ISSN No:-2456-2165

Modelling a Smart Structure with Sliding Mode Controller for Vibration Suppression

Sunith. S[1]
Dept. of Electrical Engineering
College of Engineering Trivandrum
Trivandrum, Kerala, India

R. Lethakumari[2]
Dept. of Electrical Engineering
College of Engineering Trivandrum
Trivandrum, Kerala, India

Abstract:- Modern satellites need sophisticated instruments for remote sensing with microlevel accuracy. This paper features the modelling and design of a cantilever beam made of aluminium with piezoelectric sensor and actuator at its free end and a comparative study of the sensor output voltage with a PID controller as well as Sliding Mode Controller is being done. In earlier methods self-tuning sensors and actuators were used. But it requires coating of the entire beam with piezo film which is much complex. Also, the output of the sensor will not be satisfactory. Since the accuracy is much important in remote sensing applications, separate piezoelectric sensors and piezoelectric actuators are used by replacing the piezo film coated beam. This work includes design of PID and SMC for suppressing the vibration with in desirable limits against an external disturbance to the system.

I. INTRODUCTION

Active vibration control can be considered as one of the major problems in the study of structures. The best way to solve this problem is by making the structure smart intelligent, self-controlling and adaptive. The automatic modification of the systems physical structure is the key point in active vibration control. The importance of suppressing the vibrations is that it may otherwise affect the stability and performance of the structure. For achieving this objective, the need of a smart structure arises. Smart structure means a structure which is self-contained such that it should have the strength to withstand dynamic forces and disturbances in order to minimize the amount of vibration [1]. Being a smart structure most preferably it should be cheaper structures exhibiting superior performance. A system designer's major challenge occurring in the field of vibration control. As an optimal solution to this problem sensor's and actuators to be integrated with the structure and hence it can be called as a smart structure. The advantage of using piezoelectric materials are, they are capable of altering response of the structure through sensing, actuation and control. These piezoelectric sensors and actuators can be directly embedded in to the host structure by mounting it to its surface.

The vibration control system mainly consists of four parts, a sensor, actuator, controller and the system. When an external force is applied to a simple cantilever beam it will undergo vibrations. These vibrations should be suppressed. To know the amount of deformation first measure the displacement using piezo sensors and actuators like piezoelectric, piezoceramics, PVDF, PZT, etc. can be used to produce a secondary vibration against the previous one due to force [4][5]. This causes a destructive interference with original response of the system which intact a result of the primary source of vibration.

The theoretical background can be explained by Euler Bernoulli beam theory and Timoshenko Beam theory. Modelling, controlling and implementation of smart structures using piezo materials can be explained only by using these two fundamental theories. For classical beam model in Euler-Bernoulli beam theory the assumption is that before and after bending, the plane cross section of the beam remains plane and normal to the neutral axis [1]. Since the axial displacement and shear force are neglected in Euler Bernoulli theory, the results may be slightly inaccurate. For overcoming these limitations Timoshenko Beam theory is used, as it considers the effect of axial and shear displacements [2][3].

In Timoshenko Beam theory, cross sections remain plane and rotate about the same neutral axis as in Euler Bernoulli model, but do not remain normal to the deformed longitudinal axis. The deviation from the normal structure

