Modification Elementary Row Operations to Determine the Inverse of Trapezoidal Fuzzy Numbers Matrix

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Abstract:- There are several algebraic solutions written by various authors for trapezoidal fuzzy numbers $\tilde{u}=(u,v,\alpha,\beta)$ where u and v are the midpoint, α is the left width, and β is the right width. Furthermore, trapezoidal fuzzy numbers are used in various arithmetic of trapezoidal fuzzy numbers. There are not many differences made by writers, especially for addition, subtraction, and scalar multiplication operations. However, there are many options made for multiplication and division operations. With many options for multiplication and division operations, it still does not produce $\tilde{u}\otimes\tilde{u}^{-1}=\tilde{\imath}$, therefore the author makes multiplication and division operations that can produce $\tilde{u}\otimes\tilde{u}^{-1}=\tilde{\imath}$. Before making multiplication and division operations, the middle value of the trapezoidal fuzzy number $\tilde{u}$ is first determined, which is symbolized by $m_q(\tilde{u})$. The middle value is used for constructing arithmetic multiplication, inverse, and divisibility of trapezoidal fuzzy numbers that can solve $\tilde{u}\otimes\tilde{u}^{-1}=\tilde{\imath}$. *Furthermore, the arithmetic of trapezoidal fuzzy numbers that have been constructed is used for are used to determine the inverse of the trapezoidal fuzzy number matrix using the modified fuzzy elementary row operations method.* Until now, there is no single article that provides an alternative to the elementary row operations process of a matrix. So in this article in addition to modifying the multiplication, inverse, and division operations for trapezoidal fuzzy numbers. There will also be a modification of elementary row operations in calculating the inverse of a trapezoidal fuzzy number matrix.

Keywords:- *Fuzzy Elementary Row Operations Method, Fuzzy Inverse, Trapezoidal Fuzzy Number*

I. INTRODUCTION

Trapezoidal fuzzy numbers are written in various forms, for example, authors [1-8] write in the form $\tilde{u}=(u_1,u_2,u_3,u_4)$, $u_1<u_2<u_3<u_4$ with u_2 and u_3 is the midpoint, u_1 is the left point, and u_4 is the right point. Other writers [9-13] write in a different form, $\tilde{u}=(u,v,\alpha,\beta)$ where u and v are the midpoint, α is the left width, and β is the right width. Furthermore, the trapezoidal fuzzy numbers are converted into interval form, namely $\tilde{u}(s)=[u_l(s),u_u(s)]$ with $u_l(s)=a-(1-s)\alpha$ and $u_u(s)=b+(1-s)\beta$, $0<s<1$. The interval form is used in various arithmetic operations of trapezoidal fuzzy numbers.

There are several options for the arithmetic operations of trapezoidal fuzzy numbers, including the arithmetic of addition, subtraction, and scalar multiplication are generally made in the same form, while the arithmetic of multiplication and division authors have been made in various forms. For example, one author defines [1-2,5,12] as the arithmetic form of multiplication, and another author defines the arithmetic form of division [4]. With many options for multiplication and division operations, it still does not result in a result $\tilde{u}\otimes\tilde{u}^{-1}=\tilde{\imath}$, therefore the author creates multiplication and division operations that can result in a result $\tilde{u}\otimes\tilde{u}^{-1}=\tilde{\imath}$.

Before modifying the arithmetic multiplication and division of trapezoidal fuzzy numbers, the middle value of trapezoidal fuzzy numbers $\tilde{u}$ is first determined, which is symbolized by $m_q(\tilde{u})$. Furthermore, the middle value is used to construct the arithmetic multiplication, inverse, and division of trapezoidal fuzzy numbers that can be solve $\tilde{u}\otimes\tilde{u}^{-1}=\tilde{\imath}$. Furthermore, the arithmetic modification is used on the trapezoidal fuzzy matrix, namely to determine the inverse of the trapezoidal fuzzy matrix using the elementary row operation method, so that using the arithmetic modification produces $\tilde{U}\otimes\tilde{U}^{-1}=\tilde{I}$ any matrix $\tilde{U}$.

