Therefore, the reduced order numerator is from eq. (16) is:

$$N(s) = N_{er}(s) + N_{or}(s) \quad (45)$$

$$N_r(s) = 185760s + 40320 \quad (46)$$

Denominator part:

Even part:

$$D_e(s) = s^8 + 546s^6 + 22449s^4 + 118124s^2 + 40320 \quad (47)$$

The eq. (46) can be written as:

$$D_{er}(s) = 546s^6\left(\frac{s^2}{546} + 1\right) + 22449s^4 + 118124s^2 + 40320 \quad (48)$$

After neglecting the factors which have large magnitudes the equation (46) becomes:

$$D_{er}(s) = 118124s^2 + 40320 \quad (49)$$

Odd part:

$$D_o(s) = 36s^7 + 64536s^5 + 67284s^3 + 109606s \quad (50)$$

Equation (49) can be written as:

$$D_{or}(s) = 64536s^5\left(\frac{36s^2}{64536} + 1\right) + 67284s^3 + 109606s \quad (51)$$

Reducing further eq. (50) we get:

$$D_{or}(s) = 109606s \quad (52)$$

From equation (17) the reduced denominator is

$$D_r(s) = D_{er}(s) + D_{or}(s) \quad (53)$$

Hence the reduced 2nd order model of the system using stability equation method is:

$$T_{2r(s)} = \frac{185760s + 40320}{118121s^2 + 109606s + 40320} \quad (54)$$

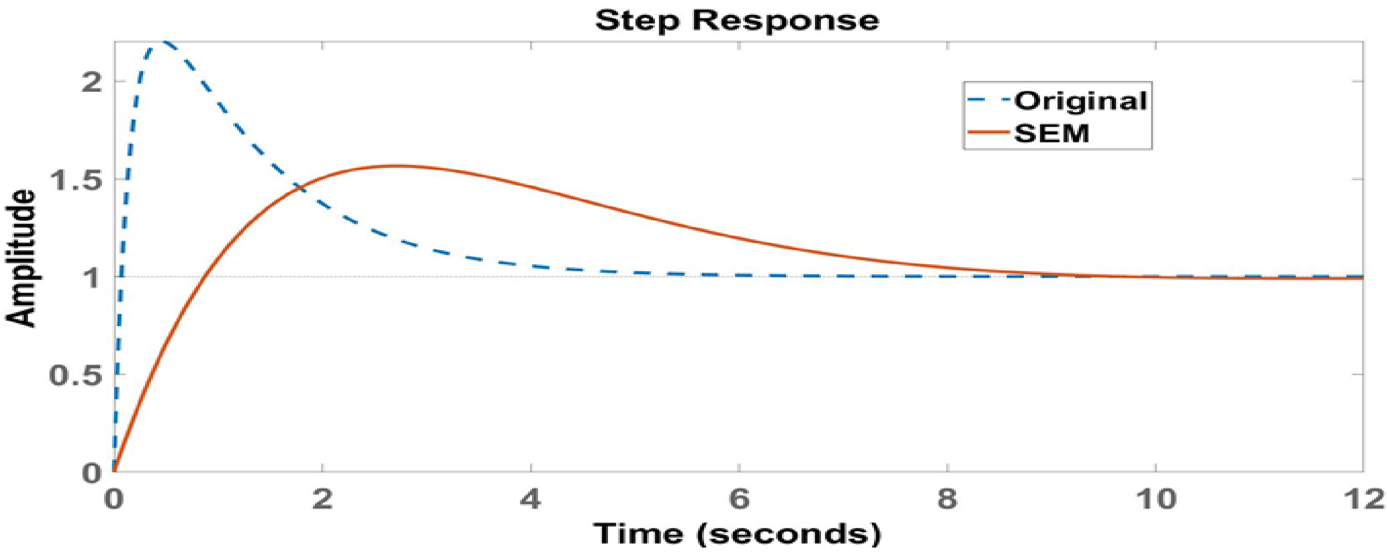

Fig 1 Comparison of Step Response of Reduced Order Model and Original Higher Order using Stability Equation Method

Example:2 Consider the equation (39) for Model order reduction using differentiation method.

$$T(s) = \frac{18s^7+514s^6+5982s^5+36380s^4+122664s^3 + 222088s^2 + 185760s + 40320}{s^8+36s^7+546s^6+4536s^5+22449s^4+67284s^3 + 118124s^2 + 109606s + 40320} \quad (55)$$

The reduction of polynomials Nominator and Denominator are separate and by taking the reciprocal of both.

