Computation of Three-Dimensional Linear Elasticity Problem using Mesh and Mesh-free Particle Modelling Techniques

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Abstract:- Mathematical models, which have already become essential tools in modern engineering, can be used to forecast and simulate the multi physical behaviour of various engineering systems and problems, whether in their simpler or complex forms. In this work, a linear elasticity problem involving a rectangular geometry of a wooden bar with an imposed load at one end and fixed at the other end was simultaneously solved by mesh and mesh-free particle methods. Elmer, a finite element program, was utilized for the mesh-based method, whereas Lattice Spring Model was used for the particle method. In the mesh-based technique, a Poisson's ratio of 0.37 was typically used. Comparatively, the Poisson's ratio for a lattice spring was discovered to always be 0.25 when using the particle technique, which is consistent with earlier findings in the literature. A numerical comparison of the data reveals that, despite the two methods' differing Poisson's ratios, they provide results that are very similar. In fact, the resulting stresses are only partially dependent on the Poisson's ratio. When, in the mesh-based method, the Poisson ratio is changed to 0.25, the values for the maximum stress are only slightly lower than those for 0.37.

***Keywords**:- Mesh; Mesh-free; Modeling; Elasticity; Young Modulus; Poisson's ratio; Elmer; LSM; LAMMPS. Particle-based modelling technique.*

I. INTRODUCTION

It was mentioned that the advent of simulation has reduced, drastically, the risk and cost by avoiding the danger and loss of life during testing. It is also true that with computer simulations many conditions can be varied and outcomes investigated without risk [1] and certain behaviours can be studied easily or closely by either speeding up or slowing down the simulation.

As already published in our previous works [2]–[4], the modelling and simulation of a particular problem can be done either by the traditional mesh-based method or mesh-free particle method (MPM). The mesh-based method considered the whole geometry as a continuum entity. The geometry is then created and the surfaces and volume gridded with either structured or unstructured mesh using a mesh generator software such as GridPro, Gmsh, Gambit etc [5]. In the particle method however, the geometry is made-up of particles which are arranged in an orderly manner to form the solid three-dimensional(3D) geometry.

Elaborately, it can be said that traditionally, computer simulations are carried out over system domains made of meshed (grided) surfaces. According to Ji et al., (2010)[7], "traditional modelling approaches make use of parametric patches, implicit surface, or subdivision surfaces that have been well integrated into 2D or 3D software". The steps in a mesh-based simulation process include creating a geometry, meshing it, and solving the model equations with a piece of software. The system is first converted into a geometry, then discretized into a mesh of a specific size, and finally the model, which is typically a differential equation, is solved. The operation is often carried out in three steps (mesh creation, solution, and postprocessing) after the geometry has been constructed, and this sometimes requires the use of three different pieces of software.On the other hand, meshfree particle methods (MPMs) or particle techniques[8], [9]generally refer to the class of meshfree methods that use a collection of a finite number of discrete particles to describe the state of a system and to record the movement of the system. Each particle in this context is either directly linked to a single discrete physical object or makes up and represents a portion of the continuum problem domain. Recently a meshfree CFD model was used by [10] to compute a linear elasticity problem, also [11]used meshfree enriched finite element formulation for micromechanical modelling of 3D particulate rubber composites. Molecular Dynamics (MD), Monte Carlo (MC), the Discrete Element Method (DEM), the Dissipative Particle Method (DPD), Smooth Particle Hydrodynamics (SPH), and others are examples of common MPMs.

Details of the theories behind the techniques were covered in [2] for the mesh-based approached and [4] and [5] for the particle-based approach, respectively.

II. METHODOLOGY

The two approaches were used to quantify the maximum von Mises stress on a rectangular wooden bar made of pine wood. The bar geometry was created and meshed using Gmsh (with a structured mesh) and then exported to Elmer for calculations. Linear Elasticity equation in Elmer was applied and the boundary conditions defined. After the computation, the Elmer result file (vtu file) is uploaded onto a post-processing software 'Paraview' for post-processing.

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For the particle-based simulations, a molecular dynamic software known as LAMMPS (Large-scale Atomic/Molecular Massively Parallel Simulator) was used and the script file was written, using MATLAB codes, for the same structure. And this time, the number of nodes obtained from the structured mesh were represented by particles instead if mesh. The principle of LSM (Lattice Spring Model) was employed where the particles are connected to each other by a means of a spring [12], [13]. With the equation relating the applied force to the distance the particle moves (1) a connection is made between the applied force and the spring property (spring constant).

$$F = k(r - r_0). \tag{1}$$

where F is the applied force, r_0 is the initial distance between two particles, r is the instantaneous distance, and k the spring constant.