II. PRELIMINARIES

A. *Trapezoidal Fuzzy Number*

Some of the theories discussed are theories related to the discussion to support the author in solving the problem. Furthermore, the definition of the set of fuzzy numbers proposed in [2-4,7,9,12-13,16,18-21,23-24,26] is given.

Definition 2.1 Suppose H is a nonempty set so that a fuzzy set $\tilde{u}$ in f is characteristic of a membership function

$$\tilde{u} = \{(f, \mu_{\tilde{u}}(f)) | f \in H,\ 0 \le \mu_{\tilde{u}}(f) \le 1\}.$$

Furthermore, with the development of mathematics, two fuzzy numbers are often used, namely triangular fuzzy numbers and trapezoidal fuzzy numbers. The definition of trapezoidal fuzzy numbers in Definition 2.2 has been explained in [9-13] as follows:

Definition 2.2 A trapezoidal fuzzy number is a trapezoidal fuzzy number if $\tilde{u} = (u, v, \alpha, \beta)$ where u and v are the midpoint, α is the left width, and β is the right width.

Trapezoidal fuzzy numbers have membership functions that have been described in [9-13] in Definition 2.3 as follows:

Definition 2.3 A trapezoidal fuzzy number has a membership function of the form

$$\mu_{\tilde{u}}(f) = \begin{cases} 1 - \dfrac{u-f}{\alpha}, & u-\alpha \le f \le u, \\ 1, & u \le f \le v, \\ 1 - \dfrac{f-v}{\beta}, & v \le f \le v+\beta, \\ 0, & others. \end{cases}$$

Definition 2.4 According to Definition 2.3, using the membership function of trapezoidal fuzzy numbers, we can determine certain values, any trapezoidal fuzzy number $\tilde{u} = (u, v, \alpha, \beta)$ has a parametric form $\tilde{u}(s) = [u_l(s), u_u(s)]$, $0 < s < 1$ that can be represented as follows:

$$u_l(s) = u - (1-s)\alpha \text{ and } u_u(s) = v + (1-s)\beta.$$

The representation of Definition 2.4 is shown in Figure 1.

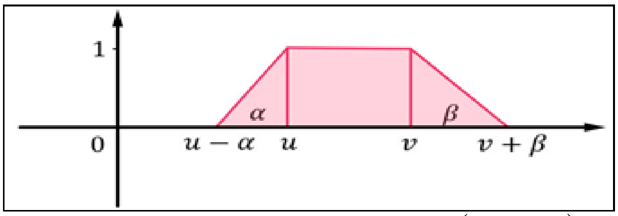

Fig 1 Trapezoidal Fuzzy Numbers $\tilde{u} = (u, v, \alpha, \beta)$.

The trapezoidal fuzzy number function is explained in [1,9-11,13,17-18,21-22,27,29] as in Definition 2.5.

Definition 2.5 A trapezoidal fuzzy number $\tilde{u} = (u, v, \alpha, \beta)$, $\tilde{u} : R \to [0,1]$ is a function that satisfies the following provisions:

- $\tilde{u}$ is the upper semicontinuity.
- $\tilde{u}(f) = 0$ outside the interval $[u-\alpha, v+\beta]$.
- There are some numbers u, v that lie within $[u-\alpha, v+\beta]$ such that

- $\tilde{u}(f)$ monotonically increasing on the interval $[u-\alpha, u]$.
- $\tilde{u}(f)$ monotonically decreasing on the interval $[v, v+\beta]$.
- $\tilde{u}(f) = 1$ for values $u \le f \le v$.

Trapezoidal fuzzy number arithmetic has been described by several different authors. Here are some of the trapezoidal fuzzy number arithmetic that have been used, described in [1-2,5,11,14,25,28].

