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How Nature Works in Four-Dimensional Space: The Untold Complex Story

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Abstract:- In this article we propose specific answers to three of the most persistent questions: i- Do probabilities and statistics belong to physics or mathematics? ii- Followed by the related question, does nature operate in 3D geometry plus time as an external controller or more specifically, does it operate in the inseparable 4D unit space where time is woven? iii-Lagrange multipliers: Is it just a classic mathematical trick that we can do without? The answers to these questions are all interconnected. Here we could say that the modern definition of probability in the transition matrix B is an interconnected thing of the three topics. Imagination is the first important common factor in mathematics, physics and especially in the probability of transition in 4D space. In a breakthrough technique, B-matrix chains are used in this article to numerically solve single, double, and triple (Hypercube) integrals as a successful preliminary answer to this question. In fact, it is the natural continuation of the success of B-chains in solving the general case of time-dependent partials of differential equations with Dirichlet Arbitrary BC and arbitrary initial conditions that have been explained in previous articles.

I. INTRODUCTION

To begin with, let's admit that nature does not see with our eyes and does not think with our brains. In this article, we try to understand how nature performs its own resolutions in x-t spacetime as an inseparable unitary block.

We suppose that B-Matrix statistical chains (or any other suitable stochastic chains) can answer this question and demonstrate, in some way, how nature works.

➢ *We Propose the Following,*

- Nature sees the 1D curve as a 2D trapezoidal area.
- Nature sees the 2D square as a 3D cube or cuboid volume.
- Nature sees the 3D cube as a 4D Hypercube and evaluates its volume as L^4.

Moreover, in all propositions i-iii, time t is the extra dimension.

However, many people still believe that x-t spacetime is practically separable and that describing x-t as an inseparable unit block is only essential when the speed of the object concerned approaches that of light.

➢ *This is the most common error in mathematics.*

Time is part of an inseparable block of space-time and therefore geometric space is the other part of the inseparable block of space-time.

Surprisingly enough, you can statistically perform numerical integration in the x-t spacetime unit with wide applicability and excellent speed and accuracy.

On the other hand, the current classical mathematical methods of numerical integration via classical mathematical techniques in a hypothetical geometric Cartesian space are still applied [1], but only in special cases and it is to be expected that their results obtain only limited success.

The universe has expanded since the time of the Big Bang at nearly the speed of light and still controlled by its universal x-t laws of physics without the slightest change in its universal constants (G, k_B, h, c . . etc).

This may be the reason why the results of classical mathematics fail and become less accurate than those of the stochastic B-matrix (or any other suitable matrix) even in the simplest situations like double integration and triple integration.

This is also the reason why the current definition of double and triple integration is incomplete. Brief,

1-It is important to understand that mathematics is only a tool for quantitatively describing physical phenomena, it cannot replace physical understanding.

2-It is claimed that mathematics is the language of physics, but the reverse is also true, physics can be the language of mathematics as in the case of numerical integration and the derivation of the normal/ Gaussian distribution law via the statistical B-matrix chains.

However, in a revolutionary technique, chains of B matrices are used to solve numerically PDE, double and triple integrals as well as the general case of time-dependent partial differential equations with arbitrary Dirichlet boundary conditions and initial arbitrary conditions.

The statement 2 that mathematics is the language of physics always was a given and widely accepted, but the idea that the reverse could be true is quite unexpected.

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- *This is one basis of the untold complex story.*

Most mathematicians believe that all mathematical equations should be accurate to the tenth digit and can have complete representations applicable to the set of generalized geometric shapes/volumes that emerge from our observations of physically existing "objects".

This is not true since nature does not operate in geometric shapes as observed with our eyes and brain, ie. in x-space alone, but it works in x-t-space now and for eternity.

II. THEORY

The object of this article is to replace simple, double and triple integration with the numerical results of the chains of the transition matrix B.

In other words, the integration $I=\int f(x)\,dx$

Which was originally defined as the limit of the sum of the product f(x).dx for a small infinitesimal incremental dx as dx tends to zero [1] can be completely ignored in numerical statistical methods as if it never existed .

