ISSN No:-2456-2165

Revisiting the Calculation of the Moment of Inertia in Stiffening Rings in Cones: A Critical Analysis and New Approach

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Abstract:- The purpose of this study is to examine the calculation of the moment of inertia in stiffening rings in pressure vessel cones, focusing on the application of the Steiner theorem and the correction of equations in the PD5500 and EN Pressure Vessel Codes. The study involves a critical analysis of the existing methods for calculating the moment of inertia, highlighting the errors and limitations in the current equations used by engineers. The research also proposes a new approach that involves the direct application of the Steiner theorem without the need for additional tools such as AutoCAD. The study finds that the third and fourth terms of the equation in the British pressure vessel code (PD5500) have errors due to the absence of the cosine term, which leads to an underestimation of the moment of inertia. By correcting this equation, this study provides a more accurate method for calculating the moment of inertia. This research is original in its critical examination of the existing methods for calculating the moment of inertia in stiffening rings in cones. It not only identifies the errors in the current methods but also proposes a more accurate and efficient approach without the need for additional tools. The findings of this study have practical implications for engineers who design pressure vessels under external pressure. The corrected equation and the proposed method can help engineers perform these calculations more accurately and efficiently, providing safer vessels.

Keywords:- *Moment of Inertia; Stiffening Rings; Cones; Steiner Theorem; Pressure vessel.*

I. INTRODUCTION

Pressure vessels are vital components in numerous industries that play a crucial role in the safe storage, transportation, and transfer of compressed gases and liquids. These vessels consist of hollow cylindrical or spherical shells specifically designed to withstand different pressures. Due to the increasing manufacturing expenses resulting from rising costs of materials and energy, there is a growing trend in specifying thinner vessel walls to reduce weight and costs (Chen et al., 2018). While this leads to lower capital expenditures, it also magnifies the need for reliable structural analysis during design. Thinner shells are more prone to deformations such as buckling if not properly reinforced. This underscores the heightened importance of reliable structural analysis during the design phase.

Engineers frequently employ ring-stiffening techniques to mitigate the risk of buckling in pressure vessels, providing additional structural support. Stiffeners, structural elements attached around a vessel's circumference, enhance its integrity under pressure loads. Proper sizing and positioning of these rings are crucial to preventing catastrophic failures (Spagnoli, 1997). Rasti et al. (2016) focused on the welding of stiffener rings to aluminum cylinders and found that welding generates residual stresses and distortions, impacting structural integrity. Fairushin et al. (2021) explored the use of locally rib-reinforced nozzle assemblies as an alternative to reinforcing rings, emphasizing their technological advantages. Limam et al. (1995) investigated the buckling behavior of thin-walled pressure vessels under complex loading conditions and examined the effects of axial stiffeners.

A fundamental parameter in stiffener sizing is the moment of inertia. This property represents an object’s resistance to bending and twisting forces based on its mass distribution. A higher moment of inertia equates to greater structural strength. Calculating the moment of inertia accurately is critical for engineers performing stress analysis on stiffened pressure vessels to ensure they withstand rated pressures safely throughout their design life (Wickline et al., 2003).

The design of pressure vessel walls adheres strictly to the guidelines provided by industry Codes. These Codes, serving as technical manuals, contain the necessary mathematical formulas for calculating and sizing pressure vessels. It is crucial to rigorously follow their application in vessel calculations, given the hazardous nature of pressure vessels. The most frequently employed codes include ASME VIII (ASME 2021), the European Standard EN 13445 for Unfired Pressure Vessels (European Standards 2021), the British Code PD5500 (BSI 2021), and the German Code AD2000 (AD2000-Merkblatt 2021).

While reviewing international pressure vessel codes for guidance on calculating the moment of inertia, the authors identified apparent shortcomings. Two major pressure vessel codes, the US ASME VIII and the German AD2000, only mention that the moment of inertia of the cone-stiffener cross-section should be calculated without providing further details (refer to Table 1). In contrast, other primary standards such as the European EN13445 and British PD5500 codes provide a formula for moment of inertia calculation.