Definition 2.6 Given two trapezoidal fuzzy numbers in interval form $\tilde{u}(s) = (u, v, \alpha, \beta) = [u_l(s), u_u(s)]$ and $\tilde{w}(s) = (w, x, \gamma, \delta) = [w_l(s), w_u(s)]$ so the following operation applies:

- *Addition*

$$\tilde{u}(s) \oplus \tilde{w}(s) = [u_l(s), u_u(s)] \oplus [w_l(s), w_u(s)]$$

$$= [u_l(s) + w_l(s), u_u(s) + w_u(s)].$$

- *Reduction*

$$\tilde{u}(s) \ominus \tilde{w}(s) = [u_l(s), u_u(s)] \ominus [w_l(s), w_u(s)]$$

$$= [u_l(s) - w_u(s), u_u(s) - w_l(s)].$$

- *Scalar Multiplication*

$$n\tilde{u}(s) = \begin{cases} [nu_u(s), nu_l(s)], & n < 0, \\ [nu_l(s), nu_u(s)], & n \ge 0. \end{cases}$$

The arithmetic multiplication of trapezoidal fuzzy numbers has been explained by several authors in different ways, namely as in Definition 2.7 described in [4,10-11,13] and Definition 2.8 described in [1-2,5].

Definition 2.7 If $\tilde{u} = (u, v, \alpha, \beta)$ and $\tilde{w} = (w, x, \gamma, \delta)$ and are two trapezoidal fuzzy numbers, then $\tilde{z} = \tilde{u} \otimes \tilde{w}.$ The chapters are some cases for the multiplication operation of trapezoidal fuzzy numbers.

- If $\tilde{u} > 0$ and $\tilde{w} > 0$, then $\tilde{z} = (uw, vx, (u\gamma + w\alpha), (v\delta + x\beta)).$
- If $\tilde{u} > 0$ and $\tilde{w} < 0$, then $\tilde{z} = (vw, ux, (v\gamma - w\beta), (u\delta - x\alpha)).$
- If $\tilde{u} < 0$ and $\tilde{w} > 0$, then $\tilde{z} = (ux, vw, (x\alpha - u\delta), (w\beta - v\gamma)).$
- If $\tilde{u} < 0$ and $\tilde{w} < 0$, then $\tilde{z} = (vx, uw, -(x\beta + v\delta), -(u\gamma + w\alpha)).$

Definition 2.8 Given the two trapezoidal fuzzy numbers $\tilde{u}(s) = (u, v, \alpha, \beta) = [u_l(s), u_u(s)]$ and $\tilde{w}(s) = (w, x, \gamma, \delta) = [w_l(s), w_u(s)]$ so the following operation applies:

$$\tilde{u}(s) \otimes \tilde{w}(s) = [\min\{u_l(s)w_l(s), u_u(s)w_u(s), u_u(s)w_l(s), u_u(s)w_u(s)\}$$

$$\max\{u_l(s)w_l(s), u_u(s)w_u(s), u_u(s)w_l(s), u_u(s)w_u(s)\}].$$

The arithmetic exponent of trapezoidal fuzzy numbers has been explained by the author [4], as in Definition 2.9.

Definition 2.9 Given a trapezoidal fuzzy number $\tilde{u}(s) = (u, v, \alpha, \beta) = [u_l(s), u_u(s)]$, the following operation applies:

$$\tilde{u}^k = (u, v, \alpha, \beta)^k \cong (u^k, v^k, -vu^{k-1}\beta, -vv^{k-1}\alpha),\ k < 0,$$

$$\tilde{u}^k = (u, v, \alpha, \beta)^k \cong (u^k, v^k, vu^{k-1}\beta, -vv^{k-1}\alpha),\ k > 0.$$

Using the exponent arithmetic in Definition 2.9, the inverse arithmetic can be formed when $k = -1$ as in Definition 2.10.

Definition 2.10 Given a trapezoidal fuzzy number $\tilde{u}(s) = (u, v, \alpha, \beta) = [u_l(s), u_u(s)]$, the following operation applies:

$$\tilde{u}^{-1} = (u, v, \alpha, \beta)^{-1} \cong (u^{-1}, v^{-1}, -vu^{-2}\beta, -vv^{-2}\alpha).$$

Using the arithmetic of multiplication in Definition 2.7 and the inverse arithmetic in Definition 2.10 the arithmetic of division can be formed as in Definition 2.11.