Similarly, the new matrix chain technique B ignores the classical expression for 3D integration in the form $I=\iiint f(x,y,z)\,dxdydz$ and evaluates it as the final result of the series of power of the matrix B.

In this article, we concentrate on some illustrative numerical results in the field of numerical integration (single, double, and triple integration) via the theory of the matrix B, which in itself is not complicated but rather long.

The transition matrix B nxn which is successfully applied in the derivation of numerical integration formulas and in the solution of time-dependent PDE is well defined in the x-t unit space and hence its time chains via the following statistical conditions [2,3,4]:

- *For Cartesian coordinates in 1,2 and 3D space, the inputs $B_{i,j}$ respect or are subject to the following conditions:*

- $B_{i,j}$ = 1/2,1/4 and 1/6 in 1D, 2D and 3D for i adjacent to j and $B_{i,j}$ = 0 otherwise. Condition (i) translates an equal a priori probability of all directions in space, i.e. no preferred direction.
- $B_{i,i}$ = RO, i.e. the main diagonal consists of equal or constant entries RO .
- RO can take any value in the interval [0,1] .
- Condition (ii) corresponds to the assumption of equal and similar residue after each jump or time step dt for all free elementary nodes.
- $B_{i,j} = B_{j,i}$, for all i, j.
- The Matrix B is symmetrical to conform to nature's symmetry and physical principles of reciprocity and detailed balance.
- The sum of $B_{i,j}$ = 1 for all rows (or columns) away from the borders and the sum $B_{i,j}$ < 1 for all the rows connected to the borders.

Condition iv means that the probability of the whole space = 1.

Obviously, the statistical matrix B is very different from the mathematical Laplacian matrix and the mathematical statistical matrix of the Markov transition probability.

The physical nature of B is clear and briefly explained above through its four conditions i-iv which support the hypothesis of being an accurate model of nature itself.

The question arises, why are statistical forms of integration faster and more accurate than mathematical forms?

We assume that the answer is inherent in the integration processes, whether they belong to the 3D geometric space or the 4D unitary x-t space (Figs 1a and 1b).

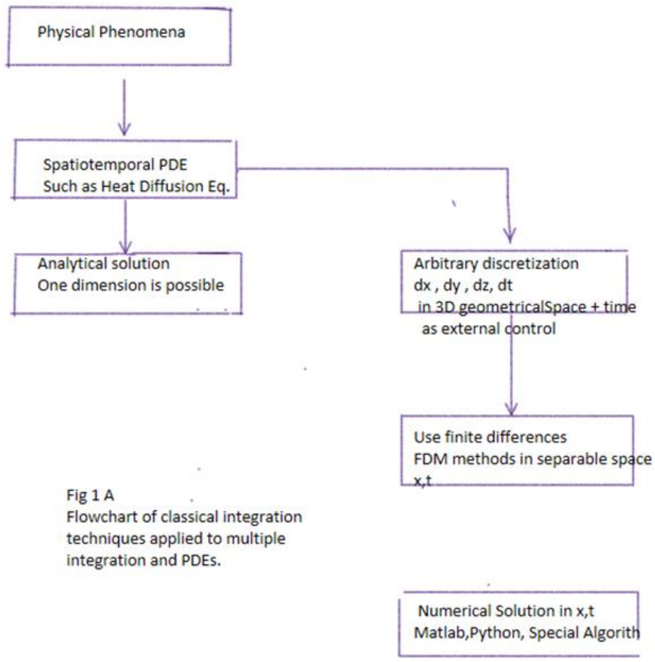

Fig 1 A Flowchart of Classical Integration Techniques Applied to Multiple Intergration And PDEs.
3D Geometrical space

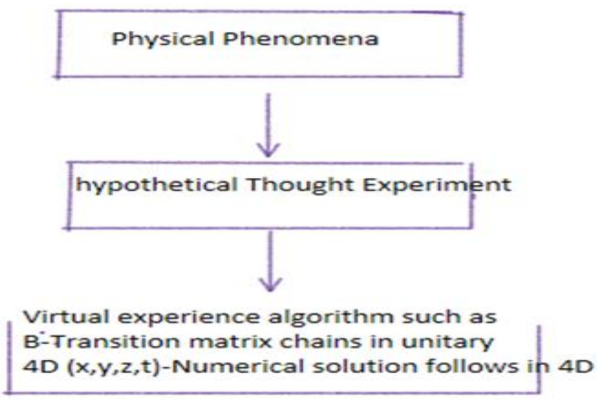

Fig 1 B Flow chart of virtual experiment techniques applied to multiple integration and PDEs. 4D unitary space

In order not to concern ourselves too much with the details of the theory let us pass directly to the illustrative applications and the numerical results.

