Three Dimensional Free Convection Heat and Mass Transfer between Two Vertical Walls filled with Porous Materials having Periodic Temperature and Concentration on a Wall

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Abstract:- The heat and mass transfer of a steady three dimensional natural convection flow of a viscous incompressible fluid between two vertical walls filled with porous materials with heat and mass transfer is investigated. The temperature and concentration of a plate is assumed to have periodic. The governing equations in non-dimensional form are solved by using the perturbation technique. Approximate solutions are derived for Velocity, Temperature and Concentration field. The effect of various non dimensional parameters on Velocity field, Temperature field, Concentration field are discussed graphically.

***Keywords:-** Free Convection, Heat and Mass transfer, Porous Medium, perturbation technique, Periodic temperature and concentration.*

I. INTRODUCTION

The fluid flow through porous media has attracted the attention of many researchers in recent years because it is encountered in many industrial applications. The phenomenon of natural convection arises in a fluid when temperature changes cause density variations leading to buoyancy forces acting on the fluid element. This process of heat transfer is encountered in aeronautics; fluid fuel, nuclear reactor and chemical engineering. Flow through porous medium have many applications in the field of Petroleum technology to study the movement of gas, oil and water through oil reservoirs, turbojets, rocket engines, like combustion chamber walls, exhaust nozzles and gas turbine blades.

The phenomenon of heat and mass transfer in porous medium has attracted many investigators. The recent books of Ingham and Pop [1] and Nield and Bejan [3] have documented exhaustive work on this area.Three dimensional free convective flow and heat transfer through porous medium has investigated by Ahmed and Sarma [2]. Heat and Mass transfer in an unsteady three dimensional mixed convection flow past an infinite vertical porous plate with cosinusoidally fluctuating temperature has studied by Arunachalam et al. [4], in which the lower plate is subjected to transverse sinusoidal suction velocity distribution of the form. Singh and Sharma [5] studied three dimensional free convective flow and heat transfer through porous medium with periodic permeability.Chaudhary and Sharma [6] studied the three dimensional unsteady convection and mass transfer flow through a porous medium. Jain and Sharma [7] and Jain and Gupta [8] have studied three dimensional coutte flow with slip boundary conditions and suction velocity vary sinusoidal. Mishra and Paul [9] have also studied free convection between two vertical walls filled with porous material and having periodic temperature on a wall.

Unsteady Three Dimensional Free Convection Heat and Mass Transfer Flow Embedded in a Porous Medium with Periodic Permeability and Constant Heat and Mass Flux studied by Jain, Chaudhary, and Vijay [10]. Heat and mass transfer free convection between two vertical walls filled with porous materials having periodic temperature on a wall investigated by Mishra and Mustaph [11].

The aim of present study to investigate the combined effect of three dimensional heat and mass transfer in the vertical channel due to the side heating span wise cosinusoidally and concentration difference. In discussing the convection phenomena in the vertical channel, we focus attention on the effect of important parameters such as thermal GrashofNumber(Gr), mass Grashof Number(Gm), wavenumber (a) and perturbation parameter (ε) on the transport phenomena.

II. MATHEMATICAL ANALYSIS

The natural convection flow of a viscous incompressible fluid has been considered in a porous region bounded by two vertical walls. The viscous and Darcy resistances are taken into account to model the momentum transfer in the porous region by taking permeability of the medium as constant. The temperature and concentration gradient between the walls occurs because the temperature of one wall is fluctuating with span wise cosinusoidally along z'.The x' is taken along one of the wall while, y' axis taken normal to the walls.

Under the usual Boussinesq's approximation the natural convection flow is governed by followingequations of conservation of momentum, thermal energy and concentration in non dimensional forms.