Definition 2.11 If $\tilde{u} = (u, v, \alpha, \beta)$ and $\tilde{u}^{-1} = (u^{-1}, v^{-1}, -vu^{-2}\beta, -vv^{-2}\alpha)$ are two trapezoidal fuzzy numbers, then $\tilde{g} = \tilde{u} \otimes \tilde{u}^{-1}$. The following are some cases for the multiplication operation of trapezoidal fuzzy numbers.

- *If $\tilde{u} > 0$, then* $\tilde{g} = (uu^{-1}, vv^{-1}, (-uvu\beta^{-2} + u^{-1}\alpha), (-vvv^{-2}\alpha + v^{-1}\beta)).$
- *If $\tilde{u} < 0$, then* $\tilde{g} = (vv^{-1}, uu^{-1}, -(v^{-1}\beta - vvv^{-2}\alpha), (uvu^{-2}\beta + u^{-1}\alpha)).$

B. Trapezoidal Fuzzy Matrix

A fuzzy matrix is a matrix in which the entries are fuzzy numbers that are in the range of [0,1]. The fuzzy matrix is summarized as $\tilde{u}$ and the elements or entries of the fuzzy matrix is summarized as $\tilde{u}_{ij}$. A writer [4] explains the definition of a trapezoidal fuzzy matrix, which is as below:

Definition 2.12 The fuzzy matrix can be expressed as $\tilde{u}$ and the elements or entries of the fuzzy matrix can be expressed as $\tilde{u}_{ij}$. The size of a fuzzy matrix is the number of rows and columns in the fuzzy matrix. So a general fuzzy matrix $\tilde{u}$ with size $m \times n$ is summarized as follows:

$$\tilde{U} = \begin{bmatrix} \tilde{u}_{11} & \cdots & \tilde{u}_{1n} \\ \vdots & \ddots & \vdots \\ \tilde{u}_{m1} & \cdots & \tilde{u}_{mn} \end{bmatrix}$$

with $\tilde{u}_{ij} \in [0,1],\ 1 \le i \le m; 1 \le j \le n.$

The trapezoidal fuzzy number matrix is summarized in this form:

$$\tilde{U} = \begin{bmatrix} (u_{11}, v_{11}, \alpha_{11}, \beta_{11}) & \cdots & (u_{1n}, v_{1n}, \alpha_{1n}, \beta_{1n}) \\ \vdots & \ddots & \vdots \\ (u_{m1}, v_{m1}, \alpha_{m1}, \beta_{m1}) & \cdots & (u_{mn}, v_{mn}, \alpha_{mn}, \beta_{mn}) \end{bmatrix}$$

Then the trapezoidal fuzzy matrix can be converted into a fuzzy interval matrix as follows:

$$\tilde{U}(s) = \begin{bmatrix} u_{l11}(s), u_{u11}(s) & \cdots & u_{l1n}(s), u_{u1n}(s) \\ \vdots & \ddots & \vdots \\ u_{lm1}(s), u_{um1}(s) & \cdots & u_{lmn}(s), u_{umn}(s) \end{bmatrix}$$

C. Fuzzy Matrix Inverse

The fuzzy matrix inverse is a new matrix that is the inverse of the original matrix which entries are fuzzy numbers in the range [0,1]. There are several ways to determine the inverse of a fuzzy matrix, including using the elementary row operations method. Unfortunately, it is very difficult to find authors who explain determining the inverse of a fuzzy matrix using the elementary row operations approach, so the author uses the elementary row operations method for real numbers described in [15].

Definition 2.13 Elementary row operations. Suppose $U = u_{ij},\ i, j \in N$ is a square matrix, then the matrix is reduced to I matrix. In the process of elementary row operation, the matrix U is converted into an enlarged matrix so $[U|I]$ that after the elementary row operations process, the final form is obtained $[I|U^{-1}]$.

On a square matrix, elementary row operation may include performed as well the following:

- Multiplying a row by a nonzero constant.
- Exchanging two rows.
- Adding/subtracting multiples of one row with another row.

III. ALTERNATIVE ARITHMETIC

The middle value of trapezoidal fuzzy numbers is first constructed to construct the arithmetic of multiplication, inverse, and division of trapezoidal fuzzy numbers.