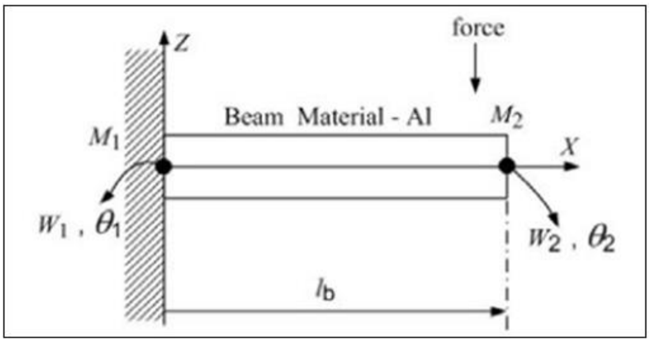

Fig 1 A Regular Flexible Beam

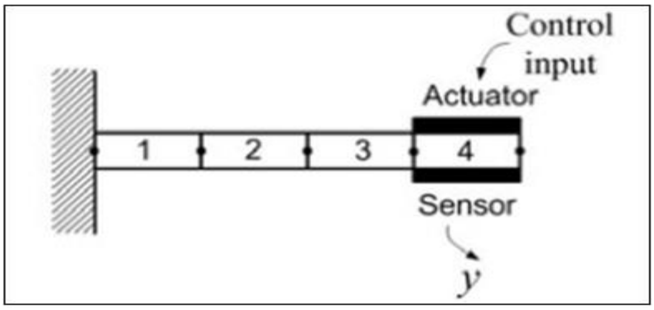

Fig 2 A Small Flexible Beam with Piezo Sensor and Actuator Placed at the Free end

Is produced by a transverse shear that is assumed to be constant over the cross section. Total slope of the beam model is contributed by bending(θ) and other due to shear(β) . Hence Timoshenko Beam theory is considered superior for precisely predicting the beam response.[5][6]

II. MATHEMATICAL MODELLING OF SMART BEAM

Consider a flexible Aluminium cantilever beam as shown in figure 1. The piezo electric sensor and actuator are bonded at the tip of the beam as shown in figure 2.

Table 1 Physical Parameters

Parameter (With Units)	Symbol	Numerical Values
Total Length(m)	*lb*	0.3
Width(m)	b	0.024
Density(kg/m^3)	δ_b	8030
Young's Modulus (GPa)	E_b	193.06
Constants used in C^*	α,β	0.001,0.0001
Thickness(mm)	t_b	1

- *Finite Element Modelling of beam Element*

The x-axis is considered as the longitudinal axis of the regular beam element as shown in figure 1. The beam element has constant modulus of elasticity, moment of inertia, length and mass density. The displacement relations of the regular beam in x,y,z direction will be [Friedman and Kos Mataka, 1993]

Table 2 Properties of the (PZT) Piezo Sensor/Actuator

Parameter (With Units)	Symbol	Numerical Values
Length(m)	*lp*	0.075
Width(m)	b	0.024
Thickness(mm)	*ta,ts*	0.5
Young's Modulus(GPa)	E_p	68
Density(kg/m^3)	δ_p	7700
Piezo stress constant ($V\,mN^{-1}$)	$g31$	10.5×10^{-13}
Piezo strain constant (m/V)	$d31$	125×10^{-12}

$$u(x,y,z,t) = z\theta(x,t) = z[\frac{\partial w}{\partial x} - \beta(x)] \quad \text{...........1}$$

$$v(x,y,z,t) = 0 \quad \text{..................................2}$$

$$w(x,y,z,t) = w(x,t) \quad \text{........................3}$$

Where w is the transverse displacement along z axis, θ is the time dependent rotation of the beam about y-axis , u is the axial displacement along x-axis, v is the lateral displacement along y-axis, which is equal to zero. Also w_1,θ_1 and w_2,θ_2 are the DOF's at the fixed end and free end respectively. θ_2 is measured in clockwise direction and calculated according to the sensor output voltage as it is being proportional with the displacement in y direction. The total slope of the beam is constituted by $\theta(x)$ and $\beta(x)$, which is bending and shear stress respectively. The strain energy of the beam element is given by

$$U = \frac{1}{2}\int_0^{lp} \begin{bmatrix} \frac{\partial \theta}{\partial x} \\ \frac{\partial w}{\partial x} + \theta \end{bmatrix}^T \begin{bmatrix} EI & 0 \\ 0 & KGA \end{bmatrix} \begin{bmatrix} \frac{\partial \theta}{\partial x} \\ \frac{\partial w}{\partial x} + \theta \end{bmatrix} dx \quad \text{...4}$$