III. APPLICATIONS AND NUMERICAL RESULTS

A. Case A- Single finite Integral

➢ *1-D Numerical Statistical Integration for 7 Free Nodes*

We first explain the numerical statistical integration procedure for the case of 7 free nodes (Fig.2) in 1D geometric space which is exactly the same for 2D and 3D geometric space and will be systematically followed in sections following in this article.

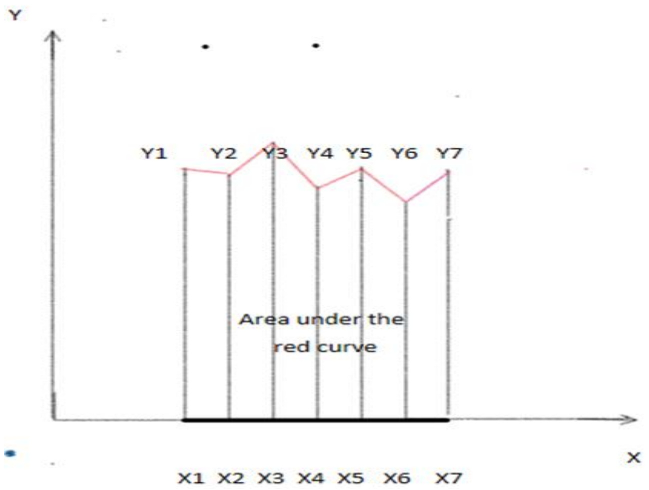

Fig 2 Numerical statistical integration for the case of 1D- 7 free nodes

- *B-Matrix 7X7*

The statistical transition matrix B is expressed by,

0	1/2	0	0	0	0	0
1/2	0	1/2	0	0	0	0
0	1/2	0	1/2	0	0	0
0	0	1/2	0	1/2	0	0
0	0	0	1/2	0	1/2	0
0	0	0	0	1/2	0	1/2
0	0	0	0	0	1/2	0

And the destination transfer matrix E which is defined as the sum of the infinite power matrix, namely,

$E= B^0 + B + B^2 + \dots\dots\dots B^N \dots\dots$ (1)

For N sufficiently large.

[It is obvious that B^0=I (unit matrix)]

Or equivalently by the expression,

$E= (I-B)^{-1}$ . . . .(2)

Equation 2 is justified by the relation,

$B0 + B + B2 + \dots\dots\dots BN =(I-B)^{\wedge -1}$, which holds for N large enough.

Further, the transfer matrix D is defined as follows:

D(N)= E(N)- I . . . . (3)

Using either Eq 1 or Eq 2 , we arrive at the numerical expression for

D=E-I which is equal to,

3/4	3/2	5/4	1	3/4	1/2	1/4	Sum of ROW 1= 6
3/2	2	5/2	2	3/2	1	1/2	Sum of ROW 2= 11
5/4	5/2	11/4	3	9/4	3/2	3/4	Sum of ROW 3= 14
1	2	3	3	3	2	1	Sum of ROW 4= 15
3/4	3/2	9/4	3	11/4	5/2	5/4	Sum of ROW 5= 14
1/2	1	3/2	2	5/2	2	3/2	Sum of ROW 6= 11
1/4	1/2	3/4	1	5/4	3/2	3/4	Sum of ROW 7= 6

The total sum of all elements of the transfer matrix D which is $\sum i \sum j$ Di,j for all i and j is given by the sum of the sum of rows, i.e.