Definition 3.1 Take any trapezoidal fuzzy number $\tilde{u}(s) = (u, v, \alpha, \beta) = [u_l(s), u_u(s)]$ and $\tilde{w}(s) = (w, x, \gamma, \delta) = [w_l(s), w_u(s)]$, then construct the middle value of the trapezoidal fuzzy number $\tilde{u}$ dan $\tilde{w}$ which is symbolized by $m_q(\tilde{u})$ and $m_q(\tilde{w})$ with values $m_q(\tilde{v})$ and $m_q(\tilde{w})$ as follows:

$$m_q(\tilde{u}) = \frac{u+v}{2} \text{ and } m_q(\tilde{w}) = \frac{w+x}{2}.$$

Using Definition 3.1 the arithmetic multiplication is constructed as follows:

$$\tilde{u}(s) \otimes \tilde{w}(s) = [u_l(s) m_q(\tilde{w}) + w_l(s) m_q(\tilde{u}) - m_q(\tilde{u}) m_q(\tilde{w}),$$

$$u_u(s) m_q(\tilde{w}) + w_u(s) m_q(\tilde{u}) - m_q(\tilde{u}) m_q(\tilde{w})]. \quad (3.1)$$

Furthermore, equation (3.1) can also be written in the form of trapezoidal fuzzy numbers, as follows:

$$\tilde{u} \otimes \tilde{w} = (u, v, \alpha, \beta) \otimes (w, x, \gamma, \delta)$$

$$\tilde{u} \otimes \tilde{w} = (u m_q(\tilde{w}) + w m_q(\tilde{u}) - m_q(\tilde{u}) m_q(\tilde{w}),$$

$$v m_q(\tilde{w}) + x m_q(\tilde{u}) - m_q(\tilde{u}) m_q(\tilde{w})). \quad (3.2)$$

Furthermore, using equation (3.2) can be constructed $\frac{1}{\tilde{u}}$ for any trapezoidal fuzzy number $\tilde{u}$ so that

$$\tilde{u} \otimes \frac{1}{\tilde{u}} = \tilde{\imath} = (1,1,0,0).$$

Theorem 3.1 For any trapezoidal fuzzy number $\tilde{u} = (u, v, \alpha, \beta)$ with $m_q(\tilde{u}) \neq 0$ and $\tilde{p} = (p, q, \gamma, \delta)$ with $m_q(\tilde{p}) \neq 0$, it can be formed

$$\tilde{u} \otimes \tilde{p} = \tilde{\imath} = (1,1,0,0).$$

Using equation (3.2) is obtained

$$\tilde{u} \otimes \tilde{p} = (u, v, \alpha, \beta) \otimes (p, q, \gamma, \delta)$$

$$\tilde{u} \otimes \tilde{p} = (u m_q(\tilde{p}) + p m_q(\tilde{u}) - m_q(\tilde{u}) m_q(\tilde{p}),$$

$$v m_q(\tilde{p}) + q m_q(\tilde{u}) - m_q(\tilde{u}) m_q(\tilde{p}),$$

$$\alpha m_q(\tilde{p}) + \gamma m_q(\tilde{u}), \beta m_q(\tilde{p}) + \delta m_q(\tilde{u})). \quad (3.3)$$

Furthermore, it is formed $\tilde{p}$ that can solve equation (3.3), namely

$$\tilde{p} = \frac{1}{\tilde{u}} = \left(\frac{2m_q(\tilde{u}) - u}{(m_q(\tilde{u}))^2}, \frac{2m_q(\tilde{u}) - v}{(m_q(\tilde{u}))^2}, \frac{-\alpha}{(m_q(\tilde{u}))^2}, \frac{-\beta}{(m_q(\tilde{u}))^2} \right).$$

Furthermore, it is proved $\tilde{p}$ that what is formed is true.