According to [12], in a regular cubic lattice structure, the spring constant is related to the bulk modulusof the material and the young modulus by (2) and (3).

$$K = \frac{5}{3}\frac{k}{r_0} \tag{2}$$

And

$$E = 3K(1 - 2b) \tag{3}$$

where K is the bulk modulus, E the Young modulus, and b the poison ratio.

For 3-dimentional spring model (cubic lattice), the Poison's ratio b is constrained to 0.25 [12] and (3) becomes

$$E = \frac{3}{2}K. \tag{4}$$

From (2) and (4), therefore, the spring constant is then related to the young modulus[14]of the material as given in (5)

$$k = \frac{Er_0}{2.5}. \tag{5}$$

Therefore, for any given value of E, the k can be evaluated and used as the spring constant in the particle simulation.

The simulation was run in LAMMPS, and the dumb file (result file) is visualized with Ovito software (another visualization software) for post-processing. In the post processing, properties such as von Mises stress and the material displacement (bending due to load) were calculated.

III. GEOMETRY AND MODEL

The geometry is that of a rectangular wood bar of dimension $x = 0.1$ m, $y = 0.05$ m and $z = 1.0$ m. Both approaches (mesh and particle-based) were used to build the geometry of that dimension with one represented by mesh and the other, by particles. A mesh generating software Gmsh was used to create the geometry and the structured mesh (grid). The surface of the geometry was grided with an equal size box (squares) of dimension 0.0125m.

For the particle geometry, however, the particles and bonds between them were created using MATLAB code with each node of the square represented by a spherical particle and the distance between the particle (bond length) is 0.0125m (which is the same size as that obtained from mesh).

The computation was done by a Finite Element Method (FEM) software Elmer where the in-built linear elasticity model was used. The model was set up with the preconditions that specified the maximum number of iteration equal 500, the steady state simulation type, and a Backward Differentiation Formula (BDF) boundary condition was employed. The equation was specified, and the material was chosen as a pine wood from the material library with a density of 550kgm^{-3}. The Young modulus of 10 x10^{19} Nm^{-2} and the Poisson's ratio of 0.37 were used. The boundary condition state that the force of 200N was applied at one end and the other end was held constant.

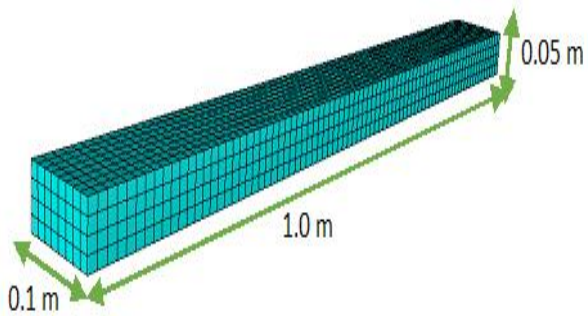

Fig. 1: A wooden bar geometry with structured mesh

In the particle method the geometry consists of 3645 number of particles and 27,100 number of bonds. The mass of particle is gotten from the specified density and the volume of the material. The same was applied for the force used and this gives the per atom mass and per atom force and the simulation boundary was set to be periodic at *x*,*y* and *z* directions. Table 1 present some of the parameters used during the simulation.

Table 1: Simulation Parameters

Mesh (Elmer)		
Parameter	***Symbol***	***Value***
Length,*z*	*z*	1.0 m
Width*x*	*x*	1.0×10^{-1} m
Thickness *y*	*y*	5.0×10^{-2} m
Applied force*F*	*F*	2,000 N
Acceleration *g*	*g*	9.88 m s^{-2}
Density of the bar	ρ	550 kg m^{-3}
Yong modulus*E*	*E*	1.0×10^{-10} N m^{-2}
Poison ratio*b*	*b*	0.37 [-]
Boundary Condition	_	BDF
Mesh-Free (LSM)		

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Parameter	*Symbol*	*Value*
Number particles	–	3,645
Number of bonds	–	27,100
Mass of each particle	–	7.55×10^{-4} kg
Particle spacing l	l	1.25×10^{-2} m
Bar thickness d	d	5.0×10^{-2} m
Elastic constant k	k	1000 kg s^{-2}
Time step	Δt	1×10^{-6} s

In the LSM, the Poisso'n ratio is always 0.25 [12] and this cannot be changed when employing the mesh-free particle method. Therefore, we tried to adjust the Poison ratio in the mesh-based method from the given 0.37 to 0.25 (similar to that of the particle method) to compare the results.

IV. RESULTS AND DISCUSSION

Here, the findings from both methods were presented and analysed critically. In order to highlight the similarities between the two and the areas where they differ, a comparative analysis was done.

