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Optimized Selective Assembly using Hungarian Algorithm

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Abstract:- Assembly of discrete parts guided by hardware design specification constitutes the final phase in product manufacturing. In the course of mass production of components, mating parts with geometric or dimensional deviation from their intended design can be made acceptable by the identification of suitable pairs after analysing design fit and tolerance limits. By transforming this application-specific problem into a unified mathematical model, an optimal solution can be achieved that minimizes the rejection of non-conforming fabricated parts. Regardless of the type and range of a design fit, the problem can be mapped into a matrix using a ranking function defined by the user. The ranking function is modifiable as per the user requirements and may vary based on the selection criteria for an assembly. Based on the type of ranking function used, the tabulated matrix is solved using the Hungarian minimization/maximization algorithm, which is a powerful combinatorial optimization algorithm that solves the classical assignment problem in mathematics. This approach ensures maximum number of suiting pairs as well as nominal suiting of parts with each other resulting in high-quality products and maximum utilization of fabricated resources.

***Keywords**:- Optimisation, Part Suiting, Hungarian Algorithm, Fit, Tolerance, Assembly.*

I. INTRODUCTION

A mechanical design comes to life after the assembly of its constituent parts according to the engineering design drawings. Assembly of parts are convenient, provided all the part geometry and dimensions conform to the original design dimensions. In the case of a non-conforming geometry or dimension, an acceptance study has to be carried out to determine the effect of deviation in the product design. If the part is not acceptable, salvage actions have to be performed which includes rework, if possible. But if the non-conformance is reported for a mating geometry or dimension, suiting or finding a matching counterpart to achieve the required tolerance fit as per the assembly design is a viable option. This matchmaking is termed as suiting of parts. Figure 1 shows the various fits in a mechanical assembly.

For deviation from the design dimensions, rework or rejection are the practical solutions manufacturers adopt. However, rework is not always a feasible option in scenarios involving mass production of components. Instead, after completion of dimensional inspections, the deviated parts (referred to as parts of type A) can be scrutinized for compatibility with the corresponding deviated mating parts, denoted as parts of type B. The dimensions of each deviated part A are systematically assessed against the dimensions of part B to ascertain a proper fit. For *n* number of part A fabricated, an appropriate pair can be identified from the set of part B to have a Selective Assembly (SA).

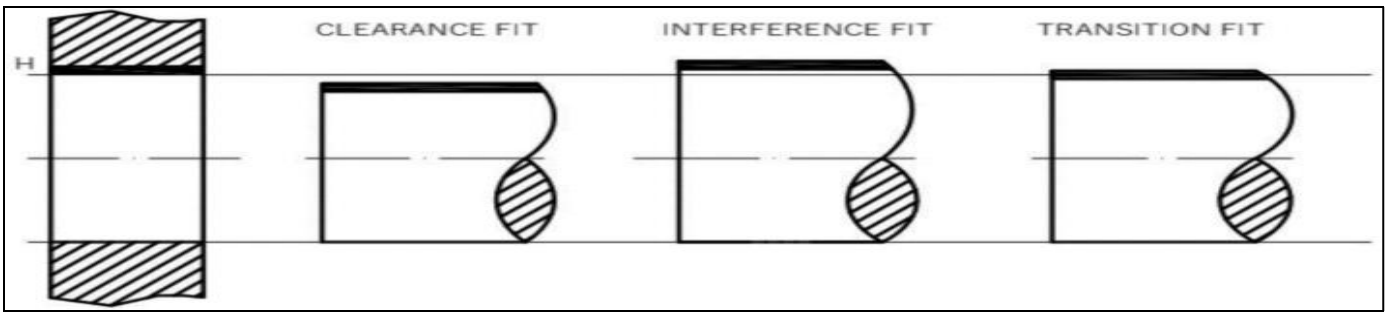

Fig 1 Various Types of Fits in a Hole-Shaft Assembly

The conventional approach of creating such a suiting list involves inspecting mating pairs to determine whether the fit falls within an acceptable range. However, this method, especially in mass production scenario, is time consuming and a complex exercise. The resultant solution may not necessarily be the optimal match where the most number of fabricated parts are made use of. Achieving an optimal solution demands iterative checking of the parts through various combinations. Additionally, when dealing with a significant number of deviated parts, the task becomes even more cumbersome.