- *Proof:* First determine the value $m_q(\tilde{p})$, that is

$$m_q(\tilde{p}) = \left(\frac{\left(\frac{2m_q(\tilde{u}) - u}{(m_q(\tilde{u}))^2} \right) + \left(\frac{2m_q(\tilde{u}) - v}{(m_q(\tilde{u}))^2} \right)}{2} \right)$$

$$m_q(\tilde{p}) = \left(\frac{\frac{4m_q(\tilde{u}) - (u+v)}{(m_q(\tilde{u}))^2}}{2} \right)$$

$$m_q(\tilde{p}) = \frac{1}{m_q(\tilde{u})}.$$

Furthermore, using equation (3.3), we can determine the value of $\tilde{u} \otimes \tilde{p}$, namely

$$\tilde{u} \otimes \tilde{p} = (u,v,\alpha,\beta) \otimes \left(\frac{2m_q(\tilde{u}) - u}{(m_q(\tilde{u}))^2}, \frac{2m_q(\tilde{u}) - v}{(m_q(\tilde{u}))^2}, \frac{-\alpha}{(m_q(\tilde{u}))^2}, \frac{-\beta}{(m_q(\tilde{u}))^2} \right)$$

Using equation (3.2) is obtained

$$\tilde{u} \otimes \tilde{p} = \left(u \frac{1}{m_q(\tilde{u})} + \frac{2m_q(\tilde{u}) - u}{(m_q(\tilde{u}))^2} m_q(\tilde{u}) - m_q(\tilde{u}) \frac{1}{m_q(\tilde{u})}, \right.$$

$$v \frac{1}{m_q(\tilde{u})} + \frac{2m_q(\tilde{u}) - v}{(m_q(\tilde{u}))^2} m_q(\tilde{u}) - m_q(\tilde{u}) \frac{1}{m_q(\tilde{u})},$$

$$\left. \alpha \frac{1}{m_q(\tilde{u})} + \frac{-\alpha}{(m_q(\tilde{u}))^2} m_q(\tilde{u}), \beta \frac{1}{m_q(\tilde{u})} + \frac{-\beta}{(m_q(\tilde{u}))^2} m_q(\tilde{u}) \right]$$

$$\tilde{u} \otimes \tilde{p} = (1,1,0,0)$$

$$\tilde{u} \otimes \tilde{p} = \tilde{\imath}.$$

It is proven that $\tilde{p}$ what is formed is true, so $\tilde{u} \otimes \tilde{p} = \tilde{\imath}$.

Furthermore, the general form of modified arithmetic division can be defined based on Theorem 3.1.

Take any trapezoidal fuzzy number $\tilde{u} = (u,v,\alpha,\beta)$ and $\tilde{w} = (w,x,\gamma,\delta)$ with $m_q(\tilde{w}) \neq 0$.

$$\frac{\tilde{u}}{\tilde{w}} = \tilde{u} \otimes \frac{1}{\tilde{w}}$$

$$\frac{\tilde{u}}{\tilde{w}} = (u,v,\alpha,\beta) \otimes \left(\frac{2m_q(\tilde{w}) - w}{(m_q(\tilde{w}))^2}, \frac{2m_q(\tilde{w}) - x}{(m_q(\tilde{w}))^2}, \frac{-\gamma}{(m_q(\tilde{w}))^2}, \frac{-\delta}{(m_q(\tilde{w}))^2} \right)$$

Using equation (3.3) is obtained

$$\frac{\tilde{u}}{\tilde{w}} = \left(u \frac{1}{m_q(\tilde{w})} + \frac{2m_q(\tilde{w}) - w}{(m_q(\tilde{w}))^2} m_q(\tilde{u}) - m_q(\tilde{u}) \frac{1}{m_q(\tilde{w})}, \right.$$

$$v \frac{1}{m_q(\tilde{w})} + \frac{2m_q(\tilde{w}) - x}{(m_q(\tilde{w}))^2} m_q(\tilde{u}) - m_q(\tilde{u}) \frac{1}{m_q(\tilde{w})},$$

$$\left. \alpha \frac{1}{m_q(\tilde{w})} + \frac{-\gamma}{(m_q(\tilde{w}))^2} m_q(\tilde{u}), \beta \frac{1}{m_q(\tilde{w})} + \frac{-\delta}{(m_q(\tilde{w}))^2} m_q(\tilde{u}) \right).$$

$$\frac{\tilde{u}}{\tilde{w}} = \left(\frac{um_q(\tilde{w}) + 2m_q(\tilde{w})m_q(\tilde{u}) - wm_q(\tilde{u}) - m_q(\tilde{u})m_q(\tilde{w})}{(m_q(\tilde{w}))^2}, \right.$$

