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The Vogel Approximation and North West Corner Transportation Models for Optimal Cost Distribution of Dangote Cement

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Abstract:- Most industrial and service organizations now include transportation logistics and management in their business decision making process. The cost of transportation and the distribution of goods from manufacturing facilities to depots raise transportation costs, which will ultimately have an impact on the selling price of the product and will then directly or indirectly affect final consumers. Several academic publications had proposed linear programming transportation models for transport logistics problems. Thispaper, models and assesses the performance of the Vogel Approximation and North West Corner transportation models on the logistics of optimal distribution bags of cement of Dangote Cement Company Plc, Nigeria, from some cement facilities, to various depots while minimizing transportation cost. The outcome of this study demonstrated a considerable decrease on the transportation cost of the distribution of the product to various depots as well as simple distribution from plants to depots. The Vogel Approximation model showed a slight improvement on distribution cost reduction by eight thousand, three hundred naira compared to the North-West Corner Rule. At the conclusion of the study, recommendations were given.

Keywords:- *Vogel Approximation, North West Corner, Transportation Model, Optimal Distribution, Dangote Cement.*

I. INTRODUCTION

Planning for sustainable mobility and management of transportation are essential for manufacturing and service provider businesses. In terms of effectiveness and environmental friendliness, a well-planned transportation system has many advantages. Over the years, plant product distributions have been carried out haphazardly without taking into account an optimal distribution pattern, which has an impact on the cost of the commodity. A transportation problem is a linear programming problem that involves choosing the best route to take finished product from plants to different locations (depots) and moving resources from one place to another while keeping costs to a minimum. The objective is to minimize the cost of distribution a product from a number of sources or origins to a number of destinations [1].Transportation problem is further describedin [2]as the most important and successful applications in the optimization), that is a special class of the linear programming (lp) in the operation research (or).The planners' mental models to comprehend the difficulties in putting sustainable transportation planning and policy into practice was examined in [3]. Three main issues: forecast-led vs vision-led planning, congestion relief, and public acceptance were considered. The study concluded that the goal of lowering car use is at odds with the ongoing investment in expanding road capacity and switching from forecast-led planning to back casting is challenging. Mishera[2] asserted that the transportation issue is regarded as being of the utmost importance and has been investigated in a number of operational and research fields. It has therefore been used to simulate a variety of issues from real life. The transportation problem is focused on the optimal distribution of units of a product from various points of origin to various destinations

A. Network flow programming

A model that is a particular instance of the more general linear problem is referred to as a "network flow program." The transportation problem, assignment problem, maximum flow problem, pure minimum cost flow problem, and generalized minimum cost flow problem are only a few examples of the problems that fall within the categories of network flow programs. The model representation is substantially more compact than the general linear problem, and many elements of real-world situations are easily recognized, making it an essential class. We frequently encounter unique linear programming issues with a fairly straightforward structure, and the transportation issue dominates this category. The study of ideal transportation and resource allocation is known as transportation theory in both mathematics and economics. Although Tolstoi was one of the first to examine the transportation problem theoretically in the 1920s, the problem was formalized by the French mathematician Gaspard in 1971. For the Soviet Union's national transportation commissariat, Volume 1 of the series Transportation Planning was published in 1930.

B. Transportation Model

The problem is represented by the network in the Figure 1. There are m sources and n destinations, each represented by a node. The arcs represent the routes linking the sources and the destinations. Arc (i,j) joining sources i to destination j carries two pieces of information: the transportation cost per unit, cij, and the amount shipped, xij. The amount of supply at source i is a_i, and the amount of demand at destination j is b_j.

