ISSN No:-2456-2165

Development of a Line based Camera Calibration Theory

Guy Blanchard Ikokou[1]
Tshwane University of Technology

Abstract:- Radial distortions can be modeled through polynomial, division and rational functions. Points based solutions to recover the coordinate of the principal point and the coefficients of radial lens distortions though polynomial, division and rational models, have been proposed in the literature. Points based calibration strategies have been criticized for the lack of accuracy of the calibrated camera parameters. Line-based calibration approaches have been hailed for their good accuracy due to strong geometric constraints imposed on calibration points such as the collinearity constraints. However, in the presence of severe radial distortions due to lens imperfections some line-based calibration strategies fail to model with accuracy the curvature of the 3D world resulting from severe lens distortions. This paper presented a new radial distortion calibration theory based on some properties of hyperbolic curves. The developed and proposed mathematical algorithms enable to recover the parameters of radial distortion as well as the coordinates of the principal point without any need for iterations or minimization of a cost function.

Keywords:- *Camera Calibration, Principal Point, Radial Lens Distortions.*

I. INTRODUCTION

With the mass production of cheap lenses non-metric cameras have been widely used for applications such as Photogrammetry. Camera calibration is a very important task for applications in Photogrammetry which require accurate measurements. Methods of camera calibration which require accurate measurements of 3D objects have been proposed in the literature **[17].** The most popular radial distortion model, the polynomial model is reported not having an inverse model and its numerous terms make the analytical solutions very difficult. The other limitation of the polynomial model is that is not suitable for large distortions **[14].** Some suggestions were made for the use of division models to handle severe distortions **[15; 16].** Others have proposed the use of rational models to correct lens distortions **[18].** Inverse models of the division and rational models have been proposed in the literature but they are reported unstable. Techniques which estimates the distortion parameters based on the straight-line constraint mostly rely on the extraction of curved lines from the imagery using some curve fitting algorithms but the challenges with such methods is firstly they are sensitive to image noise which can compromise the accuracy of the estimated parameters then secondly some of these algorithms are model specific and cannot perform efficiently with another model. In addition, some cannot handle distortion profiles such as barrel and pincushion distortions at the same time due to their mathematical formulations. In this paper we are proposing a radial distortion theory based on some properties of hyperbolic curves. The mathematical formulation of the model enables to estimate the distortion parameters analytically as well as the coordinate of the distortion centre in addition to the potential of the model to handle various distortion profiles. The model does not require the use of any optimization algorithm or knowledge of 3D points coordinates to correct image distortions.

II. RELATED WORK

Consumer grade digital cameras have been widely used in applications such as Photogrammetry. However, several studies have reported measurement errors originating from the misalignment of their optical systems **[3, 4].** Camera calibration consist of estimating the camera intrinsic and extrinsic parameters **[2].** Several line-based approaches have dealt with the estimation of radial lens distortions and addressed the limitations of points correspondence approaches which are highly error prone **[1]**. The technique proposed by **[1]** estimate distortion parameters based on geometric constraints of straight lines. The line extraction technique gathers pixels with similar texture properties based on their local gradient measures then using the orientation gradient refine their locations to form line support regions used to generate straight lines **[5].** The identified line support regions are used to estimate the parameters of distorted lines which are then utilized in a wrapping function to produce the radial distortion coefficients as well as the coordinates of the distortion center using an optimization process. One limitation of the approach is its sensitivity to image noise which could be mistakenly estimated as a line edge and be associated with the wrong pixels. **[2]** proposed a line based dynamic calibration technique based on the equivalent between an image line carried by the image plane and its corresponding carried by the 3D object plane. The approach of image line detection starts with setting gradient magnitude thresholds for each line edge. Pixels with gradient magnitude greater than the threshold are associated with the corresponding edge and all the pixels satisfying the threshold of a given edge are combined together to form a line **[6].** The line detection approach differs from that of **[5]** with the use of a Kalman filter to refine the point's coordinates on the distorted lines. A least squares adjustment technique is used to fit the straight line to the identified edge points. One limitation of the approach is that distortion parameters are estimated following a batch procedure which implies that any error with the initial values would propagate into the next output parameters. The other limitation of the approach is that the computation of the rotation matrix is not dissociated from the estimation of distortion parameters, resulting in residual effects on the intrinsic parameters estimates. **[7]** proposed a line-based calibration procedure based on the line straightness. To quantify the distortions within the image the authors estimated an error measure from the slops of the distorted curves and the straight line. To prevent the curve estimates from noise contamination the authors employed a Least-Median of squares technique. **[10]** proposed a five-step calibration approach which first extract curves in the image based on the pixel location and angular orientation with reference to candidate edge pixel. Once the curves extracted from the image the authors generated the distortion parameters using the Lavenberg-Marquardt algorithm which are then used to correct the distorted curves extracted in the initial phase of the calibration process. Although the technique produced promising distortion estimates it requires good initial points coordinate of the curve pixels and any wrong extraction of the distorted curves would produce an erroneous distortion center as the well as distortion parameters. **[11]** earlier proposed a calibration approach similar to **[10]** but instead of relying on pixel's coordinate and orientation the technique used distance thresholds to combine the different distorted curves to form straight lines and any curve segments located outside of the threshold distance is not considered for the merge. The approach also accounts for line orientation as the deviation from a straight-line model should not exceed a certain angular measure in order for segments to be merged into straight lines. To improve the accuracy of the inverse polynomial radial distortion model derived from the curves the technique employs a Random Sample Consensus technique **[12].** The technique estimates the final distortion parameters through a cost function minimized by the Lavenberg Marquardt algorithm. The most popular radial distortion model considered in remote sensing applications is the polynomial model given by the equation:

$$x_u = x_d + x_d\left(k_1 r_d^2 + k_2 r_d^4 + k_3 r_d^6\right) \tag{1}$$

$$y_u = y_d + y_d\left(k_1 r_d^2 + k_2 r_d^4 + k_3 r_d^6\right) \tag{2}$$

With k_1, k_2, k_3 the coefficients of radial distortion, x_d, y_d the measurable distorted coordinates of the image point and x_u, y_u the undistorted coordinates of the image point while r_d is the distance from the distortion center to the distorted point. It has been suggested to use lower order parameters as the higher order terms would create model instabilities **[8].** Moreover, it is reported that the polynomial distortion model is radially symmetric around the image center **[9].** The polynomial radial distortion model performs best with very small distortion and may require a large number of terms to deal with more severe distortions with the inconvenient of not being solved analytically. Remedial models were proposed to address this inconvenient of not having an inverse distortion model which include the division model **[16]** which is given by the expression as follows:

$$x_u = \frac{r_d}{1 + \kappa_1 r_d^2 + \kappa_2 r_d^2 \ldots} \tag{3}$$

It is reported that for many lenses types one parameter model variant can suffice to hand more severe distortions (Wang et al., 2009). For our approach, we will use a one-parameter model since it is reported sufficient to handle severe distortions as follows:

$$r_u = \frac{r_d}{1 + \kappa_1 r_d^2} \tag{4}$$

- *Thus, an Undistorted Point is Related to its Corresponding Distorted Point by the following Equations:*

$$x_u = \frac{x_d}{1 + \kappa_1 r_d^2} \tag{5}$$

$$y_u = \frac{y_d}{1 + \kappa_1 r_d^2} \tag{6}$$

ISSN No:-2456-2165

III. METHODOLOGY

➢ *Proposed Camera Model*

Let Υ be a 3D plan of equation $aX + bY + cZ = 0$ **(7),** and $p\left(x_p, y_p, z_p\right)$, $m\left(x_m, y_m, z_m\right)$ two image points on the camera reference system. The points p and m belong to two lines l_1 and l_2 on the camera plan with respective directional vector $\vec{u}\left(x_u, y_u, z_u\right)$ and $\vec{v}\left(x_v, y_v, z_v\right)$. The parametric equations of the lines l_1 and l_2 are respectively given by:

$$\begin{cases} x = x_p - k_1 x_u \\ y = y_p - k_1 y_u \\ z = z_p - k_1 z_u \end{cases} \tag{8}$$

With x, y, z the coordinates of a point t on the line l_1. Multiplying the first equation in **(8)** by a, the second equation in **(8)** by b and the third equation in **(8)** by c ,we obtain the linear equation as follows:

$$k_1 = \frac{ax_p + by_p + cz_p}{ax_u + by_u + cz_u} \tag{9}$$

And substituting **(9)** into the three equations in **(8)** leads to the following:

$$\begin{cases} x = \dfrac{bx_p y_u + cx_p z_u - bx_u y_b - cx_u z_p}{ax_u + by_u + cz_u} \\ y = \dfrac{ax_u y_p + cy_p z_u - ax_p y_u - cy_u z_p}{ax_u + by_u + cz_u} \\ z = \dfrac{ax_u z_p + by_u z_p - ax_p z_u - by_d z_u}{ax_u + by_u + cz_u} \end{cases} \tag{10}$$