A is the area of cross-section of the beam, K is the shear coefficient which is equal to $\frac{5}{6}$ [cooper, 1966]. Total kinetic energy can be written as

$$T = \frac{1}{2}\int_0^{lp} \begin{bmatrix} \frac{\partial w}{\partial t} \\ \frac{\partial \theta}{\partial t} \end{bmatrix}^T \begin{bmatrix} \delta A & 0 \\ 0 & \delta I \end{bmatrix} \begin{bmatrix} \frac{\partial w}{\partial t} \\ \frac{\partial \theta}{\partial t} \end{bmatrix} dx \quad \text{................(5)}$$

Where δ is the mass density. The total work due to external forces in the system is given by

$$W_e = \int_0^{lb} \begin{bmatrix} w \\ \theta \end{bmatrix}^T \begin{bmatrix} q_d \\ m \end{bmatrix} dx \quad \text{..............................(6)}$$

Where q_d is distributed force at the free end and m represent moment along the beam length.

$$\delta \prod = \int_{t1}^{t2} (\delta U - \delta T - \delta W_e)dt = 0 \quad \text{.....................(7)}$$

Substituting the values of strain energy from Eq.(4), kinetic energy from Eq.(5) and external work done from Eq.(6)in Eq.(7) and integrating by parts , the result obtained will be the differential equations of motion of a general shaped beam modelled with Timoshenko beam theory as

$$\frac{\partial KGA(\frac{\partial w}{\partial x}+\theta)}{\partial x}+q_d=\delta A\frac{\partial^2 w}{\partial t^2} \quad \text{...(8)}$$

$$\frac{\partial EI\frac{\partial \theta}{\partial x}}{\partial x}-KGA(\frac{\partial w}{\partial x}+\theta)+m=\delta I\frac{\partial^2 \theta}{\partial t^2} \quad \text{...(9)}$$

For static case having no external force applied to the beam RHS of Eq.(8) and Eq.(9) will be zero, then differential equation of motion will be

$$\frac{\partial KGA(\frac{\partial w}{\partial x}+\theta)}{\partial x}+q_d=0 \quad \text{...(10)}$$

$$\frac{\partial EI\frac{\partial \theta}{\partial x}}{\partial x}-KGA(\frac{\partial w}{\partial x}+\theta)+m=0 \quad \text{...(11)}$$

The Eq.(11) obeys Timoshenko beam theory only when the polynomial order for w is selected one order higher than the polynomial order for θ[Manjunath and Bandyopadhyay, 2005] For the modelling of regular beam in FEM method the inevitable thing is the mass matrix of the beam , which will be the sum of rotational mass and translational mass and is given by,

$$[M^b]=\int_0^{l_b}\begin{bmatrix}[N_w]\\ [N_\theta]\end{bmatrix}^T\begin{bmatrix}\delta A & 0\\ 0 & \delta I_{YY}\end{bmatrix}\begin{bmatrix}[N_w]\\ [N_\theta]\end{bmatrix}dx \quad \text{...12}$$

Where $[N^w]$, $[N^\theta]$ are shape functions from [1] and from this by integrating we get

$$[M^b]=[M_{\delta A}]+[M_{\delta I}] \quad \text{...13}$$

The next foremost important thing is the stiffness matrix, which is the sum of shear stiffness and bending stiffness and is given by

$$[K^b]=\frac{EI}{(1+\phi)l_b^3}\begin{bmatrix}12 & 6L & -12 & 6l_b\\ 6l_b & (4+\phi)l_b^2 & -6l_b & (2-\phi)l_b^2\\ -12 & 6l_b & 12 & -6l_b\\ 6L & (2-\phi)l_b^2 & -6l_b & (4+\phi)l_b^2\end{bmatrix} \quad \text{...14}$$

Where φ is the ratio of the beam bending stiffness to shear stiffness and is given by

$$\phi=\frac{12EI}{l_b^2KGA} \quad \text{...(15)}$$

➢ *FEM of Piezoelectric Beam Element*

The regular beam with piezo sensor and actuator is shown in figure 2. The bottom layer acts as the sensor and upper layer will be the actuator. Due to very thin size the effect of the shear is negligible in these piezo patches. So piezo modelling is purely based on Euler-Bernoulli beam theory. From [1][2] the mass matrix is given by

$$[M^p]=\frac{\delta_p A_p l_p}{420}\begin{bmatrix}156 & 22l_p & 54 & -13l_p\\ 22l_p & 4l_p^2 & 13l_p & -3l_p^2\\ 54 & 13l_p & 156 & -22l_p\\ -13l_p & -3l_p^2 & -22l_p & 4l_p^2\end{bmatrix} \quad \text{...16}$$