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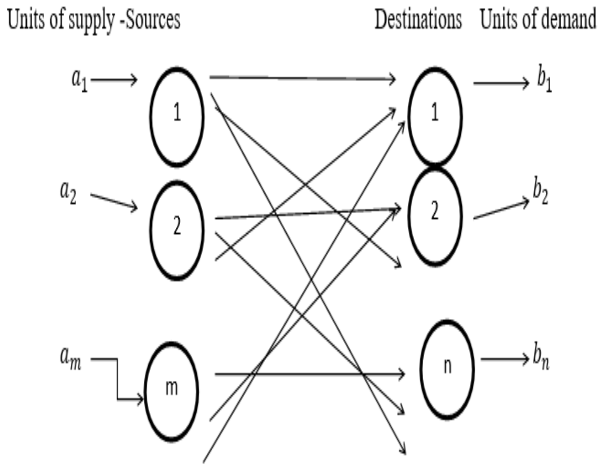

Fig. 1: A network of transport model

Essentially, a transportation problem is one that involves determining the optimal strategy to use the resources of m supply points to meet the needs of n demand sites. While attempting to determine the optimum course of action, the variable cost of shipping the goods from one supply location to another or a comparable constraint should typically be taken into account. A transportation issue is deemed to be balanced if total supply and total demand are equal.

C. Transportation tableau

The costs of shipping, demand, and supply all contribute to defining the transportation dilemma (problem). Hence, the relevant information can be distilled using a transportation tableau. The transportation tableau implicitly expresses the supply and demand thresholds as well as the shipping costs between each demand and supply point. Hence, Transportation Tableau is a unique, statistically-based strategy to the transportation problem (problem).

II. RELATED WORK

An all-encompassing framework efficiency model for moving commodities was created by [4]. It is designated for the delivery of regional less-than-truck loads. Using a semi-Delphi process, specialists in the field provided quantitative data on the size of the partial efficiencies, their dispersion, and their potential for improvement. This has shown a number of elements of road freight transportation that have significant prospects for efficiency growth. The findings of this study demonstrate that a whole spectrum of partial efficiencies can provide a theoretical foundation for an overall physical efficiency model of goods transportation.Similar to other mathematical programming problems, the sensitivity analysis of fuzzy transportation problems has not been studied in the literature. The same problem is addressed using a novel approach in this work. The proposed method is used to solve a numerical example to demonstrate how it works. The sensitivity analysis of fuzzy transportation problems is addressed using a novel method that is based on tabular representation of the problems.

In [2], three models of transportation problem, namely North West Corner Method, Minimum Cost Method and Vogel's approximation Method, wereanalysed. The results of the analysis show that the minimum-cost method finds a better starting solution by concentrating on the cheapest routes, and that the cost of transportation with Vogel's approximation method and Minimum-cost method is less than North-West corner method. Two new methods (based on fuzzy linear programming formulation and classical transportation methods) are proposed to find the fuzzy optimal solution of unbalanced fuzzy transportation problems by representing all the parameters as trapezoidal fuzzy numbers in [5]. The proposed methods are easy to understand and to apply for finding the fuzzy optimal solution of fuzzy transportation problems occurring in real life situations. Ebrahimnejad & Verdegay [6] presented a network-structured transportation problem (TP) considered in an uncertain environment. The main contributions of the paper are fivefold: (1) they convert the formulated IFTP into a deterministic classical LP problem based on ordering of TrIFNs using accuracy function; (2) in contrast to most existing approaches, which provide a crisp solution, they propose a new approach that provides an intuitionistic fuzzy optimal solution; (3) in contrast to existing methods that include negative parts in the obtained intuitionistic fuzzy optimal solution and intuitionistic fuzzy optimal cost, we propose a new method that provides non-negative intuitionistic fuzzy optimal solution and optimal cost; (4) we discuss about the advantages of the proposed method over the existing methods for solving IFTPs; (5) we demonstrate the feasibility and richness of the obtained solutions in the context of two application examples.In [7], a novel ranking function is proposed to finding an optimal solution of fully LR-intuitionistic fuzzy transportation problem by using the distance minimizer of two LR-IFNs. It is shown that the proposed ranking method for LR-intuitionistic fuzzy numbers satisfies the general axioms of ranking functions. Further, we have applied ranking approach to solve an LR-intuitionistic fuzzy transportation problem in which all the parameters (supply, cost and demand) are transformed into LR-intuitionistic fuzzy numbers. The proposed method is illustrated with a numerical example to show the solution procedure and to demonstrate the efficiency of the proposed method by comparison with some existing ranking methods available in the literature.