The reciprocated numerator is:

$$\frac{\mathrm{dN(s)}}{\mathrm{ds}} = 18 + 514s + 5982s^2 + 36380s^3 + 122664s^4 + 222088s^5 + 185760s^6 + 40320s^7 \quad (56)$$

Differentiating with respect to 's' the numerator is:

$$\frac{\mathrm{DN_r(s)}}{ds} = 514 + 11964s + 109140s^2 + 490656s^3 + 1110440s^4 + 1114560s^5 + 282240s^6 \quad (57)$$

Similarly, differentiating the numerator up to desired reduced order, the second order numerator is:

$$N_{2r}(s) = 133747200 + 203212800 \quad (58)$$

The original reduced numerator is obtained by again reciprocal of $N_{2r}(s)$

$$N_{2r} = 133747200s + 203212800 \quad (59)$$

The reciprocal of denominator is:

$$D_r(s) = 1 + 36s + 546s^2 + 4536s^3 + 22449s^4 + 67284s^5 + 118124s^6 + 109606s^7 + 4032057) \quad (60)$$

Differentiating with respect to 's'

$$\frac{dD_r(s)}{ds} = 36 + 1092s + 13608s^2 + 89796s^3\ 3364420s^4 + 708744s^5 + 767242s^6 + 322560s^7 \quad (61)$$

To obtain the desired reduced order the denominator is differentiated again and again, the reduced 2nd order denominator is:

$$\frac{dD_r(s\,)}{ds} = 85049280 + 552414240s + 812851200s^2 \quad (62)$$

Again, the reduced order denominator is reciprocated:

$$D_{2r}(s) = 85049280s^2 + 552414240s + 812851200 \quad (63)$$

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The reduce second order transfer function of the original higher order system from equation (50) and (54) is:

$$T_2(s) = \frac{133747200s+202312800}{85049280s^2+552414240s+812851200} \quad (64)$$

Applying steady state correction to reduced order model

$$SSO = \frac{40320}{40320} = 1 \quad (65)$$

$$SSR = \frac{203212800}{812851200} = 0.25 \quad (66)$$

$$K_2 = \frac{1}{0.25} = 4 \quad (67)$$

So, the final reduced second order transfer function is shown below:

$$T_2(s) = \frac{534988800s+812851200}{85049280s^2+552414240s+812851200} \quad (68)$$