$$\frac{vm_q(\tilde{w}) + 2m_q(\tilde{w})m_q(\tilde{u}) - xm_q(\tilde{u}) - m_q(\tilde{u})m_q(\tilde{w})}{(m_q(\tilde{w}))^2},$$

$$\left. \frac{\alpha m_q(\tilde{w}) - \gamma m_q(\tilde{u})}{(m_q(\tilde{w}))^2}, \frac{\beta m_q(\tilde{w}) - \delta m_q(\tilde{u})}{(m_q(\tilde{w}))^2} \right). \tag{3.4}$$

Theorem 3.2 Suppose $\tilde{u}(s)$, $\tilde{w}(s)$, $\tilde{x}(s)$ is a trapezoidal fuzzy number in interval form, respectively obtained

- $\tilde{u}(s) \otimes \tilde{0}(s) = \tilde{0}(s)$.
- $\tilde{u}(s) \otimes \tilde{\imath}(s) = \tilde{u}(s)$.
- $\tilde{u}(s) \otimes \tilde{w}(s) = \tilde{w}(s) \otimes \tilde{u}(s)$.
- $(\tilde{u}(s) \otimes \tilde{w}(s)) \otimes \tilde{x}(s) = \tilde{u}(s) \otimes (\tilde{w}(s) \otimes \tilde{x}(s))$.
- $(\tilde{u}(s) \oplus \tilde{w}(s)) \otimes \tilde{x}(s) = \tilde{u}(s) \otimes \tilde{x}(s) \oplus \tilde{w}(s) \otimes \tilde{x}(s)$.
- If $\tilde{u}(s) \otimes \tilde{t}(s) = \tilde{w}(s)$ where $\tilde{u}(s) \neq \tilde{0}(s)$, then $\tilde{t}(s) = \frac{\tilde{w}(s)}{\tilde{u}(s)}$.
- If $\tilde{u}(s) \otimes \tilde{w}(s) = \tilde{0}(s)$, then $\tilde{u}(s) = \tilde{0}(s)$ or $\tilde{w}(s) = \tilde{0}(s)$.
- If $\tilde{u}(s) \otimes \tilde{w}(s) = \tilde{u}(s) \otimes \tilde{x}(s)$ where $\tilde{u}(s) \neq \tilde{0}(s)$, then $\tilde{w}(s) = \tilde{x}(s)$.
- If $\tilde{u}(s) \neq \tilde{0}(s)$, then $\frac{1}{\tilde{u}(s)} \neq \tilde{0}(s)$ and $\frac{1}{\frac{1}{\tilde{u}(s)}} = \tilde{u}(s)$.
- If $\tilde{u}(s) \neq \tilde{0}(s)$ and $\tilde{w}(s) \neq \tilde{0}(s)$, then $\frac{1}{\tilde{u}(s) \otimes \tilde{w}(s)} = \frac{1}{\tilde{u}(s)} \otimes \frac{1}{\tilde{w}(s)}$.

Definition 3.2 Determining the inverse of a trapezoidal fuzzy matrix there is a square fuzzy matrix cannot use the method of elementary row operations on real numbers, so a modification of elementary row operations is needed as follows:

- Multiplying a row by a nonzero trapezoidal fuzzy number.
- Exchanging two rows.
- Adding/subtracting the result of multiplying a row with a trapezoidal fuzzy number to another row.

- *Example: Since the inverse calculation process is done by modifying the elementary row operations, the steps are the same for any matrix of any order. For example, here is given a matrix of degree* 2×2.

$$\tilde{U}=\begin{bmatrix}(-1,2,1,2) & (0,1,1,3)\\ (-2,1,2,1) & (2,3,1,0)\end{bmatrix}$$

Furthermore, the matrix $\tilde{U}$ is converted into interval form.

$$\tilde{U}(s)=\begin{bmatrix}[-2+s,4-2s] & [-1+s,4-3s]\\ [-4+2s,2-s] & [1+s,3]\end{bmatrix}$$

Furthermore, the inverse value of the matrix is determined $\tilde{U}(s)$ using the elementary row operations method.