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$$\frac{\partial u}{\partial y}+\frac{\partial w}{\partial z}=0 ----------------------------- (1)$$

$$v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}=Rv(\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2})-\frac{u}{Da}+Gr\theta+GmC=0 -------- (2)$$

$$v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}=Rv(\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2})-\frac{v}{Da} ------------------- (3)$$

$$v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}=Rv(\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2})-\frac{w}{Da} ----------------- (4)$$

$$v\frac{\partial \theta}{\partial y}+w\frac{\partial \theta}{\partial z}=\frac{1}{\Pr}(\frac{\partial^2 \theta}{\partial y^2}+\frac{\partial^2 \theta}{\partial z^2}) ---------------------- (5)$$

$$v\frac{\partial C}{\partial y}+w\frac{\partial C}{\partial z}=\frac{1}{Sc}(\frac{\partial^2 C}{\partial y^2}+\frac{\partial^2 C}{\partial z^2}) --------------------- (6)$$

In non-dimensionalisation process of the governing equations, the following dimensionless quantities are used

$$y=\frac{y'}{L},\quad z=\frac{z'}{L},\quad a=a'L,\quad Da=\frac{K}{L^2},$$

$$Rv=\frac{\mu_{eff}}{\mu_f},\quad u=\frac{u'L}{\upsilon_f Gr},\quad v=\frac{v'L}{\upsilon_f},\quad w=\frac{w'L}{\upsilon_f},$$

$$\theta=\frac{(T'-T'_c)}{(T'_w-T'_c)},\quad C=\frac{C'-C'_c}{C'_w-C'_c},\quad Gr=g\beta(T'_w-T'_c)\frac{L^3}{\upsilon_f^2},$$

$$Gm=g\beta(C'_w-C'_c)\frac{L^3}{\upsilon_f^2},\quad \Pr=\frac{\mu_f C_p}{k},\quad Sc=\frac{\upsilon}{D} --------------------(7)$$

where u', v' and w' are the velocity components in x', y' and z' directions respectively. K is the permeability of the porous medium μ_{eff}, effective viscosity of the porous medium; μ_f dynamic viscosity of the fluid; υ_f kinematic viscosity of the fluid; u' fluid velocity; g acceleration due to gravity; β , coefficient of thermal expansion the temperature of the fluid; density; k, the thermal conductivity of the fluid, D*, thermal diffusivity of the fluid; ρ , is density; T' and T'_c are the respective temperature of the fluid and cold wall; C' and C'_c are the respective concentration of the fluid and cold wall and C_p , is the specific heat at the constant pressure.

Where

L is the distance between the walls; ε the perturbation parameter; a' , the wave length.

Sc= Schmidt Number,
Gr = Thermal Grashof Number,
Gm = Mass Grashof Number,
Pr =Prandle number

The boundary conditions for above set of equations in none—dimensional form are obtained as

$$u=v=w=0,\quad \theta=(1+\varepsilon\cos az),\quad C=(1+\varepsilon\cos az),\quad \text{at } y=0$$
$$u=v=w=0,\quad \theta=0,\quad C=0,\quad \text{at } y=1 \qquad (8)$$

A. solution of the problem

In order to solve the equation (1) to (6) we apply the regular perturbation method due to their non-linearity. However, the fact that the perturbation parameter is small in most practical problems. By taking so, the solution of equation (1) to (6) can be obtained by assuming

$$S=S_0(y)+\varepsilon S_1(y,z)+\varepsilon^2 S_2(y,z) ----------------- (9)$$

Where S stands for u, v, w, θ and C in equation (1) to (6) and (8) and comparing the coefficients of like powers of ε on both sides, we get

B. ZEROTH ORDER EQUATIONS

$$\frac{dv_0}{dy} = 0 \text{------------------------------} (10)$$

$$Rv\frac{d^2u_0}{dy^2} - v_0\frac{du_0}{dy} - \frac{u_0}{Da} + Gr\,\theta_0 + Gm\,C_0 = 0 \text{----------} (11)$$

$$Rv\frac{d^2w_0}{dy^2} - v_0\frac{dw_0}{dy} - \frac{w_0}{Da} = 0 \text{------------------} (12)$$

$$\frac{d^2\theta_0}{dy^2} - v_0 \Pr\frac{d\theta_0}{dy} = 0 \text{----------------------} (13)$$

$$\frac{d^2C_0}{dy^2} - v_0 Sc\frac{dC_0}{dy} = 0 \text{----------------------} (14)$$