A. Stress Distribution

The von Mises stress is computed, and the distribution is presented by colour coding the structure using ParaView (for mesh-based method) and Ovito (for the particle method). With a Poisson's ratio of 0.37 and 0.25 for Elmer and LSM respectively, the results obtained for the maximum stress and the maximum displacement were very similar. The area of maximum stress can be seen as the most reddish colour while the lowest stress is the most blueish as shown in Fig. 2 (see the colour guide in the Figure). The maximum stress in (A) is 5.2 x 10^7 N m^{-2} while it is 5.24 x 10^7 N m^{-2} in (B).

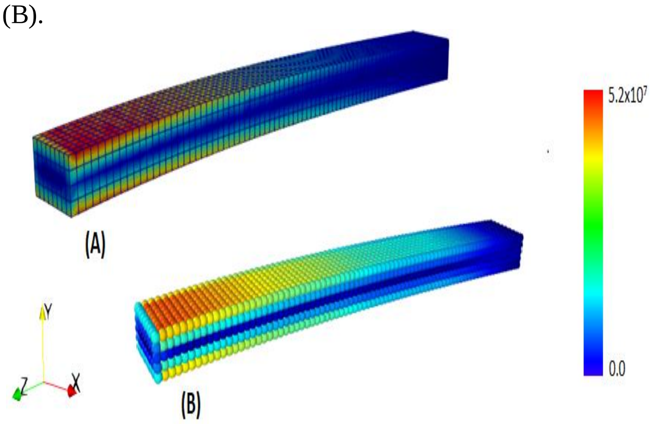

Fig. 2: The von Mises stress from (A) the ParaView (mesh-based) and (B) Ovito (particle-based).

When a Poisson's ratio of 0.25 was used in the Elmer, the result shows a small decrease in value for displacement as well as the stress. The maximum von Mises stress became 5.0 x 10^7 N m^{-2}. This is expected because the maximum stress increase with increasing the Poisson's ratio [15] and vice vasa.

When we compare the mesh-based method with Poisson's ratio 0.37, which correspond to the given Young modulus, and the particle method (with Poisson's ratio always 0.25) it is interesting to note that, despite the difference in the Poisson's ratio between the two methods, the values obtained are very close. This shows that differences in Poisson ratios result in relatively modest differences in stresses. This fact will be useful whenever the LSM is used to model solid structures. In fact, even if, by using the LSM, we are limited equal to 0.25, which can be different from the actual Poisson ration of the material under investigation, we expect only small differences due to this limitation.

B. Displacement

The displacement was also provided in terms of colour coding, similar to what was stated in section IV(A). The maximum displacements for the two methods are essentially identical, with the particle method having a maximum displacement of 6.10x10-2 m and the mesh-base method having a maximum displacement of 6.13x10-2 m (Fig. 3). Even though these values were produced using Poisson's ratios of 0.25 and 0.37 respectively, as was steted in the section above, the discrepancies are barely noticeable.

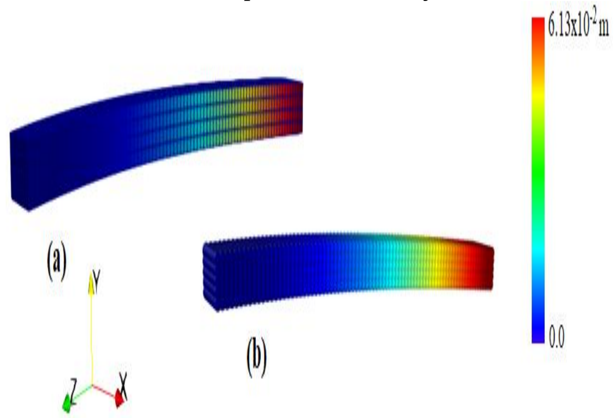

Fig. 3: Maximum displacement (a) ParaView (mesh-based) and (b) Ovito (particle-based).

Again, this shows how both methods, for a given simple geometry, can give almost same result with little variation. When the problem involves a simple geometry, one has a choice of using either of the method because there is no significant difference or advantage in term of convenience, but when the problem involves fluid structure interactions, it is easy to handle it with particle method as we can see in the next chapters.

V. CONCLUSION

In this work, a linear elasticity problem was simulated with both mesh and particle-based modelling approach. The result shows that there is no significant difference between the two methods except little variation. The LSM equation always used 0.25 as Poisson's ratio [12] whereas the Poisson's ratio given for this problem in Elmer was 0.37 [16]. Though there is freedom of choosing any Poisson's ratio with mesh-based method, however, there is not such freedom when using the LSM (particle) method. Moreover, the displacement of the bar due to the applied force was more precise with the particle method because each particle's deformation can be accounted for (deformation at any position), unlike the mesh-based method that only gives the overall deformation.

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