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In this study, the problem of identifying suitable pairs is transformed into a minimization assignment problem by assigning ranks to fit values. Subsequently, the Hungarian algorithm, also known as the Munkres algorithm, is employed to solve this assignment problem.

The fit value, denoting the compatibility of a suiting pair, is calculated based on measured dimensions of mating surfaces. The acceptability of a fit value is contingent upon both the type of fit (Refer Figure 1) and the predetermined range of acceptable levels specified for the given assembly.

II. LITERATURE REVIEW

Over the last 40 years, a lot of fundamental research effort has been dedicated to explore the mathematical basis of tolerance analysis. Due to the ever increasing requirements on high precision manufacturing technology, SA has got attraction over the past years. The idea to deal and reduce the dimensional and geometric deviations in product manufacturing buds from 1960s. Since this paper is aimed to apply optimization to a SA problem of 2 parts, the related works are only presented here. Mansoor (1961)[1] in his work, has explored the relation of manufacturing machine tolerance to achievable part tolerances. He used an example of the nozzle unit of a fuel injection pump and categorized the components based on bore diameter. Segregation of mating parts of an assembly into selective groups based on their deviations in dimensions were previously discussed by Mansoor 1961[1]; Fang and Zhang, 1995[2]; Kannan et al., 2003[3]. Fang and Zang (1995)[2] firstly elaborated establishing dimensional parameter relations and the possibility of SA to minimize rejections and avoid loss. The concept of Process Capability Indices (PCIs) is studied and proposed as an intermediary to ensure quality and statistical process control (SPC) parameters by Zhang and Fang, 1999[4]. Kannan et al. (2003)[5] has attempted the problem of SA using an optimization technique of genetic algorithm. This is one among evolutionary algorithms in operational research which starts with a set of random matches and subsequently crossover and mutates to find derived matches to reach near acceptable criteria. Kannan et al. (2008)[6] applied the concept of Taguchi's quality loss function in hole shaft assembly and developed the mathematical models for clearance range in terms of the quality loss function. They used a genetic algorithm to obtain the best combination in the SA. In the work by Tan and Wu, 2012[7], SA for multiple parts making a single assembly is discussed. The problem of direct SA is studied to be of two variants: Direct SA (DSA) and Fixed Bin SA (FBSA). The former is SA using information from measurements on component characteristics directly, whereas the latter is SA of components sorted into bins. The component matching problem for DSA is found as an axial multi-index assignment problem, whereas for FBSA, is an axial multi-index transportation problem. Dantan et al., 2012 [8] in his review work, gives an overview of available mathematical models to solve SA problems. He also elaborates the limitations to extend the problem to a solvable form. Babu and Asha (2015)[9] in their work has employed an artificial immune system algorithm which is a class of computationally intelligent, rule-based machine learning system to solve the SA by taking Taguchi's Loss function as criteria of acceptance along with the achieved tolerance.

As seen above, for component manufacturing with bigger batch size, preparation of suiting pairs for a set of deviated parts is time consuming and the suiting list so arrived does not guarantee an optimal solution. All the present mathematical models and work existing guarantee acceptance but not in its optimum. Here in this work, an optimal solution of part level suiting with product quality adherence is ensured.

III. ASSSIGNEMENT PROBLEM

The assignment problem represents a fundamental combinatorial optimization challenge, commonly depicted through a graphical model known as a complete bipartite graph, as illustrated in Figure 2.