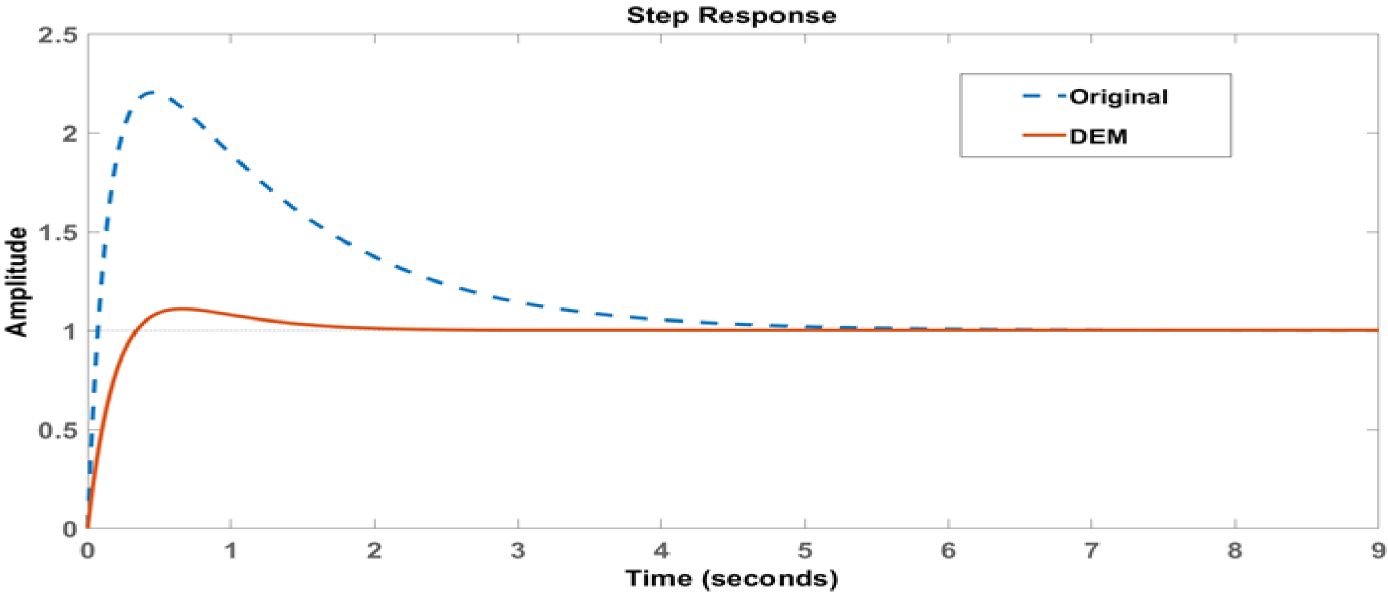

Fig 2 Comparison of the step response of the original system with the reduced system using differentiation equation method

Example: -3 considering the equation (39) for balanced truncation reduction technique

$$T(s) = \frac{18s^7 + 514s^6 + 5982s^5 + 36380s^4 + 122664s^3 + 222088s^2 + 185760s + 40320}{s^8 + 36s^7 + 546s^6 + 4536s^5 + 22449s^4 + 67284s^3 + 118124s^2 + 109606s + 40320} \quad (69)$$

With the use of equation (28) and (29) the Lyapunov controllability Gramians (60) and observability Gramians (69) equations are shown in table1.

The reduced order state space matrices of the system are

$$A_1 = \begin{bmatrix} -7.3260 & 2.1350 \\ -2.1350 & -0.0379 \end{bmatrix} \quad B_1 = \begin{bmatrix} -4.2220 \\ -0.2378 \end{bmatrix} \quad C_1 = [-4.222 \quad 0.2378] \quad D_1 = [0] \quad (70)$$

The equation(x) is used to calculate the second order reduced model:

$$T_b(s) = D_1 + C_1(sI - A)^{-1}B_1 \quad (71)$$

$$T_2(s) = \frac{17.77s+4.548}{s^2+7.364s+4.836} \quad (72)$$

Applying steady state correction to reduced order model.

From Eqn.(62) the SSO is:

$$SSO = \frac{40320}{40320} = 1 \tag{73}$$

$$SSR = \frac{4.548}{4.836} = 0.94 \tag{74}$$

$$K_2 = \frac{1}{0.94} = 1.06 \tag{75}$$

So the final reduced order transfer function system is shown below and fig.(3) shows the step response of reduced and original system.

$$T_{2(s)} = \frac{\{18.8362s+4.820\}}{\{s^2+7.364s+4.836\}} \tag{76}$$

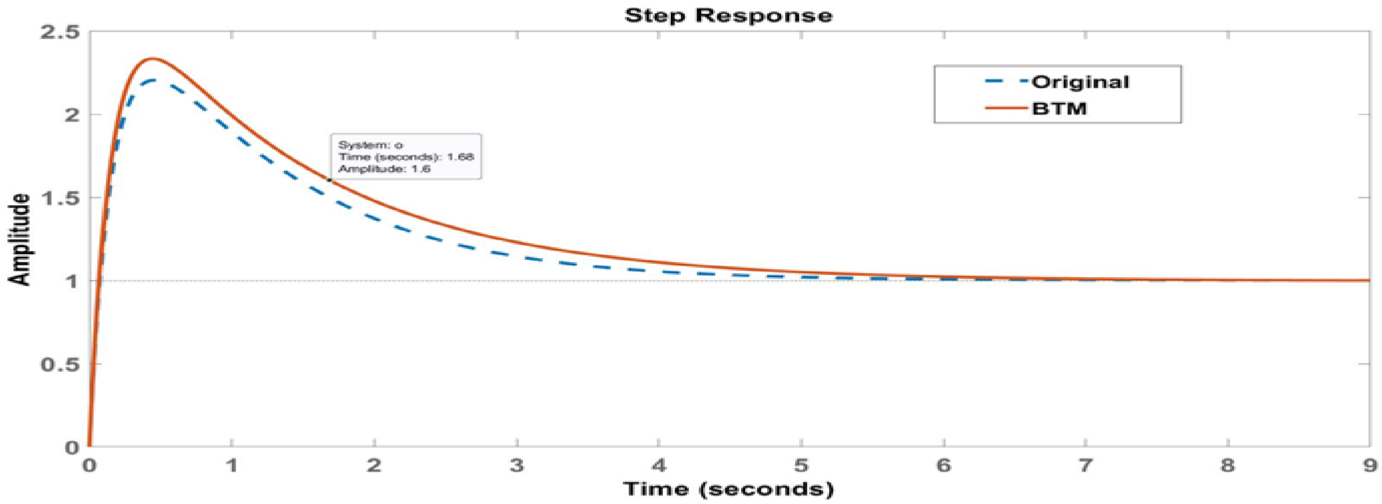

Fig 3 Comparison of the step response of the original system with the reduced system using balanced truncation method