The boundary conditions for corresponding order are:

$$u_0 = v_0 = w_0 = 0, \quad \theta_0 = 1, \quad C_0 = 1, \quad \text{at } y = 0$$
$$u_0 = v_0 = w_0 = 0, \quad \theta_0 = 0, \quad C_0 = 0, \quad \text{at } y = 1 \qquad (15)$$

By solving equations (10) to (14) subject to boundary conditions (15), the solution for zero order are obtained as follows:

$$u_0 = A(e^{-my} - e^{m(y-2)}) + Da(Gr(1-y) + Gm(1-y)) \text{------} (16)$$

$$v_0 = 0 \text{--------------------------------} (17)$$

$$w_0 = 0 \text{--------------------------------} (18)$$

$$\theta_0 = 1 - y \text{------------------------------} (19)$$

$$C_0 = 1 - y \text{------------------------------} (20)$$

where

$$m = 1/\sqrt{DaRv}, \quad \text{and} \quad A = \frac{-Da(Gr + Gm)}{1 - e^{-2m}}$$

C. FIRST ORDER EQUATIONS

$$\frac{\partial v_1}{\partial y} + \frac{\partial w_1}{\partial z} = 0 \text{---------------------------} (21)$$

$$v_0\frac{\partial u_1}{\partial y} + v_1\frac{\partial u_0}{\partial y} + w_0\frac{\partial u_1}{\partial z} = Rv(\frac{\partial^2 u_1}{\partial y^2} + \frac{\partial^2 u_1}{\partial z^2}) - \frac{u_1}{Da} + Gr\,\theta_1 + Gm\,C_1 = 0. \text{----} (22)$$

$$v_0\frac{\partial v_1}{\partial y} + v_1\frac{\partial v_0}{\partial y} + w_0\frac{\partial v_1}{\partial z} = Rv(\frac{\partial^2 v_1}{\partial y^2} + \frac{\partial^2 v_1}{\partial z^2}) - \frac{v_1}{Da} \text{------------} (23)$$

$$v_0\frac{\partial w_1}{\partial y} + v_1\frac{\partial w_0}{\partial y} + w_0\frac{\partial w_1}{\partial z} = Rv(\frac{\partial^2 w_1}{\partial y^2} + \frac{\partial^2 w_1}{\partial z^2}) - \frac{w_1}{Da} \text{------------} (24)$$

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$$v_0\frac{\partial\theta_1}{\partial y}+v_1\frac{\partial\theta_0}{\partial y}+w_0\frac{\partial\theta_1}{\partial z}=\frac{1}{\Pr}\left(\frac{\partial^2\theta_1}{\partial y^2}+\frac{\partial^2\theta_1}{\partial z^2}\right)\text{-------------}(25)$$

$$v_0\frac{\partial C_1}{\partial y}+v_1\frac{\partial C_0}{\partial y}+w_0\frac{\partial C_1}{\partial z}=\frac{1}{Sc}\left(\frac{\partial^2 C_1}{\partial y^2}+\frac{\partial^2 C_1}{\partial z^2}\right)\text{------------}(26)$$

The boundary conditions for corresponding order are as follows:

$$u_1=v_1=w_1=0,\quad \theta_1=\cos(az),\quad C_1=\cos(az)\quad at\ y=0$$
$$u_1=v_1=w_1=0,\quad \theta_1=0,\quad C_1=0\quad at\ y=1\text{----------------}(27)$$

using the transformations

$$u_1(y,z)=u_{11}(y)e^{iaz}$$
$$v_1(y,z)=v_{11}(y)e^{iaz}$$
$$w_1(y,z)=w_{11}(y)e^{iaz}$$
$$\theta_1(y,z)=\theta_{11}(y)e^{iaz}$$
$$C_1(y,z)=C_{11}(y)e^{iaz}\text{----------------------}(28)$$