In graph theory, a bipartite graph, also known as a bigraph, is defined as a graph in which the set of vertices can be partitioned into two distinct and independent sets, denoted as U and V[3][10].In this partition, every edge of the graph connects a vertex in set U to a vertex in set V. In a weighted bipartite graph, the edges between the two disjoint sets of vertices (U and V) have associated weights. Each edge is assigned a numerical value, indicating a certain measure or cost associated with the connection between the corresponding vertices in sets U and V. Optimal assignment entails matchmaking of a given size, where the sum of edge weights is either minimized or maximized in a weighted bipartite graph.

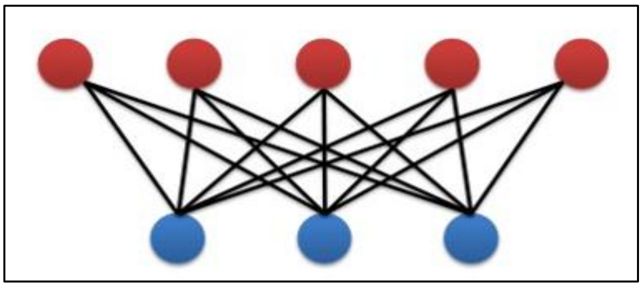

Fig 2 Complete Bipartite Graph

If the number of vertices in set U equals the number of vertices in set V, the problem is termed a balanced assignment problem. However, if the counts differ, resulting in an unbalanced scenario, a conversion to a balanced form becomes necessary before solving.

IV. FORMULATION OF SUITING PROBLEM

The suiting problem is formulated into a matrix form to facilitate the application of optimization methods, allowing users the flexibility to choose their preferred optimization approach. Consider a scenario with n instances each of part A and part B. The fitness values for all conceivable pairings can be systematically organized into a matrix, as exemplified in Table 1, demonstrating the concept using

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three instances of both part types. Subsequently, using the available inspection reports for an array of parts, these fitness values are populated within the matrix, constituting a valuable dataset for optimization.

To elevate this matrix into a mathematically optimized problem, a ranking system is introduced based on predetermined selection criteria. This ranking process, an integral precursor to optimization, allows for the categorization of data such that optimal values assume low ranks in the case of minimization objectives, while unacceptable fits are assigned elevated ranks. The converse holds for maximization algorithms, where superior fits garner higher ranks, and undesirable fits are relegated to lower ranks.

In scenarios where the quantity of parts available for pairing is uneven, dummy parts are introduced to convert the problem into a square matrix. Notably, the rank values assigned to dummy parts mimic those of unacceptable fits. Following the establishment of a balanced ranked matrix, optimization procedures can be instituted.

During a dimensional inspection, various locations of the same dimension in a given part undergo measurement, and the resulting minimum and maximum values are documented in the inspection reports. Consequently, a single dimension for a particular part is represented by two distinct values, denoting its minimum and maximum extents. In the assessment of fits, particularly for interferences, both of these values demand consideration.

For instance, in the case of evaluating interference between two parts, denoted as A (the shaft) and B (the hole), the minimum possible interference is calculated as $\boldsymbol{B_{max}} - \boldsymbol{A_{min}}$, while the maximum possible interference is determined as $\boldsymbol{B_{min}} - \boldsymbol{A_{max}}$.

As each fit encompasses two distinct values arising from the minimum and maximum measurements of a given dimension, two rank matrices are constituted to address the minimum & maximum acceptability range. The effective ranking matrix which is used for optimization is determined by the summation of both rank matrices. To ensure the positivity of the cumulative value and prevent nullification due to cancellation, the magnitude of each value is considered for the summation process.

Table 1 Matrix Formulation Based on Fit Values for a Two-Part Suiting

Parts	B1	B2	B3
A1	Fit for A1 with B1	Fit for A1 with B2	Fit for A1 with B3
A2	Fit for A2 with B1	Fit for A2 with B2	Fit for A2 with B3
A3	Fit for A3 with B1	Fit for A3 with B2	Fit for A3 with B3

V. RANKING FUNCTION

Any selection problem can be reformulated into an optimization assignment problem through a systematic conversion to its optimizable matrix form. This transformation holds significant import, as the criteria for selection or assignment may vary from one problem to another. The nuanced variations in these criteria must be effectively captured through a mapping function.

