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On a Generalization of Nelly Distribution and its Statistical Properties

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Abstract:- Statistical distributions can be used to describe a number of real-world occurrences. Numerous academics have thoroughly examined their theory and developed new distributions as a result of the usefulness of statistical distributions. There continues to be a strong drive in probability theory and statistics to develop probability distributions that are more adaptable and powerful. In this study, we introduce the Nelly distribution a brand-new probability distribution and develop suitable expressions for its statistical characteristics.

Keywords:- *Nelly Distribution, Moment Generating Function, Characteristics Function, Probability Density Function.*

I. INTRODUCTION

The description of phenomena in the real world is typically done using statistical distributions. The value of statistical distributions has prompted in-depth research into their theory and the development of new distributions. In the area of probability theory and statistics, there is still a lot of work being done to produce more useful and flexible probability distributions [5]. Being more effective and flexible when representing real-world data is the obvious rationale for generalizing a standard distribution.

In an effort to increase the flexibility of probability distributions, many academics have combined different continuous distributions over the years, including [13],[6], [1], [4], [7], [8], and [2] more so, [12] have shown that distributions using the Bayesian approach are flexible, perform better, and have a wider applicability. We introduce the Nelly distribution, which was motivated by the need for constant extension and generalization to more complex situations, as well as current developments in developing novel distributions.

II. METHODS

A. Developing of Nelly Distribution

The probability density function of Nelly distribution is developed using Bayesian approach and Exponential-Gamma distribution by Oguwale et. al (2019) as prior

According to Oguwale et al. (2019) theorem, if X_1 and X_2 be a continuous independent random variables such that $X_1 \sim E(X,\lambda)$ and $X_2 \sim G(X,\propto,\lambda)$ then their probability density functions are given as

$$f(X_1) = \lambda e^{-\lambda x} \qquad (1)$$

$$f(X_2) = \frac{\lambda^{\propto} X^{\propto-1} e^{-\lambda x}}{\Gamma(\propto)} \qquad X,\alpha,\lambda > 0 \qquad (2)$$

Therefore, the joint probability density function of Exponential-Gamma distribution is obtained as $f(X_1, X_2) = f(X_1)f(X_2)$ where $f(X_1, X_2)$ is the product of $f(X_1)$ and $f(X_2)$. then the pdf of Exponential-Gamma distribution is given as

$$f(X) = \frac{\lambda^{\propto+1} X^{\propto-1} e^{-2\lambda x}}{\Gamma(\propto)} \qquad X,\alpha,\lambda > 0 \qquad (3)$$

Finding the integral of equ. (3) with respect to X we have

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$$\int_0^{\infty} \frac{\lambda^{\propto+1}\, X^{\propto-1}\, e^{-2\lambda x}}{\Gamma(\propto)}\, dx \qquad (4)$$

Let $u = 2\lambda x, X = \frac{u}{2\lambda}$ and $dx = \frac{du}{2\lambda}$

Substituting we have

$$\int_0^{\infty} \frac{\lambda^{\propto+1} \left(\frac{u}{2\lambda}\right)^{\propto-1} e^{-u}}{\Gamma(\propto)} \frac{du}{2\lambda}$$

$$= \frac{\lambda^{\propto+1}}{(2\lambda)^{\propto}} \qquad (5)$$

Then the posterior which will be the new Nelly distribution will be

$$\left(\frac{\lambda^{\propto+1}\, X^{\propto-1}\, e^{-2\lambda x}}{\Gamma(\propto)}\right) \Big/ \left(\frac{\lambda^{\propto+1}}{(2\lambda)^{\propto}}\right)$$

$$\left(\frac{\lambda^{\propto+1}\, X^{\propto-1}\, e^{-2\lambda x}}{\Gamma(\propto)}\right)\left(\frac{(2\lambda)^{\propto}}{\lambda^{\propto+1}}\right)$$

$$f(X) = \frac{(2\lambda)^{\propto}\, X^{\propto-1}\, e^{-2\lambda x}}{\Gamma(\propto)} \qquad (6)$$