While conducting a comprehensive review of international pressure vessel codes to gather guidance on the calculation of moment of inertia, it became evident that certain shortcomings exist. Notably, two prominent pressure vessel codes—ASME VIII and the German AD2000—merely mention calculating the moment of inertia for cone-stiffener cross sections without providing sufficient details. In contrast, standards such as European EN13445 and British PD5500 offer more comprehensive formulas for this crucial calculation. However, the formulas provided by EN13445 and PD5500 only apply to T-type cone stiffeners and not to other geometries commonly encountered in practice (Kumar et al., 2018; Zhu et al., 2015). To address these limitations and present a more comprehensive approach, the authors of this scientific paper aimed to develop a generalized methodology for calculating the moment of inertia in stiffening rings within pressure vessel conical sections.

II. THEORETICAL BACKGROUND

Accurate calculation of the moment of inertia is essential to ensure the structural integrity and safety of pressure vessels. The moment of inertia, a property of an object, describes its resistance to changes in rotational motion. In the context of pressure vessels, determining the moment of inertia is particularly crucial for assessing the stiffness and strength of stiffening rings in conical sections. These rings play a vital role in maintaining the structural integrity of the pressure vessel, especially under external loads and pressure fluctuations.

To accurately calculate the moment of inertia in stiffening rings in pressure vessel conical sections, the authors reviewed existing codes and handbooks for guidance. Additionally, the authors conducted a thorough analysis of the geometric properties and structural behavior of stiffening rings in conical sections. Recent improvements in the EU pressure vessel Codes for cones have focused on providing more detailed guidance for stress analysis, as well as improving accuracy and consistency in design calculations. Table 1 compares the cone section stiffener calculation methods provided in ASME VIII, AD2000, EN13445, and PD5500.

Table 1: Comparison of Cone Section Stiffener Calculation Methods in Pressure Vessel Codes

Component	**ASME VIII**	**EN13445-3**	**AD2000**	**PD5500**
Cone sections	UG-32, UG-33, UG-36 & Appendix 1	7.6 & 8.6	B2	3.5.3 & 3.6.3
Stiffener calculation	None	MoA formula	None	MoA formula

The primary concern identified in calculating cone sections under external pressure lies in the limitation of the formulas provided by EN13445 and PD5500. These formulas are only applicable to a specific type of stiffener, namely the tee-type as illustrated in Figure 1. Consequently, a comprehensive investigation was necessary to determine accurate methodologies for calculating the moment of inertia of other cross-sections of stiffeners on conical parts, as depicted below. Through extensive analysis, the application of Steiner's Theorem emerged as a suitable approach to effectively address this issue.

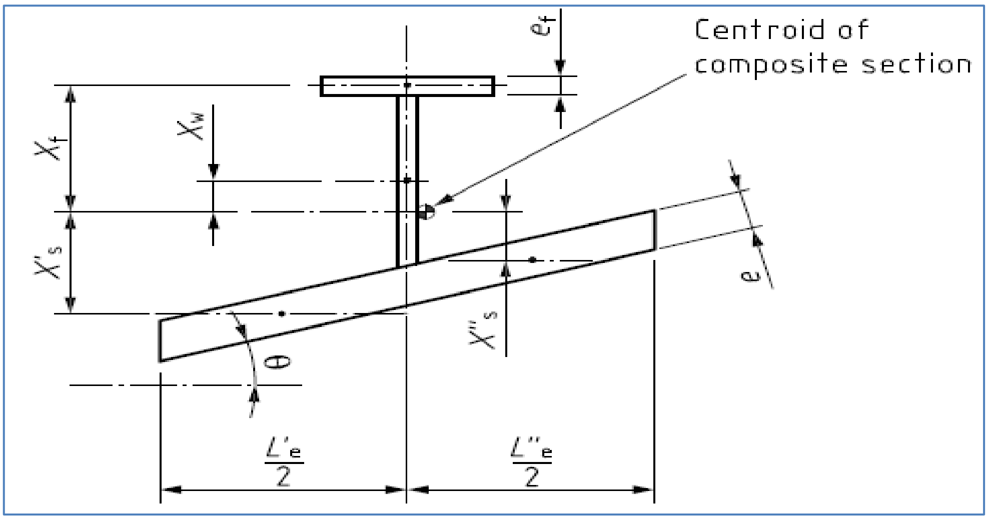

Fig. 1: Tee-type stiffener cross-section. The inclined parallelogram represents the cone section, the vertical rectangular section represents the web of the tee-type stiffener and the horizontal rectangular section represents the flange of the tee-type stiffener