First, increase the size of the matrix $\tilde{U}(s)$ as follows:

$$\left[\begin{matrix}[-2+s,4-2s] & [-1+s,4-3s]\\ [-4+2s,2-s] & [1+s,3]\end{matrix}\right|\left.\begin{matrix}[1,1] & [0,0]\\ [0,0] & [1,1]\end{matrix}\right]$$

Furthermore, row 1 is multiplied by $\dfrac{1}{[-2+s,4-2s]}$, thus obtained

$$\left[\begin{matrix}[1,1] & [3,1-2s]\\ [-4+2s,2-s] & [1+s,3]\end{matrix}\right|\left.\begin{matrix}[12-4s,-12+8s] & [0,0]\\ [0,0] & [1,1]\end{matrix}\right]$$

Then row 2 is added with the multiplication $[4-2s,-2+s]$ of row 1, thus obtained

$$\left[\begin{matrix}[1,1] & [3,1-2s]\\ [0,0] & [6-s,1]\end{matrix}\right|\left.\begin{matrix}[12-4s,-12+8s] & [0,0]\\ [13-6s,-11+6s] & [1,1]\end{matrix}\right]$$

Next multiply row 2 by $\dfrac{1}{[6-s,1]}$, thus obtained

$$\left[\begin{matrix}[1,1] & [3,1-2s]\\ [0,0] & [1,1]\end{matrix}\right|\left.\begin{matrix}[12-4s,-12+8s] & [0,0]\\ \left[4-\frac{17}{9}s,\frac{-31}{9}+2s\right] & \left[\frac{1}{9}s,\frac{5}{9}\right]\end{matrix}\right]$$

The last step of row 1 is added with the multiplication $[-3,-1+2s]$ of row 2, thus obtained

$$\left[\begin{matrix}[1,1] & [0,0]\\ [0,0] & [1,1]\end{matrix}\right|\left.\begin{matrix}\left[\frac{22}{3}-\frac{19}{9}s,\frac{-77}{9}+\frac{20}{3}s\right] & \left[\frac{-2}{3}-\frac{1}{9}s,\frac{-5}{9}+\frac{2}{3}s\right]\\ \left[4-\frac{17}{9}s,\frac{-31}{9}+2s\right] & \left[\frac{1}{9}s,\frac{5}{9}\right]\end{matrix}\right]$$

Using the elementary row operation method on the matrix $\tilde{U}(s)$ the inverse value symbolized by $\tilde{U}^{-1}(s)$ is obtained as follows:

$$\tilde{U}^{-1}(s)=\begin{bmatrix}\left[\frac{22}{3}-\frac{19}{9}s,\frac{-77}{9}+\frac{20}{3}s\right] & \left[\frac{-2}{3}-\frac{1}{9}s,\frac{-5}{9}+\frac{2}{3}s\right]\\ \left[4-\frac{17}{9}s,\frac{-31}{9}+2s\right] & \left[\frac{1}{9}s,\frac{5}{9}\right]\end{bmatrix}$$

Using the arithmetic in Definition 2.6, Definition 2.13, Equation (3.2), Equation (3.3), and Equation (3.4), it can be easily proved that $\tilde{U}(s)\otimes\tilde{U}^{-1}(s)=\tilde{I}(s)$.

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V. CONCLUSION

- Using arithmetic multiplication, as in Equation (3.2), the inverse of the trapezoidal fuzzy number is obtained, such that $\tilde{u}=(u,v,\alpha,\beta)$ and $\tilde{u}^{-1}=\left(\dfrac{2m_q(\tilde{u})-u}{(m_q(\tilde{u}))^2},\dfrac{2m_q(\tilde{u})-v}{(m_q(\tilde{u}))^2},\dfrac{-\alpha}{(m_q(\tilde{u}))^2},\dfrac{-\beta}{(m_q(\tilde{u}))^2}\right)$ produces $\tilde{u}\otimes\tilde{u}^{-1}=\tilde{\iota}$.
- In addition, the inverse of the trapezoidal fuzzy matrix using the elementary row operations method is calculated to obtain the identity value.

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