In order to solve transportation issues, [8]divided the original issue into a number of two-dimensional optimization problems. The goal function's monotonic integer-valued solution method constrains the size of the necessary computation. We now have a system of limitations that can offer all optimal solutions to the original transportation problem, rather than just one. The suggested method performs effectively for transportation issues with throughput restrictions. Unlike the conventional plan improvement method, we simply need to translate the algorithm's fundamental structures in this case. A key network-structured linear programming (LP) topic that arises in many contexts and has received a lot of attention in the literature is the transportation problem. The existing transportation problems only account for one arc's worth of shipping expenses or revenue. Yet, in many real-world applications, many factors are typically taken into consideration while resolving a transportation issue.While each arc contains numerous, out-of-phase inputs and outputs, the work in progress proposes a solution to this issue. The relative efficiency of each possible transportation concept is described. There are two suggested linear programming models for determining the most efficient transportation design. A numerical example was provided to illustrate the strategy's viability. The transportation problem, a significant network-structured LP problem, has been applied to a number of other problems. The cost or profit along an arc is the sole contributing component to the current transportation problems.However, many real-world solutions to transportation problems consider a range of inputs and outputs. The typical transportation issue was broadened in the current work by taking into account a number of inputs and outputs. The suggested method takes relative efficiency into account rather than only cost or profit in order to measure performance for various inputs and outputs of each arc. To find the transportation design with the maximum relative efficiency, a DEA-based strategy is presented. When a decision-maker has multiple goals to achieve for each potential shipment, some of which may conflict with one another, the suggested strategy can be useful.

This study models and assesses the Vogel Approximation and North West Corner transportation models on the logistics of optimal distribution of bags of cement of Dangote Cement Company Plc, Nigeria, from some cement facilities, to various depots while minimizing transportation cost. Optimum reduction in distribution costwould consequently lead to reduction of the price of the commodity to consumers.

III. METHODOLOGY

A. Data Collection and Description

The Data for this study was collected from Dangote Cement, Plc., Nigeria. The data consist of estimate of the distribution quantities and cost of a bag cement from various factories (Ibese, Gboko, Obajana) to some depots (Ibafo, Otta, Idiroko and Abeokuta) as at 17th June, 2014. Dangote Cement Plc was reported to be the largest cement producer in Sub-Saharan Africa, as at time 8th May 2023, with 51.6 MTA total capacity across Africa and 32.25MTA in Nigeria[9].

B. Transportation Model

Subject to supply and demand constraints, transportation models determine the least expensive flow from an origin through a network to a destination. Every node is either a source or a drain in one of the most straightforward lowest cost flow network flow problems (demand). For instance, we may picture a distributor with a number of warehouses and a number of clients, in this case the Dangote Cement mill. Each consumer that is served from a specific plant comes at a price.

➢ *Assumptions in the transportation model*

- Total quantity of the item available at different sources is equal to the total requirement at different destinations.
- Item can be transported conveniently from all sources to destinations.
- The unit transportation cost of the item from all sources to destinations is certainly and precisely known.
- The transportation cost on any given route is directly proportional to the number of units shipped on that route.
- The objective is to minimize the total transportation cost for the organization as a wide and not for individual supply and distribution centers.

➢ *General description of a transportation problem: In general, a transportation problem is specified by the following information:*

- A set of *m supply points* from which a good is shipped. Supply point *i*can supply at most a_i units.
- A set of *n demand points* to which the good is to be shipped.
- Each unit produced at supply point *i*and shipped to demand point *j* incurs a variable cost of c_{ij}.

Let a_i be the number of supply units available at source i (i = 1, 2, 3,… ,m) and let b_j be the number of demand units required at destination j (j = 1, 2, 3,… ,m).