The EN13445 and PD5500 codes exclusively furnish a moment of inertia formula for tee-type stiffeners, as depicted in the following equation. It is crucial to acknowledge, however, that the formula stipulated in these codes possesses limitations in its applicability. Notably, it does not encompass other varieties of stiffeners frequently employed in pressure vessel conical sections.

$$I_c = A_f X_f^2 + A_w X_w^2 + \left(\frac{eL'_e}{2}\right)(X'_s)^2 + \left(\frac{eL''_e}{2}\right)(X''_s)^2 + I_f + I_w + \frac{e}{12}\sin^2\theta\left[\left(\frac{L'_e}{2}\right)^3 + \left(\frac{L''_e}{2}\right)^3\right] + \frac{e^3}{12}\cos^2\theta\left(\frac{L'_e}{2} + \frac{L''_e}{2}\right)$$

Where Af represents the area of the flange, Aw is the area of the web, If is the second moment of area of the flange about its own centroid, and Iw is the second moment of area of the web about its own centroid. The remaining geometric parameters are illustrated in Figure 1.

Steiner's Theorem, also recognized as the parallel axis theorem, offers a methodology for determining the moment of inertia of an object concerning an axis parallel to, and at a specified distance from, the object's centroid or principal axis. When utilized in conjunction with axis transformation equations, it establishes a comprehensive framework that empowers engineers to precisely calculate the moment of inertia for various geometries. The application of Steiner's Theorem and axis transformation equations facilitates the accurate computation of the moment of inertia for complex stiffener geometries. This approach is particularly valuable for the precise evaluation of the moment of inertia in pressure vessel conical sections featuring stiffening rings.

According to the Steiner theorem, a tee stiffener cone section can be represented by three distinct simple shapes: a parallelogram representing the cone wall, and two rectangles – one for the web of the stiffener and one for the flange of the stiffener. For each of these shapes, it is necessary to calculate the moment of inertia to its center of gravity, as well as its area and the distance between the axes of the center of gravity of the shape and the axis of calculation. Along with determining the angle between the center of gravity axis and the axis of calculation, a comprehensive set of equations is established to calculate the total moment of inertia of the stiffener in relation to the cone axis, as stipulated by the pressure vessel code.

$$I = I_{centroid} + Ad^2$$

$$I_{x'} = I_x \cos^2\theta + I_y \sin^2\theta - I_{xy} \sin 2\theta$$

$$I_{y'} = I_x \sin^2\theta + I_y \cos^2\theta - I_{xy} \sin 2\theta$$

$$I_{x'y'} = \frac{1}{2}(I_x - I_y)\sin 2\theta + I_{xy} \cos 2\theta$$

Where,

- I is the moment of inertia about any axis.
- $I_{centroid}$ is the moment of inertia about a parallel axis through the centroid.
- A is the area of the shape.
- d is the distance between the two axes.
- θ is the cone angle

III. METHODOLOGY

The methodology employed in this study involves utilizing Steiner's Theorem and axis transformation equations to calculate the moment of inertia in stiffening rings within conical sections of pressure vessels. The initial phase of the analysis entails identifying various shapes constituting the stiffening ring, such as the cone wall, the stiffener's web, and flange. This identification process is followed by assigning each term in the PD5500 moment of area formula to the tee-section geometry. A closer examination of the PD5500 moment of area formula reveals the connection between Steiner's theorem and the formula itself.

A more in-depth examination of the PD5500 moment of area formula reveals the connection between the Steiner theorem and the formula itself. Specifically, the first term of the PD5500 formula represents the flange's parallel transformation moment of inertia, while the second term represents the web's parallel transformation moment of inertia. The third and fourth terms signify the parallel transformation of the two symmetrical pieces of the cone section. The remaining terms denote the primary moments of area with respect to each component's centroid (term 5 for the flange, term 6 for the web, and terms 7 and 8 for the cone sections).