Testing Nelly distribution as a probability distribution

$$f(X) = \int_0^{\infty} \frac{(2\lambda)^{\propto}\, X^{\propto-1}\, e^{-2\lambda x}}{\Gamma(\propto)}\, dx$$

Let $u = 2\lambda x, X = \frac{u}{2\lambda}$ and $dx = \frac{du}{2\lambda}$

Substituting we have

$$f(X) = \int_0^{\infty} \frac{(2\lambda)^{\propto} \left(\frac{u}{2\lambda}\right)^{\propto-1} e^{-u}}{\Gamma(\propto)} \frac{du}{2\lambda} \qquad (7)$$

$$f(X) = \frac{(2\lambda)^{\propto}}{\Gamma(\propto)} \int_0^{\infty} \left(\frac{u}{2\lambda}\right)^{\propto-1} e^{-u} \frac{du}{2\lambda}$$

$$f(X) = \frac{(2\lambda)^{\propto}}{\Gamma(\propto)} \int_0^{\infty} \frac{u^{\propto-1}\, e^{-u}}{(2\lambda)^{\propto}}\, du \qquad (8)$$

$$f(X) = \frac{(2\lambda)^{\propto}}{(2\lambda)^{\propto}} \int_0^{\infty} \frac{u^{\propto-1}\, e^{-u}}{\Gamma(\propto)}\, du$$

Where $\int_0^{\infty} \frac{u^{\propto-1}\, e^{-u}}{\Gamma(\propto)}\, du = 1$, we have

$$f(X) = 1$$

Which implies that Nelly distribution is a probability distribution

III. MOMENTS

Theorem 1: if X is a continuous random variable distributed as a Nelly distribution (x, α, λ) then the r[th] non-central moment is given by $\mu_r = \frac{(2\lambda)^{\propto}}{\Gamma(\propto)} \frac{1}{(2\lambda)^{\propto + r}} \Gamma(\propto + r)$

Proof:

$$\mu_r = \int_0^\infty X^r f(X, \propto, \lambda)\ dx$$

$$= \int_0^\infty \frac{X^r (2\lambda)^{\propto} X^{\propto - 1} e^{-2\lambda x}}{\Gamma(\propto)}\ dx$$

$$= \int_0^\infty \frac{(2\lambda)^{\propto} X^{r+\propto-1} e^{-2\lambda x}}{\Gamma(\propto)}\ dx \qquad (9)$$

Let $u = 2\lambda x, \quad X = \frac{u}{2\lambda}, \qquad dx = \frac{du}{2\lambda}$

Substituting, we have

$$= \frac{(2\lambda)^{\propto}}{\Gamma(\propto)} \int_0^\infty \left(\frac{u}{2\lambda}\right)^{r+\propto-1} e^{-u} \frac{du}{2\lambda}$$

$$= \frac{(2\lambda)^{\propto}}{\Gamma(\propto)} \int_0^\infty \frac{u^{r+\propto-1}\ e^{-u}}{(2\lambda)^{r+\propto}}\ du$$

$$= \frac{(2\lambda)^{\propto}}{(2\lambda)^{r+\propto}} \int_0^\infty \frac{u^{r+\propto-1}\ e^{-u}}{\Gamma(\propto)}\ du$$

$$= \left(\frac{(2\lambda)^{\propto}}{(2\lambda)^{r+\propto}}\right)\left(\frac{\Gamma(\propto+r)}{\Gamma(\propto)}\right) \qquad (10)$$

Therefore when r = 1, we have the mean of the probability distribution

$$= \left(\frac{(2\lambda)^{\propto}}{\Gamma(\propto)}\right)\left(\frac{\Gamma(\propto+1)}{(2\lambda)^{\propto+1}}\right)$$

$$= \left(\frac{(2\lambda)^{\propto}}{\Gamma(\propto)}\right)\left(\frac{\propto \Gamma(\propto)}{(2\lambda)^{\propto} . (2\lambda)}\right)$$

$$\text{Mean} = \mu_1 = \frac{\propto}{2\lambda} \qquad (11)$$

Finding the second moment when r = 2

$$= \left(\frac{(2\lambda)^{\propto}}{\Gamma(\propto)}\right)\left(\frac{\Gamma(\propto+2)}{(2\lambda)^{\propto+2}}\right)$$