Where δ_p is the mass density of piezoelectric beam element, A_p is the area of the piezoelectric patch and l_p is the length of piezoelectric patch. The stiffness matrix can be obtained as [1][2]

$$[K^p]=\frac{E_pI_p}{l_p}\begin{bmatrix}\frac{12}{l_p^2} & \frac{6}{l_p} & \frac{12}{l_p^2} & \frac{6}{l_p}\\ \frac{6}{l_p} & 4 & \frac{-6}{l_p} & 2\\ \frac{-12}{l_p^2} & \frac{-6}{l_p} & \frac{12}{l_p^2} & \frac{-6}{l_p}\\ \frac{6}{l_p} & 2 & \frac{6}{l_p} & 4\end{bmatrix} \quad \text{...(17)}$$

Where E_p is the modulus of elasticity of piezo material and I_p is the moment of inertia of the piezoelectric layer.

➢ *Piezoelectric Sensors and Actuators*

- *Piezoelectric Sensor Equation*

$$D=dT+\varepsilon^T E \quad \text{...(18)}$$

Where T is the Stress, E is the electric field and S will be the Strain. D can be considered as the di-electric displacement. The other parameters are ϵ, S^E and d which is the permittivity of the medium, compliance of the medium and piezoelectric constant respectively.

The strain in the structure is calculated by using the direct piezoelectric equation. The strain acting on the sensor will be directly proportional to the electric displacement developed on the sensor surface, since no external field is applied to the sensor layer. If we assume that the poling is done along the z-direction of the sensor. i.e. along the thickness direction of the sensor. Then the electric displacement can be written as,

$$D_z = d_{31}E_p\epsilon_x = e_{31}\epsilon_x \quad \ldots\ldots(19)$$

Where E_p is the young's modulus, e_{31} is the piezo electric stress/charge constant and $_x$ is the strain of the testing structure at a point on the beam. The total charge developed on the sensor surface Q(t) will be the summation of all point charges developed in the sensor layer. The current developed is given by

$$i(t) = ze_{31}c\int_0^{l_p} N_a^T \dot{q}dx \quad \ldots\ldots(20)$$

Where $z = \frac{t_b}{2} + t_a$, N_a^T is the second spacial derivative of the mode shape function of the beam. Open circuit voltage V^s can be obtained from the current with the help of a signal conditioning device with gain G_c and applies to an actuator with a controller gain K_c. hence actuator input voltage will be

$$V^s = K_cG_cze_{31}c\int_0^{l_p} N_a^T \dot{q}dx \quad \ldots\ldots(21)$$

- *Piezoelectric Actuator Equation*

$$S = S^ET + dE \quad \ldots\ldots(22)$$

Strain developed by the application of electric field E_f on actuator layer is given by

$$\epsilon_d = d_{31}E = d_{31}\frac{V^a(t)}{t_a} \quad \ldots\ldots(23)$$

The stress developed in the thickness direction will be

$$T_A = E_pd_{31}\frac{V^a(t)}{t_a} \quad \ldots\ldots(24)$$

The resultant moment is given by

$$M_A = E_pd_{31}\bar{z}V^a(t) \quad \ldots\ldots(25)$$

Where $\bar{z}$ is the distance between neutral axis and piezoelectric layer of the beam. The control force applied by the actuator is given by

$$f_{ctrl} = E_pd_{31}\bar{z}\int_0^{l_p} N_\theta dxV^a(t) \quad \ldots\ldots(26)$$