Expending the equations, the first, second and third equations in (10) gives the following transformation from the 3D space onto the camera space:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x_p \\ y_p \\ z_p \end{bmatrix} \begin{bmatrix} \left(\dfrac{by_u - cz_u}{ax_u + by_u + cz_u}\right) & \left(\dfrac{-bx_u}{ax_u + by_u + cz_u}\right) & \left(\dfrac{-cx_u}{ax_u + by_u + cz_u}\right) \\ \left(\dfrac{-ay_u}{ax_u + by_u + cz_u}\right) & \left(\dfrac{ax_u + cz_u}{ax_u + by_u + cz_u}\right) & \left(\dfrac{-cy_u}{ax_u + by_u + cz_u}\right) \\ \left(\dfrac{-az_u}{ax_u + by_u + cz_u}\right) & \left(\dfrac{-bz_u}{ax_u + by_u + cz_u}\right) & \left(\dfrac{ax_u + by_u}{ax_u + by_u + cz_u}\right) \end{bmatrix} \tag{11}$$

The nine elements of the projection matrix can be estimated analytically by dividing the first, second equations in **(11)** by the third equation as follows:

$$x_i = \frac{bx_p y_u + cx_p z_u - bx_u y_b - cx_u z_p}{ax_u z_p + by_u z_p - ax_p z_u - by_p z_u} \tag{12}$$

$$y_i = \frac{ax_u y_p + cy_p z_u - ax_p y_u - cy_u z_p}{ax_u z_p + by_u z_p - ax_p z_u - by_p z_u} \quad (13)$$

➢ *Proposed Radial Distortion Model*

The hypothesis underlying our new approach is that two parallel lines in the 3D space can be projected through a perspective transformation as curves and the most distorted points on both lines and their corresponding undistorted locations belong to the circle centered at the distortion center as illustrated in the figure1.

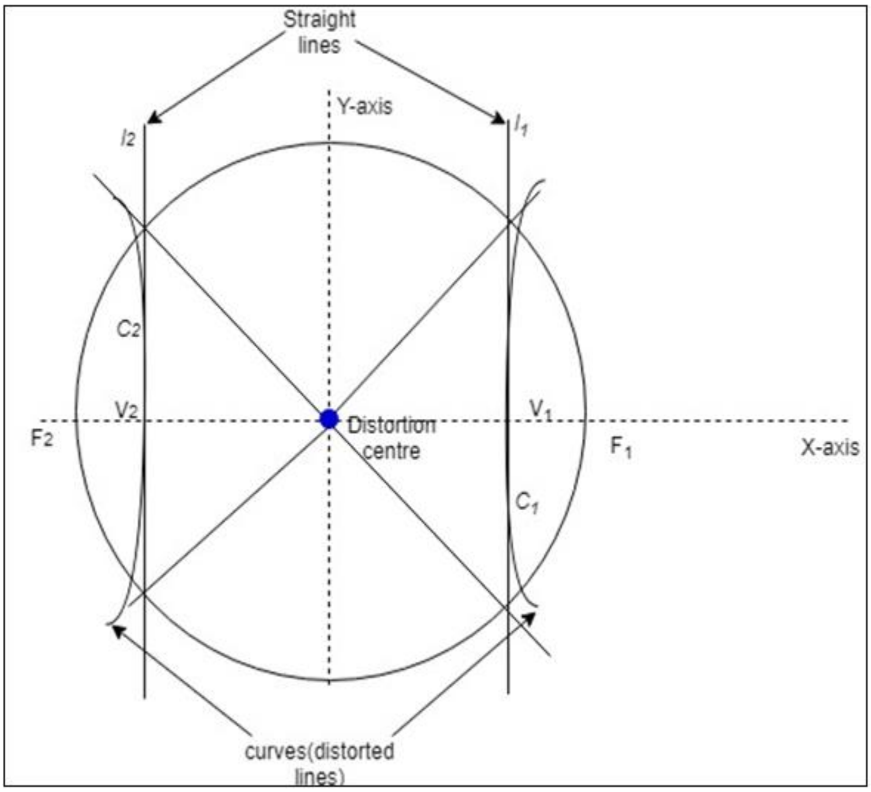

Fig 1 Projection of Two Parallel Lines with their Distorted Curves

From the figure above, we can derive the following equation of a hyperbola given:

$$\frac{y_d^2}{a^2} - \frac{x_d^2}{b^2} = 1 \quad (14)$$

Isolating the distorted coordinates x_d and y_d from (12) and (13) then substituting the results into (14) produces the inverse distortion model, which correspond to the equation of the straight lines (undistorted) as follows:

$$\frac{x_u^2\left(1+\kappa_1 r_d^2\right)^2}{a^2} - \frac{y_u^2\left(1+\kappa_1 r_d^2\right)^2}{b^2} = 1 \quad (15)$$

Let $h(x, y)$ be the following equation represent the curve l_1 given by:

$$h(x,y): y = mx + n \quad (16)$$

Computing the partial derivatives of the equation (16) gives:

$$dh = \frac{\partial h}{\partial x}(x,y)dx + \frac{\partial f}{\partial y}(x,y)dy \quad (17)$$

Thus, the equation of the tangent to the curve l_1 is given by:

$$\left(x-x_0\right)\frac{\partial h}{\partial x}+\left(y-y_0\right)\frac{\partial h}{\partial y}=0 \tag{18}$$

Similarly, by considering $g\left(x,y\right)$ the equation of the curve l_2 we have the equation of the tangent to l_2 given by:

$$\left(x-x_1\right)\frac{\partial g}{\partial x}+\left(y-y_0\right)\frac{\partial g}{\partial y}=0 \tag{19}$$

By intersecting both tangents given by the equation (18) and (19), we can solve the parameters a and b with knowledge of at least two points on each curve then we have our complete inverse distortion model.

➢ *Estimating the Distortion Parameters*

To estimate the radial distortion parameters, we are proposing a system of two objective functions given by:

$$\begin{cases} \square : \dfrac{x_u^2\left(1+\kappa_1 r_d^2\right)^2}{a^2}-\dfrac{y_u^2\left(1+\kappa_1 r_d^2\right)^2}{b^2}-\left(x-x_0\right)\dfrac{\partial h}{\partial x}-\left(y-y_0\right)\dfrac{\partial h}{\partial y}-1=0 \\ \square : \dfrac{x_u^2\left(1+\kappa_1 r_d^2\right)}{a^2}-\dfrac{y_u^2\left(1+\kappa_1 r_d^2\right)}{b^2}-\left(x-x_1\right)\dfrac{\partial g}{\partial x}+\left(y-y_1\right)\dfrac{\partial g}{\partial y}-1=0 \end{cases} \tag{20}$$

The distortion parameters from the model in (20) can be solved analytically considering the coordinates of the image center at $\left(x_c=0;y_c=0\right)$ and that the undistorted coordinates x_u and y_u can be solved analytically from the equation (19) and (19) through the intersection points of the curves with the circle as illustrated in Fig.1.

➢ *Refining the Coordinates of the Distortion Centre.*

Rewriting the equation (15) gives:

$$\left(\frac{1+\kappa_1 r_d^2}{a}x_u\right)^2-\left(\frac{1+\kappa_1 r_d^2}{b}y_u\right)^2=1 \tag{21}$$

Factorizing (21) gives the equation (22) as follows:

$$\left(x_u\frac{1+\kappa_1 r_d^2}{a}+y_u\frac{1+\kappa_1 r_d^2}{b}\right)\left(x_u\frac{1+\kappa_1 r_d^2}{a}-y_u\frac{1+\kappa_1 r_d^2}{b}\right)=1 \tag{22}$$

From (22) we can extract the equations of the two asymptotes of the hyperbolic curve as follows:

$$y=\frac{-b}{a}x \tag{23}$$

$$y=\frac{b}{a}x \tag{24}$$

By intersecting the equations in (23) and (24) of the asymptotes we obtain the coordinates of the image center. If $x_c\neq 0$ and $y_c\neq 0$ the distortion parameters can be refined in equations in (20).