$\sum i \sum j$ Di,j =6+11+14+15+14+11+6 = 77

Therefore,

The statistical integration formula or area under the curve for 7 nodes is given by,

$I=[6h^2/77]*(6.Y1 +11.Y2 + 14.Y3 +15.Y4 +14.Y5 + 11.Y6 + 6.Y7)$ . . . (3)

- *Which is the Statistical Equivalence of Simpson's Rule for 7 Nodes*

Now consider the special limited integral,

$I=\int y\, dx$ from x=2 to x=8, where $y=X^2$.

That's to say,

X = 2 3 4 5 6 7 8

Y = 4 9 16 25 36 49 64

Numerical result via Trapezoidal ruler (I_t),

I_t = Y1/2+Y2+Y3+Y4 +Y5+Y6+Y7/2=

2+9+16 +25+ 36+ 49+32=169 square units.

Analytic integration expression (I_a)

I_a=X^3/3

Ia = (512-8)/3= 168 square units.

Finally, the statistical integration formula for 7 nodes is given by,

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Is=6 h^2/77 (6*4 +11* 9 + 14* 16+15* 25 +14* 36 + 11*49 + 6*64)

I = 167.455 square units.

This means that statistical integration is quite fast and accurate.

B. Case B- Single finite Integral

➢ *1-D numerical statistical integration for 19 free nodes*

Following exactly the same procedure as that followed for 7 nodes, we arrive at,

Is= h2 (0.2571Y1+0.4872Y2+ 0.6902Y3+ 0.8662Y4+1.0150Y5+1.1368Y6+1.2316Y7 +1.2992Y8+ 1.339Y9+ 1.3534Y10+ 1.339Y11+1.2992Y12+1.2316Y13+1.1368Y14 +1.0150Y15+ 0.8662Y16+ 0.6902Y17+0.4872Y18+ 0.2571Y19) . . . . . . (4)

- *Equation 4 is the statistical equivalence of Simpson's rule for 19 nodes.*

We can compare the statistical weights given in Equation 4 by their corresponding values offered by the normal/Gaussian distribution formula y=c1exp-c2(x-Mue)^2 as shown in Table I.

The constants c1 and c2 are conveniently set to 1.3534 and 0.01 and the median Mue is obviously equal to 10.

Table 1 Statistical equivalence of Simpson's integration for 19 nodes vs Gaussian distribution

I	x=.1*I	Gauss Formula	Statistical equivalence (Eq 4)	Absolute error e_a
1	0.1	0.602070928	0.2571	0.343
2	0.2	0.713637590	0.4872	0.226
3	0.3	0.829128623	0.6902	0.14
4	0.4	0.944235206	0.8662	0.088
5	0.5	1.05402899	1.0150	0.039
6	0.6	1.15329134	1.1368	0.016
7	0.7	1.23691452	1.2316	0.0063
8	0.8	1.30033243	1.2992	0.001
9	0.9	1.33993340	1.339	0.0009
10	1.0	1.35339999	1.3534	0.000
11	1.1	1.33993340	1.339	0.0009
12	1.2	1.30033243	1.2992	0.001

Table 1 shows a striking agreement between the outcomes of the Normal/Gaussian distribution curve and the statistical weights of the proposed statistical single integration .

The absolute error e_a is practically less than 0.001 for N = 19 and should decrease for higher N.

Moreover, the well-known normal/Gaussian distribution law can be directly derived without the need for Lagrange multipliers.

It worth mention that the same statement was mentioned in references 4 and 5.

- *Now the Lasting Questions Arise:*

✓ *Do probabilities and statistics belong to physics or mathematics?*
✓ *Followed by the related question, does nature operate in 3D geometry plus time as an external controller or more specifically, does it operate in the inseparable 4D unit space where time is woven?*
✓ *Lagrange multipliers: Is it just a classic mathematical trick that we can do without?*

C. CASE C

Double finite integral I=∬ f(x,y) dx dy over the domain a<=x<=b and c<=y<=d Figs 3a and 3b.