In the context of the current part suiting problem, a specific range of acceptable fits must be allotted a lower rank, while unacceptable fit ranges are assigned a higher rank. Given the adoption of a minimization algorithm, the chosen ranking function should map the most optimal fit to the minimum rank. Within the acceptable fit band, the optimal fit is defined as the mid-value, and the rank incrementally increases towards both sides within the band limits. Conversely, the unacceptable fit ranges are assigned an exceptionally high rank. The adopted mapping for this particular problem is illustrated in Figure 3 and is mathematically expressed by the following equation:

$$R = \left\{ \begin{matrix} \left|\boldsymbol{F} - \boldsymbol{R_{avg}}\right|, \\ for\ F \in [Min_{limit}, Max_{limit}] \\ \boldsymbol{1000} * \boldsymbol{Max_{limit}}, \\ for\ F \in (-\infty, Min_{limit}) \cup (Max_{limit}, +\infty) \end{matrix} \right\} \ldots\ldots (1)$$

R - Rank

F - Fit obtained for given pair

R_{avg}- Theaverage of accepted fit band limits

Min_{limit} *is the minimum limit in the accepted fit band*

Max_{limit} *is the maximum limit in the accepted fit band*

The above function is similar to the famous Taguchi's Loss function, $L = k(y - m)^2$, where m is the theoretical 'target value' and y is the actual size of the product, k is a constant and L is the loss[6]. So in equation (1) loss is similar to Rank (R), theoretical target value is similar to average of accepted fit band limits (R_{avg}), and the actual size of the product is similar to the fit obtained for the given pair (F). Hence on minimization of rank, we are essentially maximizing the quality of the fit. Once the minimum and maximum fit values are processed through the mapping function, the resulting rank matrices for both the minimum and maximum fits are combined using summation to determine the cumulative effect of the fit combination's acceptability. In the consolidated matrix, all elements exceeding or equal to 1000 times the maximum limit are preserved to uphold uniformity in rejection criteria. The conclusive rank matrix is subsequently subjected to the Hungarian optimization technique.

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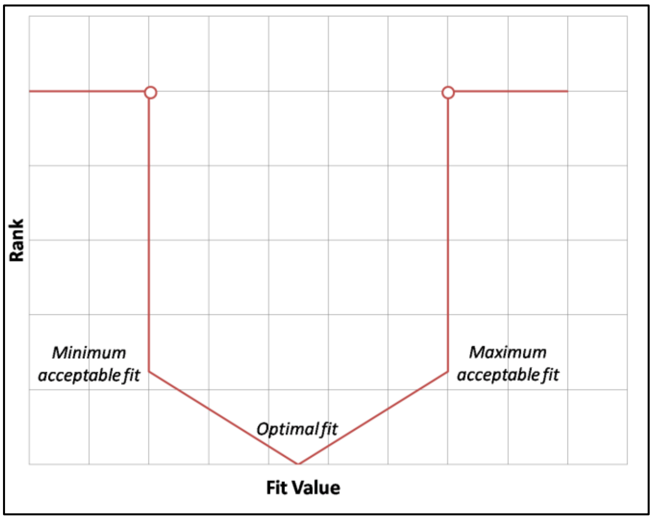

Fig 3 Minimization Mapping (Piece-Wise Linear Graph)

Typical minimization mapping using a piece-wise linear graph is illustrated in Figure 3. The mapping process assigns optimal and acceptable values to a lower rank, resembling a dip, while unfavourable or unacceptable values are allocated to a higher rank. The best fit (R_{avg}) receives the lowest rank in this ranking scheme.

In cases for 2 parts having multiple mating interfaces (Eg: pitch circle diameter holes) ranking can be extended by adding ranks of individual mating dimensions.