Choosing the right number of units to move from source *i*to destination *j* will ensure that the overall transportation costs are kept to a minimum. Also, the supply restrictions at the sources and the demand specifications at the destinations must both be met precisely.

x_{ij}= number of units shipped from supply point *i*to demand point *j*, then the general formulation of the transportation problem is

$$\min \sum_{i=1}^{i=m} \sum_{j=1}^{j=n} c_{ij} x_{ij} \quad (1)$$

such that

$$\sum_{j=1}^{j=n} x_{ij} \leq a_i \qquad (i = 1, 2, \dots, m) \quad \text{(supply constraints)} \quad (2)$$

$$\sum_{i=1}^{i=m} x_{ij} \geq b_j \qquad (i = 1, 2, \dots, m) \quad \text{(Demand constraints)} \quad (3)$$

$$\text{where } x_{ij} \geq 0 \ (i = 1, 2, \dots, m;\ j = 1, 2, \dots, n) \quad (4)$$

The two set of constraints will be consistent i.e., the system will be balanced if:

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j \quad (5)$$

C. Matrix Terminology

We take the straightforward example of a distributor with three warehouses and six customers to demonstrate the solution method. The transportation problem has a straightforward structure, making it simpler to picture the issue in matrix form as seen below.

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Table 1: Sample Transportation Matrix Tableau

Sources (Plants)	Destinations 1		2		3		4		5		6		Supply
A	x_{11}	c_{11}	x_{12}	c_{12}	x_{13}	c_{13}	x_{14}	c_{14}	x_{15}	c_{15}	x_{16}	c_{16}	a_1
B	x_{21}	c_{21}	x_{22}	c_{22}	x_{23}	c_{23}	x_{24}	c_{24}	x_{25}	c_{25}	x_{26}	c_{26}	a_2
C	x_{31}	c_{31}	x_{32}	c_{32}	x_{33}	c_{33}	x_{34}	c_{34}	x_{35}	c_{35}	x_{36}	c_{36}	a_3
Demand	b_1		b_2		b_3		b_4		b_5		b_6		

D. Northwest Corner Rule (NCR)

To find a basic feasible solution by the Northwest Corner method, we begin in the upper left (or northwest) corner of the transportation tableau and set x_{11} as large as possible. Clearly, x_{11}can be no larger than the smaller of a_1 and b_1. If $x_{11}= a_1$, cross out the first row of the transportation tableau; this indicates that no more basic variables will come from row 1 of the tableau. Also change b_1.to b_1.- a_1. If $x_{11} = b_1$, cross out the first column of the transportation tableau; this indicates that no more basic variables will come from the first column of the tableau. Also changea_1 to $a_1 - b_1$. if $x_{11}= a_1 = b_1$,cross out either row 1 or column 1 (but not both) of the transportation tableau. If you cross out row 1, change b_1 to 0; if you cross out column 1, change a_1 to 0.

Apply this process once more to the tableau's northwest most cell that is not located in a crossed-out row or column. You will eventually reach a situation where there is just one cell that can be given a value. Cross out the cell's row and column and give this cell a value equal to either requirement. Now that a fundamental workable solution has been found.

E. The Vogel Approximation Method (VAM)

This method takes costs into account in allocation. Five steps are involved in applying this heuristic:

- Step 1: Determine the difference between the lowest two cells in all rows and columns including the dummies as the case may be.
- Step 2: Identify the row and column with the largest difference. ties may be broken arbitrarily
- Step 3: Allocate as much as possible to the lowest cost cell in the row or column with the highest difference. if two or more differences are equal allocate as much as possible to the lowest cost cell in these rows or columns.
- Step 4: Stop the process if all row and column requirements are met if not, go to the next step.
- Step 5: Recalculate the differences between the two lowest cells remaining in all rows and columns. Any row and column with zero supply or demand should not be used in calculating further differences, then go to step 2.

The Vogel approximation method (VAM) usually produces an optimal or near optimal solution.

IV. RESULT AND DISCUSSION

The experimental results are presented and discussed.

A. Experimental Results

For effective and a well detailed transportation model we start by showing the costs of a bag of cement which is 50kg in some selected destinations (deports) from plants (sources). Here, it is assumed that the cost of transportation from plants to deports is imbedded in the cost of receiving a bag of cement from plants.