$[N_\theta]^T$ is the first spacial derivative of mode shape function of the beam. If an external impulse disturbance acts on the beam then total force

$$ft = fext + fctrl \quad \ldots\ldots(27)$$

- *Dynamic Equation and State Space Model of the Smart Structure*

The dynamic equation can be found out from the matrices in Eq.(13), Eq.(14), Eq(16) and Eq(17). Hence the assembled matrices M and K are obtained. The equation of motion will be

$$M\ddot{q} + Kq = f_{ext} + f_{ctrl} = f \quad \ldots\ldots(28)$$

Where M is the global mass matrix, K is the global stiffness matrix ,f_{ext} is the external force applied to the beam, f_{ctrl} is the controlling force by the actuator. the generalised coordinate transformation is given by $q = Tg$, which gives the first two vibratory modes. T is the modal matrix containing the eigen vectors representing the first two vibratory modes. q and g are the generalised and principal co-ordinates respectively. Now Eq.(28) becomes

$$MT\ddot{g} + KTq = f_{ext} + f_{ctrl} \quad \ldots\ldots(29)$$

Multiplying by T^T on both sides and simplifying we get

$$M^*\ddot{q} + c^*\dot{g} + K^*g = f^*_{ext} + f^*_{ctrl} \quad \ldots\ldots(30)$$

$$g = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \dot{g} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} x_3 \\ x_4 \end{bmatrix} \quad \ldots\ldots(31)$$

Hence from Eq,(31) $\dot{x}_1 = x_3, \dot{x}_2 = x_4$ which can be further simplified as

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & I \\ -(M^*)^{-1}K^* & -(M^*)^{-1}C^* \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ (M^*)^{-1}T^Th \end{bmatrix}u(t) + \begin{bmatrix} 0 \\ (M^*)^{-1}T^Tf \end{bmatrix}r(t) \quad ..(32)$$

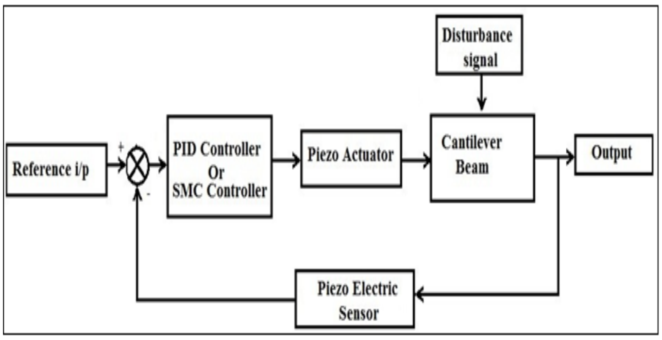

Fig 3 System Block Diagram with PID and SMC

$$\dot{X} = Ax(t) + Bu(t) + Er(t) \quad \ldots\ldots(33)$$

Sensor voltage is taken as the output of the system

$$y(t) = \begin{bmatrix} 0 & P^T & T \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \quad \ldots\ldots(34)$$

$$y(t) = C^Tx(t) + Du(t) \quad \ldots\ldots(35)$$

From Eq.(33) and Eq.(35), we get state equation and the output equation with

$$A = \begin{bmatrix} 0 & I \\ -(M^*)^{-1}K^* & -(M^*)^{-1}C^* \end{bmatrix} B = \begin{bmatrix} 0 \\ (M^*)^{-1}T^T h \end{bmatrix} \qquad \text{..........(36)}$$

$$C^T = [0 \quad P^T] \; D = Null\ matrix \; E = \begin{bmatrix} 0 \\ (M^*)^{-1}T^T f \end{bmatrix} \qquad \text{..........(37)}$$

Where $r(t)$, $u(t)$, A, B, C, D, E, $x(t)$, $y(t)$ represents the external force input, the control input, system matrix, input matrix, output matrix, transmission matrix, external load matrix, state vector and the system output (sensor output).