$$= \left(\frac{(2\lambda)^{\propto}}{\Gamma(\propto)}\right)\left(\frac{\propto(\propto+1)\Gamma(\propto)}{(2\lambda)^{\propto} . (2\lambda)^2}\right)$$

$$= \frac{\propto(\propto+1)}{4\lambda^2}$$

Variance = $\mu_2 - (\mu_1)^2$

$$= \frac{\propto(\propto+1)}{4\lambda^2} - \frac{\propto^2}{4\lambda^2}$$

Therefore, the variance of Nelly distribution is

$$variance = \frac{\propto}{4\lambda^2} \qquad (12)$$

Finding the third moment when r =3

$$\mu_3 = \left(\frac{(2\lambda)^{\propto}}{\Gamma(\propto)}\right)\left(\frac{\Gamma(\propto+3)}{(2\lambda)^{\propto+3}}\right)$$

$$= \left(\frac{(2\lambda)^{\propto}}{\Gamma(\propto)}\right)\left(\frac{\propto(\propto+1)(\propto+2)\Gamma(\propto)}{(2\lambda)^{\propto}.(2\lambda)^{3}}\right)$$

$$\mu_3 = \frac{\propto(\propto+1)(\propto+2)}{8\lambda^3} \qquad (13)$$

Finding the fourth moment when r = 4

$$\mu_4 = \left(\frac{(2\lambda)^{\propto}}{\Gamma(\propto)}\right)\left(\frac{\Gamma(\propto+4)}{(2\lambda)^{\propto+4}}\right)$$

$$= \left(\frac{(2\lambda)^{\propto}}{\Gamma(\propto)}\right)\left(\frac{\propto(\propto+1)(\propto+2)(\propto+3)\Gamma(\propto)}{(2\lambda)^{\propto}.(2\lambda)^{4}}\right)$$

$$\mu_4 = \frac{\propto(\propto+1)(\propto+2)(\propto+3)}{16\lambda^4} \qquad (14)$$

Theorem: if X is a continuous random variable distributed as Nelly distribution. Then the moment generating function is defined as $\left(1-\frac{t}{2\lambda}\right)^{-\propto}$

Proof:

$$M_x(t) = E(e^{tx}) = \int_0^\infty e^{tx} f(X,\lambda,\propto)\ dx \quad if\ t<\lambda, x>0$$

$$= \int_0^\infty \frac{e^{tx}(2\lambda)^{\propto}X^{\propto-1}e^{-2\lambda x}}{\Gamma(\propto)}\ dx$$

$$= \int_0^\infty \frac{(2\lambda)^{\propto}X^{\propto-1}e^{-(2\lambda-t)x}}{\Gamma(\propto)}\ dx \qquad (15)$$

Let $u = x(2\lambda - t),\ X = \frac{u}{2\lambda-t}, dx = \frac{du}{2\lambda-t}$

Substituting we have

$$= \frac{(2\lambda)^{\propto}}{\Gamma(\propto)}\int_0^\infty \left(\frac{u}{2\lambda-t}\right)^{\propto-1} e^{-u}\frac{du}{2\lambda-t}$$

$$= \frac{(2\lambda)^{\propto}}{\Gamma(\propto)}\int_0^\infty \frac{u^{\propto-1}\ e^{-u}}{(2\lambda-t)^{\propto}}\ du$$

$$= \frac{(2\lambda)^{\propto}}{(2\lambda - t)^{\propto}} \int_0^{\infty} \frac{u^{\propto -1}\ e^{-u}}{\Gamma(\propto)}\ du$$

Where $\int_0^{\infty} \frac{u^{\propto -1}\ e^{-u}}{\Gamma(\propto)}\ du$ is a gamma function which sums to 1

$$M_x(t) = \left(\frac{2\lambda}{2\lambda - t}\right)^{\propto}$$

$$= \left(\frac{2\lambda - t}{2\lambda}\right)^{-\propto}$$

$$= \left(1 - \frac{t}{2\lambda}\right)^{-\propto} \qquad (16)$$

Theorem: if a random variable distributed as a Nelly distribution $(X, \propto, \lambda)$ then the characteristic function $\varphi_x(it)$ is defined as $\left(1 - \frac{it}{2\lambda}\right)^{-\propto}$