Equations (21) – (26) are transformed in the following linear differential equations

$$Rv\,u_{11}''-(Rv\,a^2+\frac{1}{Da})u_{11}+Gr\,\theta_{11}+Gm\,C_{11}+v_{11}\left(m\,A(e^{-my}+e^{m(y-2)})-Da(Gr+Gm)\right)=0\text{--}(29)$$

$$Rv\,v_{11}''-(Rv\,a^2+\frac{1}{Da})v_{11}=0\text{-----------------------------}(30)$$

$$Rv\,w_{11}''-(Rv\,a^2+\frac{1}{Da})w_{11}=0\text{-----------------------------}(31)$$

$$\theta_{11}''-a^2\theta_{11}+\Pr v_{11}=0\text{---------------------------------}(32)$$

$$C_{11}''-a^2C_{11}+Scv_{11}=0\text{---------------------------------}(33)$$

ε= perturbation parameter and all the other symbols are in their usual notations

Subject to the boundary conditions

$$u_{11}=v_{11}=w_{11}=0,\quad \theta_{11}=1,\quad C_{11}=1,\quad at\ y=0$$
$$u_{11}=v_{11}=w_{11}=0,\quad \theta_{11}=0,\quad C_{11}=0,\quad at\ y=1\qquad(34)$$

Where dashes differentiation with respect to y.

Solving (29) to (31) subject to the conditions (34), we have obtained the following solutions:

$$u_{11} = C_1 e^{m_1 y} + C_2 e^{-m_1 y} + A_1\left(e^{-ay} - e^{-a(2-y)}\right) ---------(35)$$

$$v_{11} = 0 ------------------------------------- (36)$$

$$w_{11} = 0 ------------------------------------- (37)$$

$$\theta_{11} = \frac{1}{(1-e^{-2a})}\left(e^{-ay} - e^{-a(2-y)}\right) -------------------(38)$$

$$C_{11} = \frac{1}{(1-e^{-2a})}\left(e^{-ay} - e^{-a(2-y)}\right) -------------------(39)$$

Where $m_1 = \sqrt{a^2 + \frac{1}{DaRv}}$, $A_1 = \frac{1}{(1-e^{-2a})}$, $A_2 = -\frac{Da\,(Gr+Gm)}{(1-e^{-2a})}$

$$C_1 = \frac{(1-e^{-2a})e^{-2m_1}A_2}{1-e^{-2m_1}}, \quad C_2 = -\frac{(1-e^{-2a})A_2}{1-e^{-2m_1}}$$

By using the above solutions the expressions for velocity and temperature are obtained as follows:

$$u = \begin{bmatrix} (A(e^{-my} - e^{m(y-2)}) + Da(Gr(1-y) + Gm(1-y))) + \\ \varepsilon\left(C_1 e^{m_1 y} + C_2 e^{-m_1 y} + A_2\left(e^{-ay} - e^{-a(2-y)}\right)\cos z_1\right) \end{bmatrix} -----(40)$$

$$\theta = 1 - y + \varepsilon A_1\left(e^{-ay} - e^{-a(2-y)}\right)\cos z_1 ---------------(41)$$

$$C = 1 - y + \varepsilon A_1\left(e^{-ay} - e^{-a(2-y)}\right)\cos z_1 ---------------(42)$$

Where $z_1 = az$

Skin frictions and Nusselt numbers in dimensionless form at both the walls are as follows:

$$\tau_1 = \left.\frac{du}{dy}\right|_{y=0} = \left[-m\,A(1+e^{-2m}) - Da(Gr+Gm) + \varepsilon\left(m_1(C_1 - C_2) - a\,A_2(1+e^{-2a})\right)\cos(z_1)\right] - (43)$$

$$\tau_2 = -\left.\frac{du}{dy}\right|_{y=1} = \left(2\,Ame^{-m} + Da(Gr+Gm)\right) - \varepsilon\left(m(C_1e^{m} - C_2e^{-m}) - 2\,aA_2e^{-a}\right) -------(44)$$

$$Nu_1 = -\left.\frac{d\theta}{dy}\right|_{y=0} = 1 + \varepsilon\,a\,A_1(1+e^{-2a})\cos z_1 ------------(45)$$

$$Nu_2 = -\left.\frac{d\theta}{dy}\right|_{y=1} = 1 + 2\varepsilon a\,A_1\,e^{-a}\cos z_1 ----------------(46)$$

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III. RESULT AND DISCUSSION

In this section, analytical computation of full mathematical problem has been carried out in order to analyse the effect of parameters Darcy number (Da), ratio of viscosities (Rv), Thermal Grashof Number (Gr), Mass Grashof Number (Gm), wave number (a) and perturbation parameter (ε) on the transport phenomena.