III. CONTROL SYSTEM DESIGN

➢ *PID Controller*

The block diagram of a specified PID controller in a closed loop system is shown in figure 3. The Output of the PID controller is given by following equations.

$$u(t) = K_p e(t) + \frac{1}{T_i}\int_0^t e(t)dt + Td\frac{e(t)}{t} \qquad \text{..................(38)}$$

The transfer function of a PID controller is

$$G_{PID(s)} = \frac{U(s)}{E(s)} = K_p + \frac{K_i}{s} + K_d s \qquad \text{......................(39)}$$

$$= K_p(1 + \frac{1}{T_i s} + T_d s) \qquad \text{....................................(40)}$$

In the design of PID controller for active vibration control of beam, three parameters are specified: proportional gain, integral gain and derivative gain. The performance of the controller directly depends on these parameters. In order to obtain the desired system response, these parameters are optimally adjusted by fine tuning in MATLAB. From this the optimal values of K_p, K_i and K_d are respectively obtained as -1.4,-6.04 and -2.817.

➢ *Sliding Mode Controller*

From the output Eq.(35), choosing y_4 as the sliding variable and differentiating

$y(\dot{t}) = C^T[Ax(t) + Bu(t) + Er(t)]$............................(41)

Multiply by $[C^TB]^{-1}$ and taking $y(\dot{t}) = 0$

$u = -[C^T Ax(t)[C^TB]^{-1}] +$....................................(42)

SMC has two phases called Reaching phase and sliding

Phase. Let $v = \frac{1}{2}\sigma^2$ be the chosen Lyapunov function at the reaching phase $\therefore \dot{v} = \sigma\dot{\sigma} = -\delta|\sigma|$ where σ is the sliding variable

$\dot{v} = \sigma[C^TAx(t)[C^TB]^{-1}] + w = \sigma w + \sigma D$.........................(43)

Where $w = -ksgn(\sigma)$ and let $t_r = 0.2s$

$\therefore w = -[\delta + D]sgn(\sigma)$..(44)

In the sliding phase from Eq.(42) the control signal by the SMC will be

$u = -[C^TAx(t)[C^TB]^{-1}] - [\delta + D]sgn(\sigma)$......................(45)

IV. SIMULATION AND RESULTS

➢ *Open Loop Beam Response*

When a step change is given at a time t=0 from 0 to 2, the cantilever beam experiences a fixed force at the free end. Due to this external force the sensor output voltage deflects in the range 0 to $2.8 \times 10^{-5}v$. The open loop disturbance voltage is measured as $1.6 \times 10^{-5}v$ and persist through out in the system after 0.3 s of initial vibrations.

➢ *Closed Loop Beam Response with PID Controller*

When a PID controller is placed with values of Kp = -1.46, Ki = -6.04, Kd = -2.817, under the same external load change, the sensor output voltage will deflect from 0 to1.42×10^{-14} and settles down to 0.8×10^{-14} in 0.235 s. From this result it is being clear that external persisting load is being reduced by the controller action through the actuator.