IV. CONCLUSION

This study proposed a radial distortion theory based on the hyperbolic properties. In contrasts to some of the existing theories it solves the undistorted coordinates analytically and does not require an optimization process. The technique estimates radial distortion parameters analytically through a system of two objective functions which is a more stable approach than through curve optimization. The other advantage of the proposed approach is it is simple to implement and uses only one parameter. The theoretical formulation of the model is stable and suitable to handle severe distortions near the edges of the image and is expected to produce reliable distortion parameters. The performance of the proposed model will be evaluated with real image and the accuracy compared to the polynomial and division models. The performance of the model could also improve with more accurate distorted and undistorted points' coordinates and we are looking at employing a Kalman filter to improve the coordinates' accuracy then refine the inverse distortion model.

REFERENCES

[1]. B. Prescott and G.F. McLean, "Line-based correction of radial lens distortion" Graphical Models and Image Processing, 19979, 59(1), pp.39-47.

[2]. A.M.G. Tommaselli and C.L. Tozzi, "Line based camera calibration in machine vision dynamic applications". Controle and Automacao.1999.

[3]. R. Wackrow, J.H.Chandler and P. Bryan, "Geometric consistency and stability of consumer-grade digital cameras for accurate spatial measurement". Photogrammetric Record, 2007, 22(118): 121–134.

[4]. E. Sanz-Ablanedo, J.R. Rodríguez-Pérez and R. Arias-Sánchez," Metric potential of a 3D measurement system based on digital compact cameras", Sensors, 2009; 9: 4178–4194.

[5]. J.B. Burns, A.R. Hanson and E.M. Riseman, "Extracting Straight Lines," IEEE Trans. Pattern Analysis and Machine Intelligence, vol. 8, no. 4, pp. 425-455, July 1986.

[6]. S. A. Dudani and A. L. Luk, "Locating straight line edge segments on outdoor scenes," Patterni Recog., vol. 10, no. 3, pp. 145-166, 1978.

[7]. M. Ahmed and A. Farag, "Nonmetric calibration of camera lens distortion: differential methods and robust estimation", IP (14) 8, 1215–1230 (2005)

[8]. V. Orekhov, B. Abidi, C. Broaddus and M. Abidi, "Universal camera calibration with automatic distortion model selection," in Proceedings of International Conference on Image Processing (IEEE, 2007), pp. 397–400.

[9]. Z. Tang, R. G. von Gioi, P. Monasse and J.-M. Morel, ''A precision analysis of camera distortion models,'' IEEE Trans. Image Process., vol. 26, no. 6, pp. 2694–2704, Jun. 2017.

[10]. G.Y. Song and J.W. Lee, "Correction of radial distortion based on line-fitting", Int. J. Control, Autom. Syst. 8(3), 615–621 (2010).

[11]. H.B. Thormaehlen and I. Wassermann, "Robust line-based calibration of lens distortion from a single view". In Mirage 2003, pages 105-112, 2003.

[12]. M.A. Fischler and R.C. Bolles, "Random sample consensus: a paradigm for model fitting with applications to image analysis and automated cartography". Communications of the ACM, 24(6), pp.381-395.

[13]. A. Wang, T. Qiu, and L. Shao, "A simple method of radial distortion correction with centre of distortion estimation," Journal of Mathematic Imaging and Vision, vol. 35, no. 3, pp. 165-172 Nov. 2009.

[14]. M. Alemán-Flores, L. Alvarez, L. Gomez and D. Santana-Cedres, "Wide-angle lens distortion correction using division models" Progress in Pattern Recognition, Image Analysis, Computer Vision and Applications, Springer, Berlin, Heidelberg, 99.415-422, 2013.

[15]. X. Mei, S. Yang, J. Rong, X. Ying, S. Huang and H. Zha,2015, September. Radial lens distortion correction using cascaded one-parameter division model. In Image Processing (ICIP), 2015 IEEE International Conference, 2015, (pp. 3615-3619).

[16]. A.W. Fitzgibbon," Simultaneous linear estimation of multiple view geometry and lens distortion". In: Proc. IEEE International Conference on Computer Vision and Pattern Recognition pp. 125–132 (2001)

[17]. N. Abdo and A. Borgeat, 3D camera calibration. Technical report, 2010

[18]. J. Heikkil and O. Silvn, "A four-step camera calibration procedure with implicit image correction," in IEEE Computer Society Conference on Computer Vision and Pattern Recognition, San Juan, Puerto Rico, 1997, pp. 1106–1112.