In figure 1, velocity distribution $(u1)$ is plotted against y, fixing Da=10^{-1},10^{-2}, and Rv=1.0, 1.5, 2.0. It is evident from this figure that the velocity increases with increase in Darcy number (Da). Interpreting physically, increase in the permeability parameter (K) increases the flow which leads to increase in velocity. Velocity decreases due to the increase of Rv for all values of Da, which is due to the high effective viscosity of the porous medium to that of fluid viscosity.

Figure 2, depicts the velocity profile (u) for Da=10^{-1},10^{-2},Rv=1.0, 1.5, 2.0 and ε=0.2. The velocity increase with increase of Darcy number and has reverse case with ratio of viscosities.

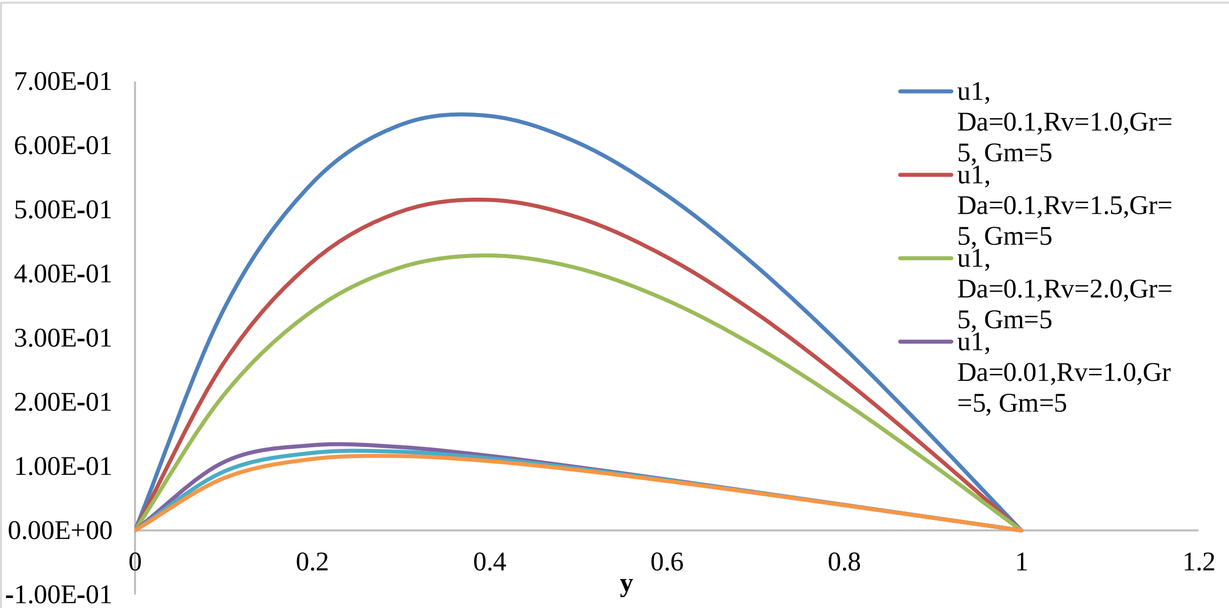

Fig. 1: Velocity Profile (u1) for the different values of Da and Rv

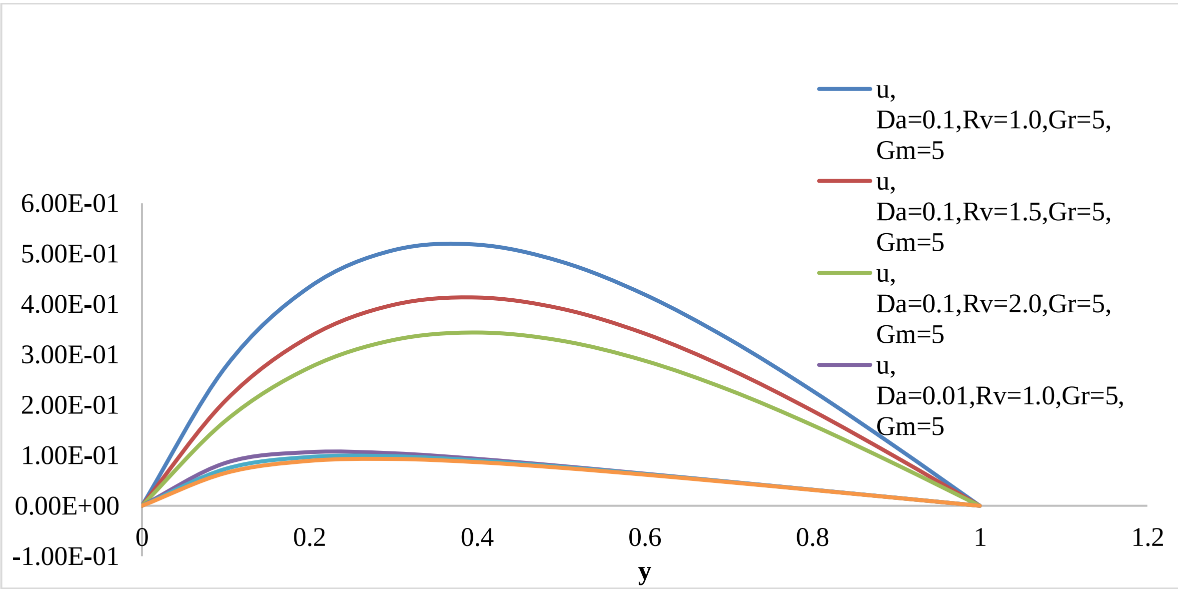

Fig. 2: Velocity Profile (u) for the different values of Da and Rv

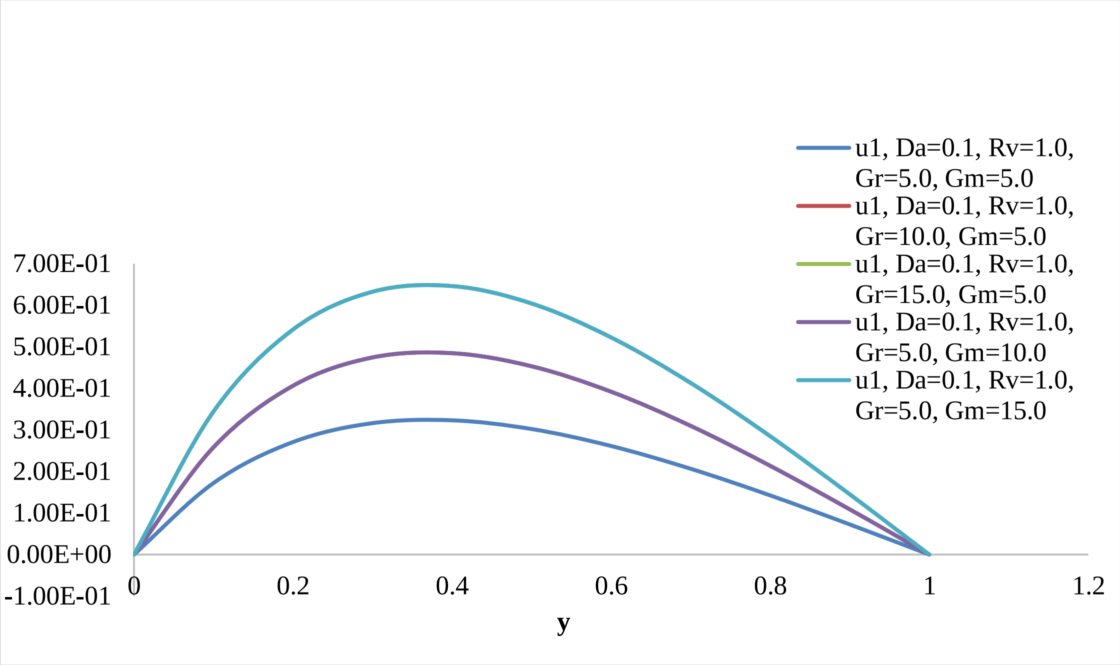

Fig. 3: Velocity Profile (u1) for the different values of Gr and Gm

Figure 3 plotted for the velocity profile with the fixed values of Da=$10^{-1}$and Rv=1.0for the different values of Gr andGm. It is evident that velocity increases with increase of thermalGrashof Number (Gr), mass Grashof Number (Gm),