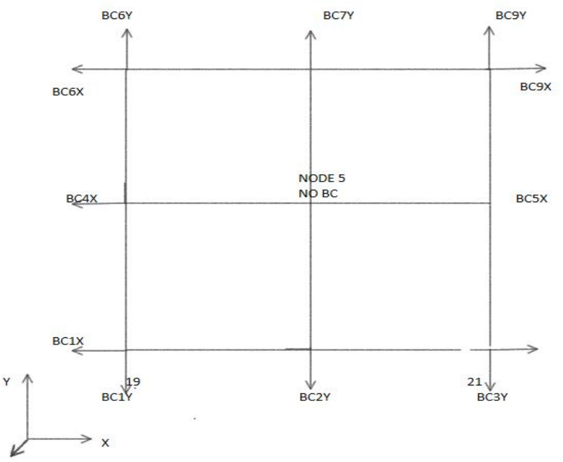

Fig 3a Double finite integral over the domain a<=x<=b and c<=y<=d

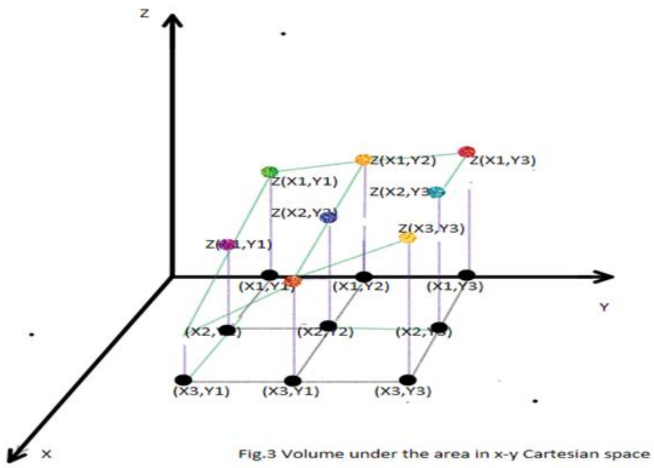

Fig 3b Double finite integral over the domain a<=x<=b and c<=y<=d

If we introduce a specific example without loss of generality where the function Z=f(x,y) is expressed by,

Z(x,y)= X^2.Y^2 + X^3 . . . . . (5)

Defined on the rectangular domain [abcd], 1<=x=>3 and 1<=y=>3 . . . Domain D(1)

The process of double numerical integration (I),

$$I=\iint f(x,y)\,dxdy$$

On the D1 domain can be achieved via three different approaches, namely, 1-analytically (a),

$$Ia=(x^3/3 * y^2 + x^4/4) + (x^2 * y^3/3 + x^3) \ldots (6)$$

2-Rule of the Double Sympson (ds),

$$I\ ds= h^3.(16f(b+a/2,d+c/2)+4f(b+a/2,d)+4f(b+a/2,c)+4f(b,d+c/2)+4f(a,d+c/2)+f(b,d)+f(b,c)+f(a,d)+f(a,c))/36 \ldots (7)$$

The statistical integration formula via the Cairo technique (ct)[6],

$$Ict = 9h^3/29.5(2.75Z(1,1)+3.5Z(1,2)+2.75.Z(1,3)+3.5Z(2,1)+4.5Z(2,2)+3.5Z(2.3)+2.75Z(3.1)+3.5Z(3.2)+2.75Z(3.3))\ (8)$$

Where h is the equidistant interval on the x and y axes.

- *Equation 8 is the statistical equivalence of double Simpson's rule for 9 nodes.*

Equation 8 is the statistical equivalence of double Simpson's rule for 9 nodes. The numerical results are as follows,

- Ia =227.25
- Ids=226.5
- I ct=227.035

Note again that the proposed statistical integration is extremely accurate.

- *CASE C:*

- *Triple Finite Integral and Hypercube*

$I=\iiint f(x,y,z)\,dx\,dy\,dz\ldots$ for the domain $a<=x<=b$ and $c<=y<=d$ & $e<=z<=f$ Fig 4.