To illustrate the calculation methodology, consider the geometrical details of a specific case outlined in Table 2. Assume that a tee-type stiffener is positioned on a cone section at a fifteen-degree angle with a thickness of 20 mm. The web portion of the stiffener measures 91 mm in length and has a thickness of 9 mm, while the flange is 15 mm thick and spans a length of 200 mm. Calculating simply, we find that the flange area totals 3000 mm², while the web area is 819 mm². In terms of centroid position (center of mass), it can be determined that this point lies approximately 41.12 mm away from the axis under consideration.

Table 2: Geometry parameters of tee-type stiffener example

Geometrical feature	Symbol	Value
Cone Angle (degrees)	θ	15°
Cone thickness (mm)	e	20
Cone length (mm)	L_e	400
Flange cross-section (mm x mm)	e_f x L_f	15 x 200
Web cross-section (mm x mm)	e_w x L_w	9 x 91

Applying the Steiner theorem and the Code formula results in a distinct calculation of the moment of inertia for both the flange and the web.

For Flange (I_f): $15^3 \times 200/12 = 56{,}250$ mm^4
For Web (I_w): $91^3 \times 9/12 = 565{,}178$ mm^4

ISSN No:-2456-2165

The positional values for the flange, the web, and the cone (see Figure 1) are calculated. By applying the Code formula, a specific moment of inertia for the entire cross-section is determined.

- **Positional values:**
  X_f: 78.09 mm X_s': 57.56 mm
  X_s'': 3.97 mmX_w: 25.09 mm

- **Applying the Code formula:**
  $I_c = 3{,}000 \times 78.09^2 + 819 \times 25.09^2 + 4{,}000 \times 66.68^2 + 4000 \times 6.12^2 + 56{,}250 + 565{,}178 + \cdots = 34{,}780{,}742.28\ mm^4$

From the previous calculation, it is evident that both the PD5500 and EN13445 Codes apply the Steiner theorem to formulate the moment of inertia calculation. After a comprehensive examination of the formula specified in the code and its comparison with the geometry, it has been determined that the pressure vessel Code simplifies cone section geometry by representing it as a rectangle (Figure 2). Despite the calculation being intended for the cone's central axis (axis X), this simplification remains valid and does not result in significant errors as long as the cone angle remains within a few degrees. Under this approach, the conical section is represented by its actual parallelogram-shaped geometry, and Steiner's theorem is applied again to calculate the moment of inertia. The application of the Steiner theorem to these shapes yields different results, as illustrated in Figure 2.