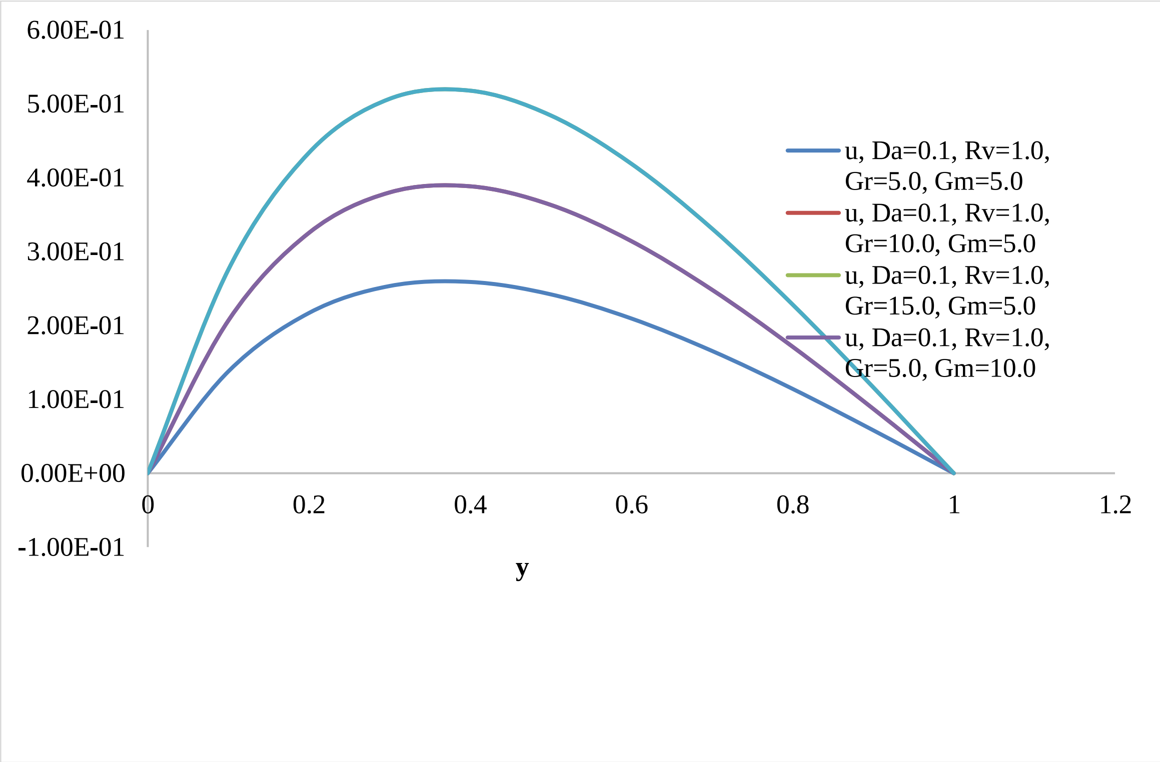

Fig. 4: Velocity Profile (u) for the different values of Gr and Gm

Figure 5-6 , indicates the temperature and concentration profilesrespectively for the different values of wave number i, e a=0.2, 0.6, 0.8, 1.2 and 1.6. It is clear from both the figures that temperature as well as concentration decreases with the increase of wave number (a)