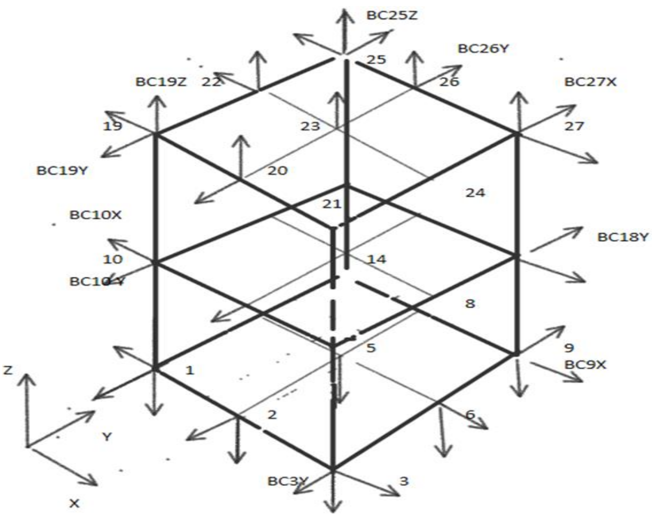

Fig 4 Triple integration on a 3D geometric cube

Again, we present the nature of the statistical B-matrix of the triple integration on the cube abcdefgh, divided into 27 equidistant nodes.

We describe below the proposition of statistical B-matrix of the triple integration on the 3D cube abcdefgh where the 27 free nodes are named (1,1,1 & 2,1,1, ...3,3,2& 3 ,3, 3)

$I=\iiint W(x,y,z)\,dxdydz$on the cubic domain of Figure 4.

$$I=27h^4/59(2.555W(1,1,1)+3.13W(1,2,1)+2.555.W(1,3,1)+3.13W(2,1,1)+3.876W(2,2,1)+3,13W(2,3,1)+2,555W(3,1,1)+2,555W(3,2,1)+3,13Z(3,3,1)\ldots etc.) \ldots (9)$$

- *Equation 9 is the statistical equivalence of Simpson's "triple" rule for 27 nodes.*

It's clear that the performance of the Eq 9 merges speed and precision**.**

Again, the question arises, why are statistical forms of integration faster and more accurate in 1D, 2D and 3D geometric spaces than mathematical formulas?

We assume that the answer is that the integration processes, whether they belong to the 1D, 2D or 3D geometric space, must be carried out via the unitary x-t space.

Note that in classical theoretical physics, mathematics, and quantum mechanics itself, time is currently understood as an external controller of all events and unwoven into 3D space to form an inseparable 4D construct block. In other words, classical time is absolute time and proper times are defined by a classical separable spacetime metric.

Perhaps this is the reason for their incompleteness in physics and mathematics.

The correct definition of transition probability in the x-t 4D is the corner stone to better understanding of the nature universal laws.

Space must relate to time in a collective system of probability of transition. Moreover, it is expected that there is a rigorous relationship between the microscopic transition probability Bij and the macroscopic transition probability of the system as a whole which is not yet defined.

IV. CONCLUSION

In general, probability and statistics are a missing part of mathematics and belong to physics rather than mathematics.

Throughout this paper, we proposed specific answers to three of the most lasting questions: i- Do probabilities and statistics belong to physics or mathematics?

Followed by the related question, does nature operate in 3D geometry plus time as an external controller or more specifically, does it operate in the inseparable 4D unit space where time is woven?

Lagrange multipliers: Is it just a classic mathematical trick that we can do without?

We assume that our answers to these questions are satisfactory and all interconnected.

Here we could say that the modern definition of probability in the transition matrix B is an interconnected thing of the three topics. Imagination is the first important common factor in mathematics, physics and especially in transition probability in 4D unit space.

In a breakthrough technique, B-matrix chains are used in this article to numerically solve single, double, and triple (Hypercube) integrals as a successful preliminary answer to these questions. It is in fact the natural continuation of the success of B-chains in solving the general case of time-dependent partials of differential equations with Dirichlet Arbitrary BC and arbitrary initial conditions that have been explained in previous articles. We also explained the statistical derivation of the conventional normal distribution and compared our numerical results with those of the well-known classical Gaussian distribution curve through an arbitrarily chosen special case without loss of generality.

The agreement between the two formulas is striking and confirms the validation of the proposed technique.

In other words, the B-Matrix chain technique is able to process and derive adequate mathematical formulas for different physical and mathematical situations, which supports the hypothesis that B-Matrix chains are the matrix chains of nature. Or how nature's energy fields live and function in 4D x-t unitary space.

NB. All calculations in this article were produced using the author's double-precision algorithm to ensure maximum accuracy, as follows by ref. 8 for example.

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