Table 2: Estimate Costs of a bag of Cement per depot as at 17th June, 2014 in Naira

Plant/Depot	Ibafo	Otta	Abule-Egba	Abeokuta	Idiroko
Ibese	₦1652	₦1655	₦1700	₦1655	₦1670
Gboko	₦1752	₦1660	₦1752	₦1700	₦1752
Obajana	₦1800	₦1850	₦1850	₦1780	₦1852

Table 3: Costs of Transportationof a bag of Cement from Plant to Depot in tens of Naira

Plant/Depot	Ibafo	Otta	Abule-Egba	Abeokuta	Idiroko
Ibese	10.16	10.18	10.45	10.18	10.27
Gboko	10.77	11.01	10.76	10.45	10.77
Obajana	11.07	11.38	11.38	10.95	11.39

We proceeded to construct a transportation tableau based on the costs of a bag of cement from a plant against a depot in matrix form taking into account the plants (sources) capacities and requirements (demands) at depots (destinations).

Table 4: Tableau 1.0 - Initial Transport Matrix for Dangote Cement Company (Nigeria)

Sources	Ibafo	Otta	Abule-Egba	Abeokuta	Idiroko	Plants Capacity (bags)
Ibese	10.16	10.18	10.45	10.18	10.27	8000
Gboko	10.77	11.01	10.76	10.45	10.77	10000
Obajana	11.07	11.38	11.38	10.95	11.39	12000
Depot requirements	8000	6000	5000	5000	6000	30000

- *The North-West Corner Rule:*

Since the total number of demand (depot requirements) is equal to the total estimated capacity for the 5 depots the transportation problem is balanced, hence we construct the initial transportation problem using the Northwest Corner Method. There would be no need for a dummy depot. Table.

Table 5: Tableau 1.1 - Allocation of bags of Cement using North-West Corner Rule

Sources	Ibafo	Otta	Abule-Egba	Abeokuta	Idiroko	Plants Capacity (bags)
Ibese	8000	10.18	10.45	10.18	10.27	8000
Gboko	10.77	6000	4000	10.45	10.77	10000
Obajana	11.07	11.38	1000	5000	6000	12000
Depot requirements	8000	6000	5000	5000	6000	30000

The first row means that a total of 8000 bags of cement per day was supplied to Ibafo from Ibese plant and since it satisfies the depot requirements, we move on to the next destination Otta and empty the remainder of the estimated plant capacity. Second row sees the supply of 6000 bags of cement to Otta depot to satisfy the depot requirements etc.

Table 6: Computing the cost of shipment of the assignment

From Plant to Depot	Bags shipped x Per Unit Cost	Total cost (tens of ₦)
Ibese to Ibafo	8000 x 10.16	81280
Gboko to Otta	6000 x 11.01	66060
GbokotoAbule-Egba	4000 x 10.76	43040
Obajana toAbule-Egba	1000 x 11.38	11380
Obajana to Abeokuta	5000 x 10.95	54750
Obajana to Idiroko	6000 x 11.39	68340
	Total Cost	324850

Summing the total cost in tens of Naira we obtain a total of 324, 850*10 = N 3, 248, 500.00.

➢ *Vogel Approximations Method (VAM):*

In using the North-West Rule, the cost of across the row was not take into consideration, it was allocated from the first cell, then to the next adjacent cell and so on until the capacity of the first plant is exhausted and then we moved to next plant down the row and do likewise, we continue this way until all the plants capacities are exhausted and all the depots are satisfied. We now take the costs into consideration by assigning good to cells with least cost in the first row and continue that way until the capacity of the first plant is exhausted and depot(s) is (are) giving allocations. This procedure is followed thereafter, until all the plants capacities are exhausted and all the depots are satisfied. See the table 8.