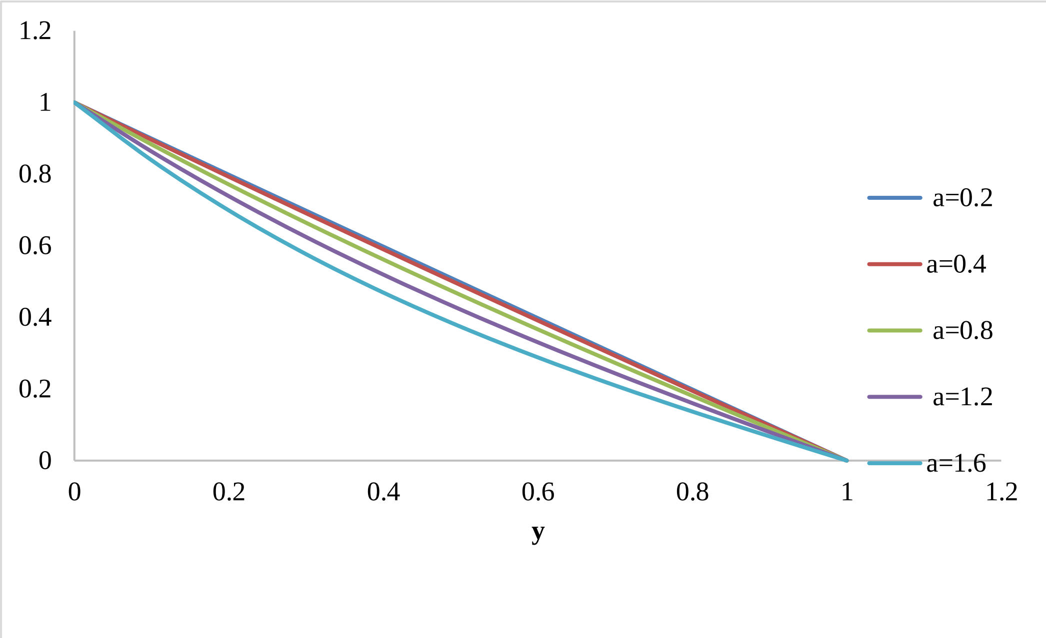

Fig. 5: Temperature Profile for the different values of a

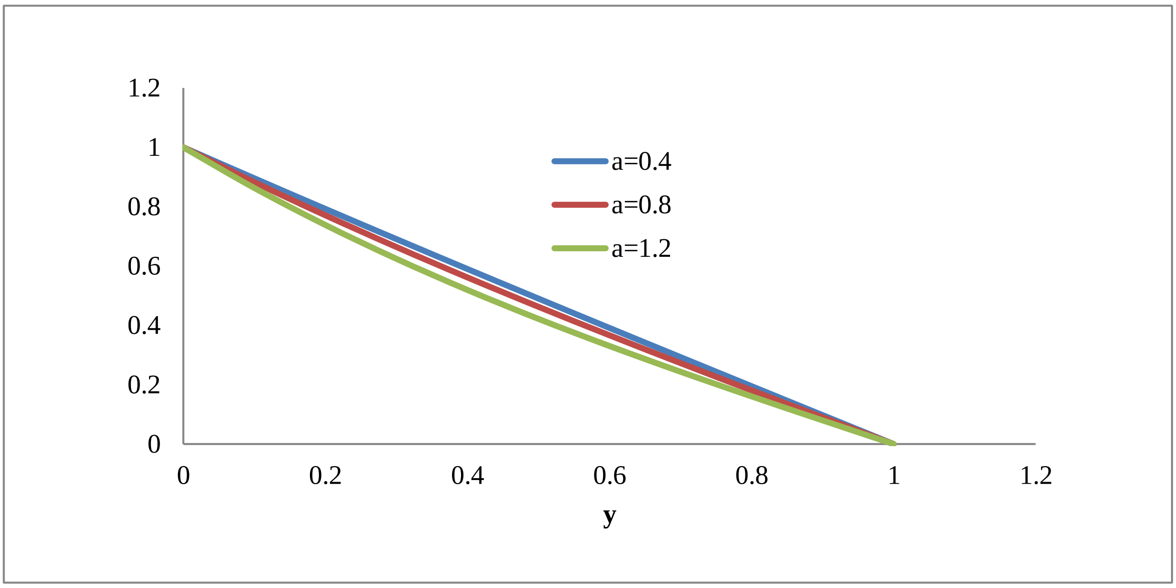

Fig. 6: Concentration Profile

IV. CONCLUSION

In the present study, fully developed flow and heat and mass transfer between two vertical walls containing porous materials is studied analytically using regular perturbation method. The Brinkman extended Darcy model is used for flow through porous media, the velocity and concentration profiles have been presented. Hence one can conclude that the Darcy number promotes the velocity and ratio of viscosities suppresses the velocity.

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