Table 1 The Lyapunov Observability and Controllability of Higher Order System

Sr. No.	Controllability and Observability	Eq. No.
1	$P = \begin{bmatrix} 0.0214 & 0.0000 & -0.0001 & -0.0000 & 0.0000 & -0.0000 & -0.0000 & 0.0000 \\ 0.0000 & 0.0001 & -0.0000 & -0.0000 & -0.0000 & 0.0000 & -0.0000 & -0.0000 \\ -0.0001 & -0.0000 & 0.0000 & 0.0000 & -0.0000 & -0.0000 & 0.0000 & 0.0000 \\ -0.0000 & -0.0000 & 0.0000 & 0.0000 & -0.0000 & -0.0000 & 0.0000 & 0.0000 \\ 0.0000 & -0.0000 & -0.0000 & -0.0000 & 0.0000 & -0.0000 & -0.0000 & -0.0000 \\ -0.0000 & 0.0000 & -0.0000 & -0.0000 & -0.0000 & 0.0000 & 0.0000 & -0.0000 \\ -0.0000 & -0.0000 & 0.0000 & 0.0000 & -0.0000 & 0.0000 & 0.0000 & -0.0000 \\ 0.0000 & -0.0000 & 0.0000 & 0.0000 & -0.0000 & -0.0000 & -0.0000 & 0.0000 \end{bmatrix}$	(60)
2	$Q=10^9 * \begin{bmatrix} 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0001 & 0.0003 & 0.0002 & 0.0000 \\ 0.0000 & 0.0000 & 0.0002 & 0.0013 & 0.0042 & 0.0074 & 0.0059 & 0.0009 \\ 0.0000 & 0.0002 & 0.0024 & 0.0148 & 0.0500 & 0.0907 & 0.0754 & 0.0153 \\ 0.0000 & 0.0013 & 0.0148 & 0.0913 & 0.3145 & 0.5888 & 0.5250 & 0.1407 \\ 0.0001 & 0.0042 & 0.0500 & 0.3145 & 1.1135 & 2.1838 & 2.1344 & 0.7392 \\ 0.0003 & 0.0074 & 0.0907 & 0.5888 & 2.1838 & 4.6028 & 5.0745 & 2.2276 \\ 0.0002 & 0.0059 & 0.0754 & 0.5250 & 2.1344 & 5.0745 & 6.5581 & 3.5768 \\ 0.0000 & 0.0009 & 0.0153 & 0.1407 & 0.7392 & 2.2276 & 3.5768 & 2.3825 \end{bmatrix}$	(61)

Table 2 Time Response Analysis of Step Responses of Reduced2nd Order System

Reduction Techniques	Reduced 2nd order	Rise Time	Settling Time	Peak	Peak Time	Overshoot
Balance Truncation	$T_{2(s)} = \frac{\{18.8362s + 4.820\}}{\{s^2 + 7.364s + 4.836\}}$	0.0528	5.9677	2.3329	0.4442	134.0648
Stability Equation Method	$\frac{185760s + 40320}{118121s^2 + 109606s + 40320}$	0.6863	8.7186	1.5659	2.6800	56.5926
Differential Equation Method	$\frac{534988800s + 812851200}{85049280s^2 + 552414240s + 812851200}$	0.2438	1.7198	1.1093	0.6541	10.9293

V. CONCLUSION

It can be seen that the balance truncation method's reduced order equation's response and the result of the original higher order equation are similar.

A model reduction method based on Stability equation method, Differentiation equation method and balance truncation method (BTM) is presented in this research. A numerical example that compares the time responses of the original system with the reduced system evaluates that the rise time, settling time, peak and peak time of the balanced truncation technique is low as compared to the SEM and DEM.

Where, the overshoot is higher in BTM technique compared to SEM and DEM.

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