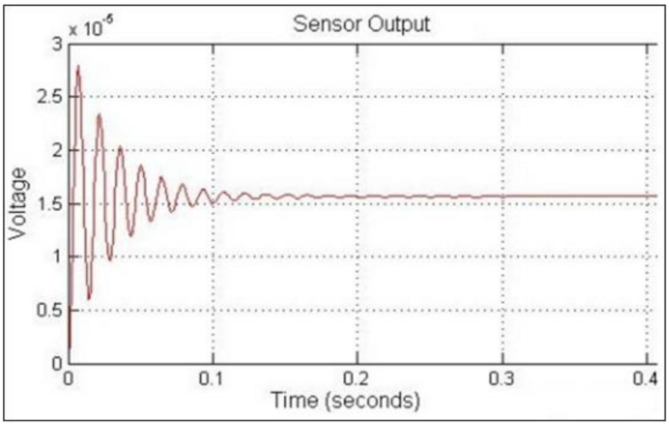

Fig 4 Open Loop Response of the Beam Without Controller

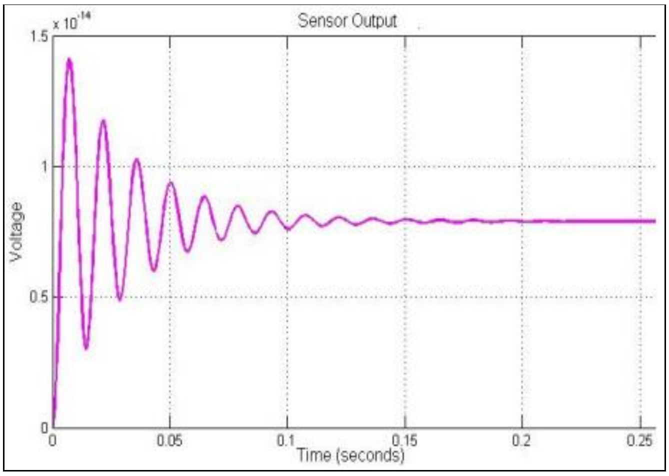

Fig 5 Closed Loop Response of the Beam with PID Controller

ISSN No:-2456-2165

- *Closed Loop Beam Response with Sliding Mode Controller*

By introducing a sliding mode controller in place of PID controller further reduces the external load to 4×10^{-16} and settles down with in 0.2 s. Our aim was to completely eliminate the external load and up to a far extent it is being achieved and the sensor output voltage almost equals to zero with in the response time assigned to the controller.

V. CONCLUSION

The Cantilever beam with a piezo sensor and piezo actuator at its free end is modelled and applied a fixed external disturbance. The beam output deflection is measured with a piezo sensor with voltage as output. Our aim is to bring back the cantilever beam to null position by eliminating the applied disturbance. For that a piezo actuator is placed and is controlled by a controller. In this paper a comparison between a PID controller and Sliding mode controller is done and by verifying the results it is found that placing a Sliding Mode Controller will be more adequate in bringing the beam back to null position in desired time. From the results it is clear that the proposed system can be applicable to micro/nano level applications such as sensing and eliminating the deformations of cantilever beams to adjust the focal length of cameras which are being used in remote sensing satellites.

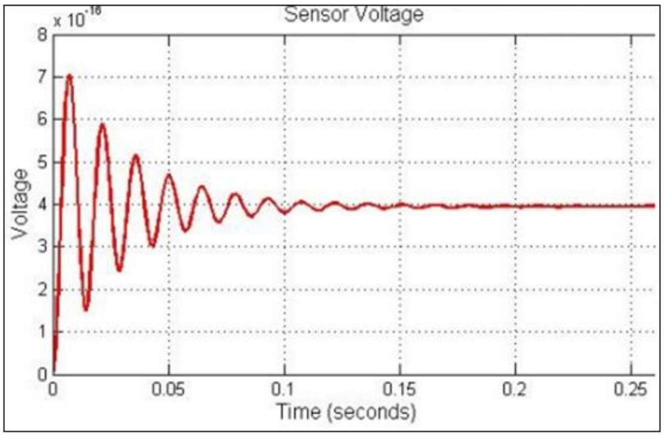

Fig 6 Closed Loop Response of the Beam with Sliding Mode Controller

REFERENCES

[1]. T. C. Manjunath and B. Bandyopadhyay, ”Smart Control of cantilever structures,” *International Journal of simulations*, Vol. 7, No.4, 2004.

[2]. Umapathy M., and Bandyopadhyay B, “Vibration control of flexible beam through smart structure concept using periodic output feedback,” *System Science Journal*, Volume 26 Issue 1, Pages 23-46, 2000

[3]. Ulrich Gabbertl,Tamara Nestorovic Trajkovl and Heinz Koppel ”Modeling , control, simulation of piezoelectric smart structures using FEM and optimal LQ control ,” *Facta Universitatis Series: Mechanics, Auto. Control and Robotics*, Volume 3 Issue 12, Pages 417-430, 2002

[4]. Hwang W., and Park H.C., ”Finite element modeling of piezoelectric sensors and actuators,” *AIAA Journal*, Volume 31 Issue 5, Pages 930-937, 1993

[5]. Young-Hun Lim, Senthil Gopinathan V., Vasundara Varadhan V., and Vijay Varadan K., ”Finite element simulation of smart structures using an optimal output feedback controller for vibration and noise control,” *Journal of Smart Materials and Structures*, Volume 8 , Pages 334-337, 1999