Proof:

$$\varphi_x(it) = E(e^{itx}) = \int_0^{\infty} e^{itx}\, f(X, \propto, \lambda) \quad dx$$

$$= \int_0^{\infty} \frac{e^{itx}(2\lambda)^{\propto} X^{\propto -1} \mathrm{e}^{-2\lambda x}}{\Gamma(\propto)} \quad dx$$

$$= \int_0^{\infty} \frac{(2\lambda)^{\propto} X^{\propto -1} \mathrm{e}^{-(2\lambda - it)x}}{\Gamma(\propto)}\ dx \quad (17)$$

Let $u = x(2\lambda - it),\ X = \frac{u}{(2\lambda - it)}, dx = \frac{du}{(2\lambda - it)}$

Substituting we have

$$= \frac{(2\lambda)^{\propto}}{\Gamma(\propto)} \int_0^{\infty} \left(\frac{u}{2\lambda - t}\right)^{\propto -1} e^{-u} \frac{du}{2\lambda - t}$$

$$= \frac{(2\lambda)^{\propto}}{\Gamma(\propto)} \int_0^{\infty} \frac{u^{\propto -1}\ e^{-u}}{(2\lambda - it)^{\propto}}\ du$$

$$= \frac{(2\lambda)^{\propto}}{(2\lambda - it)^{\propto}} \int_0^{\infty} \frac{u^{\propto -1}\ e^{-u}}{\Gamma(\propto)}\ du$$

$$= \left(\frac{2\lambda}{2\lambda - it}\right)^{\propto}$$

$$= \left(\frac{2\lambda - it}{2\lambda}\right)^{-\propto}$$

$$= \left(1 - \frac{it}{2\lambda}\right)^{-\propto} \qquad (18)$$

Theorem: if X is a continuous random variable from the Nelly distribution, the cumulative density function (cdf) is given as $\left(1 - \frac{t}{2\lambda}\right)^{-\propto}$

Proof:

$$F(x) = \int_0^x \frac{(2\lambda)^{\propto} X^{\propto-1} e^{-2\lambda x}}{\Gamma(\propto)} \quad dx \qquad (19)$$

Let $u = 2\lambda x,\ X = \frac{u}{2\lambda}, dx = \frac{du}{2\lambda}$

Substituting we have

$$= \frac{(2\lambda)^{\propto}}{\Gamma(\propto)} \int_0^x \left(\frac{u}{2\lambda}\right)^{\propto-1} e^{-u} \frac{du}{2\lambda}$$

$$= \frac{(2\lambda)^{\propto}}{\Gamma(\propto)} \int_0^x \frac{u^{\propto-1}\ e^{-u}}{(2\lambda)^{\propto}}\ du$$

$$= \int_0^x u^{\propto-1}\ e^{-u}\, du \qquad (20)$$

Where (20) is an incomplete lower gamma function. Therefore, the cumulative density function of a Nelly distribution is given as

$$F(x) = \frac{\gamma(\propto, X)}{\Gamma(\propto)} \qquad (21)$$

Coefficient of variation (C.V): it is the standardized measure of dispersion of a probability distribution. It is given as

$$\text{C.V} = \frac{Standard\ deviation}{mean} \qquad (22)$$

Therefore, the coefficient of variation of Nelly distribution is given as

$$\text{C.V} = \frac{\sqrt{\frac{\propto}{4\lambda^2}}}{\frac{\propto}{2\lambda}} = \frac{1}{\sqrt{\propto}} \qquad (23)$$

IV. CONCLUSION

In the study of probability and statistics, a variety of distributions have been defined and shown to be used extensively. In this study, the first four moments, the moment generating function, the characteristics function, and the cumulative distribution function of a new distribution called the Nelly distribution are defined, investigated, and established.

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