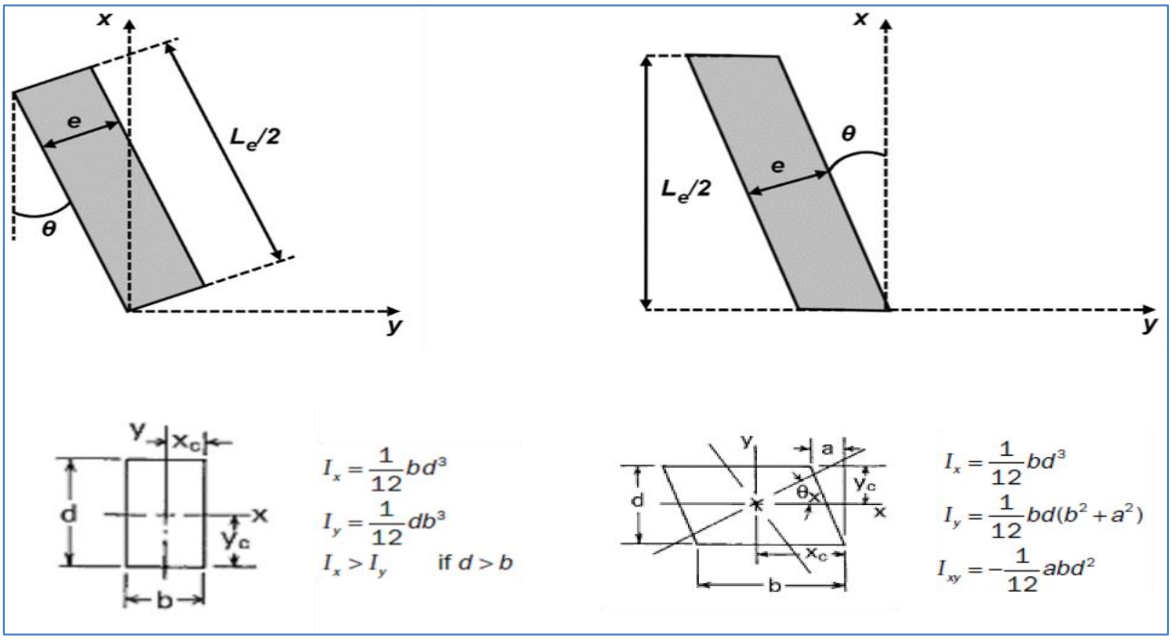

Fig. 2: (Left) Simplified cone section geometry representation by Code and (right) and the actual geometry as proposed in this study along with their respective moment of area equations

Through a detailed analytical examination of the application of Steiner's theorem in pressure vessel codes, differences have been identified in the treatment of conical sections. Specifically, it was observed that terms three and four of the Code formula omit an essential cosine factor associated with the true parallelogram cross-section shape of cones. These terms are indicative of the Cone Section parallel transformation.

The implication of this simplification is the underestimation of the moment of inertia due to the absence of the cosine term. In order to rectify this issue, a modification is proposed, involving the direct application of Steiner's theorem. This modification utilizes the accurate geometry as defined by the cone angle, rather than relying on a simplified rectangular approximation. Furthermore, it is noteworthy that the formula provided by the British and European codes is exclusively valid for tee-sections. Consequently, current design practices involve meticulous cross-section design, with designers utilizing CAD tools such as AutoCAD's MASSPROP function (see MASSPROP 2023 in the Help section) to calculate the accurate moment of inertia.

IV. RESULTS AND DISCUSSION

After thoroughly examining the disparities between the Code formula and the suggested approach, we conducted a comprehensive quantitative analysis to compare their performance. The comparison was executed by utilising the geometric parameters outlined in Table 2. The proposed approach replaces the British/European formula with a direct application of the Steiner theorem. By considering all aspects of the cone section's geometry, including its mitered cone edges, we achieve significantly more accurate results.

In the methodology section, an example is considered wherein a cone angle of 15° is examined. The utilization of AutoCAD's MASSPROP function enables the calculation of the moment of inertia based on actual measurements conducted by engineers employing this well-established engineering tool. A comparison between the calculated moment of inertia and that obtained using the Code formula reveals an absolute relative error exceeding 2%. However, by employing the proposed solution, which yields results analogous to those obtained through advanced CAD tools like AutoCAD but without the approximation errors introduced by existing formulas or algorithms commonly employed elsewhere, the respective absolute relative error is lower than 0.006% (see Table 3 and Table 4). Based on these outcomes, it can be confidently stated that this study presents an analytically exact solution comparable to what industry-standard software offers.

Table 3: Moment of Area calculation for the tee-type stiffener according to Steiner theorem for the geometry example described in Table 2

Stiffener part	**Area A**	**c.m X**	**X²**	**AX**	**AX²**	**Inertia I**
Web	819.00	45.50	2,070	37,265	1,695,535	565,178
Flange	3,000.00	98.50	9,702	295,500	29,106,750	56,250
Σ { A } = As	**3,819.00**	mm²	Sums:	**332,765**	**30,802,285**	621,428
c.m = Xs = Sum AX / As =		**87.13**		mm		
Is = Sum AX² + Sum I - C Sum AX		**2,428,632**		mm⁴		

Table 4: Entire cross-section calculation for the cone-stiffener composite geometry according to geometry example described in Table 2.