Table 7: Tableau 1.2 - VAM Matrix for Dangote Cement Company (Nigeria)

Sources	Ibafo	Otta	Abule-Egba	Abeokuta	Idiroko	Plants Capacity (bags)
Ibese	3000	4000	10.45	1000	10.27	8000
Gboko	3000	2000	5000	10.45	10.77	10000
Obajana	2000	11.38	11.38	4000	6000	12000
Depot requirements	8000	6000	5000	5000	6000	30000

Table 8: Computing the cost of shipment of the assignment by VAM

From Source to Destination	**Bags shipped x Per Unit Cost**	**Total cost (tens of N)**
Ibese to Ibafo	3000 x 10.16	30480
Ibese to Otta	4000 x 10.18	40720
Ibese to Abeokuta	1000 x 10.41	10410
Gboko to Ibafo	3000 x 10.77	32310
Gboko to Otta	2000 x 11.01	22020
Gboko to Abule-Egba	5000 x 10.76	53800
Obajana to Ibafo	2000 x 11.07	22140
Obajana to Abeokuta	4000 x 10.95	43800
Obajana to Idiroko	6000 x 11.39	68340
	Total Cost	324020

When we add up the whole transportation costs in Naira, we get 324, 020*10 = N 3, 240, 200.00. We can observe that the total cost of transportation based on the North-West Corner Rule differs by ₦ 8, 300.00 in favor of VAM.

Table 9 and Fig. 1 show the models cost of the distribution of the bags of Dangote Cement from the various plants to depots.

Table 9: Model distribution cost

Model	cost (Naira)
NCR	3248500.00
VAM	3240200.00
Difference	8300.00

The Vogel Approximations Method (VAM) produced lesser cost of distribution than the North-West Corner Rule, with a diffrence of eight thosand, three hundred naira (₦8300.00).

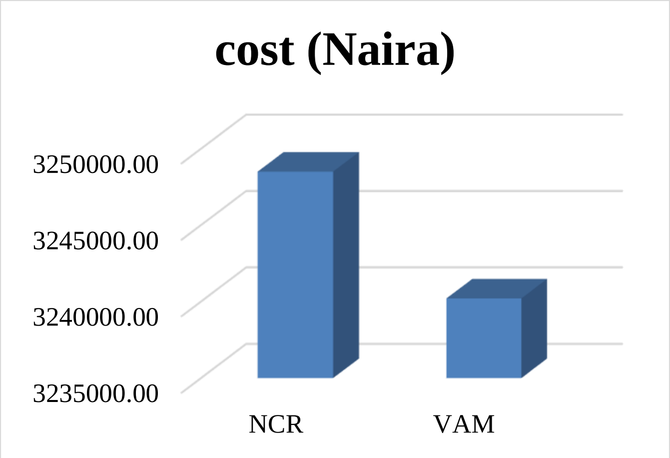

Fig. 1: Model distribution cost

B. *Discussion of Results*

When comparing the initial solution with the ideal solution produced by the Northwest Corner rule and the Vogel Approximation method, we discovered a decrease in the cost of transportation between the two solutions, which is calculated at N8, 300.00. Ibafo Cement Depot should receive 3,000 cement bags from the Ibese facility, as well as Otta receiving 4,000 and Abeokuta receiving 1,000 from the same plant. Gboko needs to distribute 3,000 cement bags to Ibafo, 2,000 to Otta, and 5,000 to Abule-Egba. While the Obajana plant supplies 2000 to Ibafo, 4,000 to Abeokuta, and 6,000 to Idiroko depots, respectively. Overall the Vogel Approximations Method showed a lesser distirbution cost than the Nourth West Corener Rule.

V. CONCLUSION

The study showed a reduction in transportation costs with the use of network flow programming models in the distribution of cement from plants to depots, which undoubtedly will affect the selling price of the commodity to end users. This study has demonstrated that the Northwest Corner solution is not always the best; therefore, it is justified to utilize the Vogel Approximation approach to provide an optimal solution given the little rise in the numbers. Cement is shipped from factories to depots using a linear programming model, which lowers costs and demonstrates the optimal distribution pattern amongst the chosen depots.The suggested optimal model is adaptable and can be changed by adding new constraints to ensure that the resulting solution is reasonable and sufficient for efficient cost distribution and determination. The update of the model to include more depots and various transportation models like the stepping-stone and modified distribution models is now advised.

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