Part	**Area A**	**Centroid X**	**X²**	**AX**	**AX²**	**Inertia I**
Cone,top	4,141.10	-16.44	270	-68,089	1,119,525	1,139,007
Cone,bottom	4,141.10	37.15	1,380	153,832	5,714,518	1,139,007
Tee-stiffener	3,819.00	107.84	11,629	411,839	44,412,484	2,428,632
Sum:	12,101.21			497,583	51,246,527	4,706,647
C = Sum AX / Sum A =			**41.118**		mm	
I = Sum AX² + Sum I - C Sum AX			**35,493,361**		mm⁴	

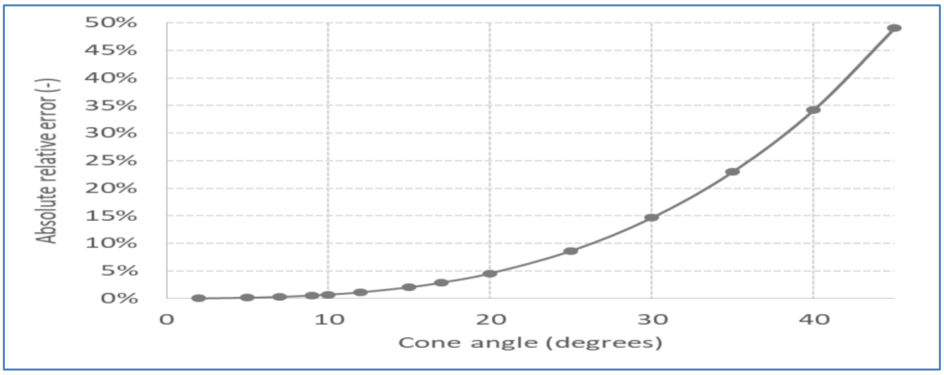

Fig. 3: Difference in the composite geometry moment of inertia calculation between the PD5500/EN13445 formula and the exact solution, expressed as absolute relative error

To demonstrate the significance of the proposed improvement in the calculation of moment of inertia, detailed CAD models of sample stiffened cones were developed across a range of typical design angles, from 2 to 45 degrees. The moments of inertia for these models were then calculated. The proposed method was applied, and the results were compared to the Code formula. Figure 3 illustrates the absolute relative error of the Code formula compared to the analytical solution, which is identical to the proposed solution. For the same tee-section stiffener, the two approaches exhibit an exponential difference in moment of inertia calculations for cone angles between 2° and 45°. For cone angles up to 10°, the absolute relative error between the two approaches is less than 1%. However, for larger cone angles, the Code formula severely underestimates the moment of inertia, by up to 50% for a 45° cone angle. This underestimation results in increased material costs and pressure vessel weight. To exemplify the material increase required for a cone angle of 20°, in order to match the moment of inertia provided by the proposed solution, the conventional approach would need to increase either the cone thickness, flange thickness, or flange length. Specifically, the cone thickness would need to increase by 6.8%, the flange thickness by 8.87%, and the flange length by 10.6%.

V. CONCLUSIONS

In conclusion, this scientific paper has presented a method for calculating the moment of inertia in stiffening rings within pressure vessel conical sections. By employing the proposed analytical solution, accurate results can be obtained without introducing the approximation errors commonly associated with existing formulas or algorithms. The analysis of the proposed method, applied to a range of sample stiffened cones, demonstrated its superiority compared to the industry-standard Code formula. The proposed method exhibited a significant improvement in the accuracy of moment of inertia calculations for cone angles ranging from 2° to 45°. The absolute relative error between the proposed method and the analytical solution is nearly zero, while the Code formula severely underestimated the moment of inertia, resulting in an absolute relative error of up to 50% for a cone angle of 45°. These findings underscore the importance of using the proposed method to avoid unnecessary material costs and weight increase in pressure vessels. With the proposed method, vessel geometries can be represented more accurately, enabling engineers to achieve performance targets more optimally without compromising safety margins through proper stiffener sizing.

VI. GRANT INFORMATION/FUNDING

This work has been co-financed by the European Union and Greek national funds through the Operational Program Competitiveness, Entrepreneurship and Innovation, under the call RESEARCH – CREATE – INNOVATE(project code: T2EDK-04885).

European Union
European Regional Development Fund
EPAnEK 2014-2020
OPERATIONAL PROGRAMME
COMPETITIVENESS
ENTREPRENEURSHIP
INNOVATION
ΕΣΠΑ 2014-2020
Partnership Agreement 2014 - 2020

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