VI. HUNGARIAN OPTIMIZATION ALGORITHM

The Hungarian method is a polynomial-time optimization algorithm designed to solve assignment problems efficiently. Widely applicable to various primal-dual optimization problems, the algorithm revolves around a series of matrix operations aimed at simplifying the matrix to its most reduced form, ultimately leading to the determination of the optimal solution[11]. Subsequently, the assignment operation is performed.

Hungarian optimization algorithm ensures the gradual refinement of the assignment until an optimal solution is achieved, covering the nuances of the assignment problem systematically.

VII. A SAMPLE SUITING PROBLEM: FORMULATION AND RESULTS

Consider a sample suiting having five numbers of each component within a mating pair, as illustrated in Figure 4. The acceptable interference fit range for the parts is 10-35µ. Φ10.8 is the mating dimension for the parts.

The available dataset comprises the inspected values of mating dimensions, as outlined in Table 2.

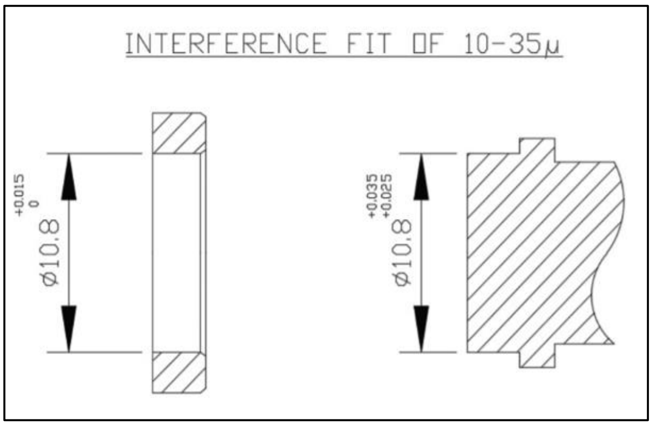

Fig 4 Typical Example of Mating Pairs with an Interference Fit

Table 2 Inspected Dimensional Values of Mating Surface for Parts A & B (5 nos. each)

Part P			Part Q		
Idn. No.	**ø10.8 +0.035/+0.025**		**Idn. No.**	**ø10.8 +0.015**	
	Min Pmin	***Max P max***		***Min Q min***	***Max Q max***
P1	**10.822**	10.825	**Q1**	**10.793**	10.801
P2	**10.823**	10.826	**Q2**	**10.798**	10.811
P3	10.825	10.827	**Q3**	**10.791**	10.807
P4	**10.824**	10.828	**Q4**	**10.792**	10.805
P5	10.826	10.829	**Q5**	**10.790**	10.804

For this particular problem, all the possible pairing combinations of the part P and part Q are listed as in Table 3. Finally 5 optimal pairs have to be identified in such a way that each part P is matched with a unique part Q. In the below combination matrix, the optimization is to be done with respect to their fit value.

Table 3 Possible Suiting Combinations

	Q1	**Q2**	**Q3**	**Q4**	**Q5**
P1	P1-Q1	P1-Q2	P1-Q3	P1-Q4	P1-Q5
P2	P2-Q1	P2-Q2	P2-Q3	P2-Q4	P2-Q5
P3	P3-Q1	P3-Q2	P3-Q3	P3-Q4	P3-Q5
P4	P4-Q1	P4-Q2	P4-Q3	P4-Q4	P4-Q5
P5	P5-Q1	P5-Q2	P5-Q3	P5-Q4	P5-Q5

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For an interference fit problem, the minimum interference is given by $\boldsymbol{P_{min} - Q_{max}}$ and maximum interference is given by $\boldsymbol{P_{max} - Q_{min}}$. For all the combinations as in Table 3, a minimum interference matrix is tabulated and ranked using equation (1) to generate M1 matrix (Table 4). Similarly, maximum interference matrix is tabulated and ranked to generate M2 matrix (Table 5).

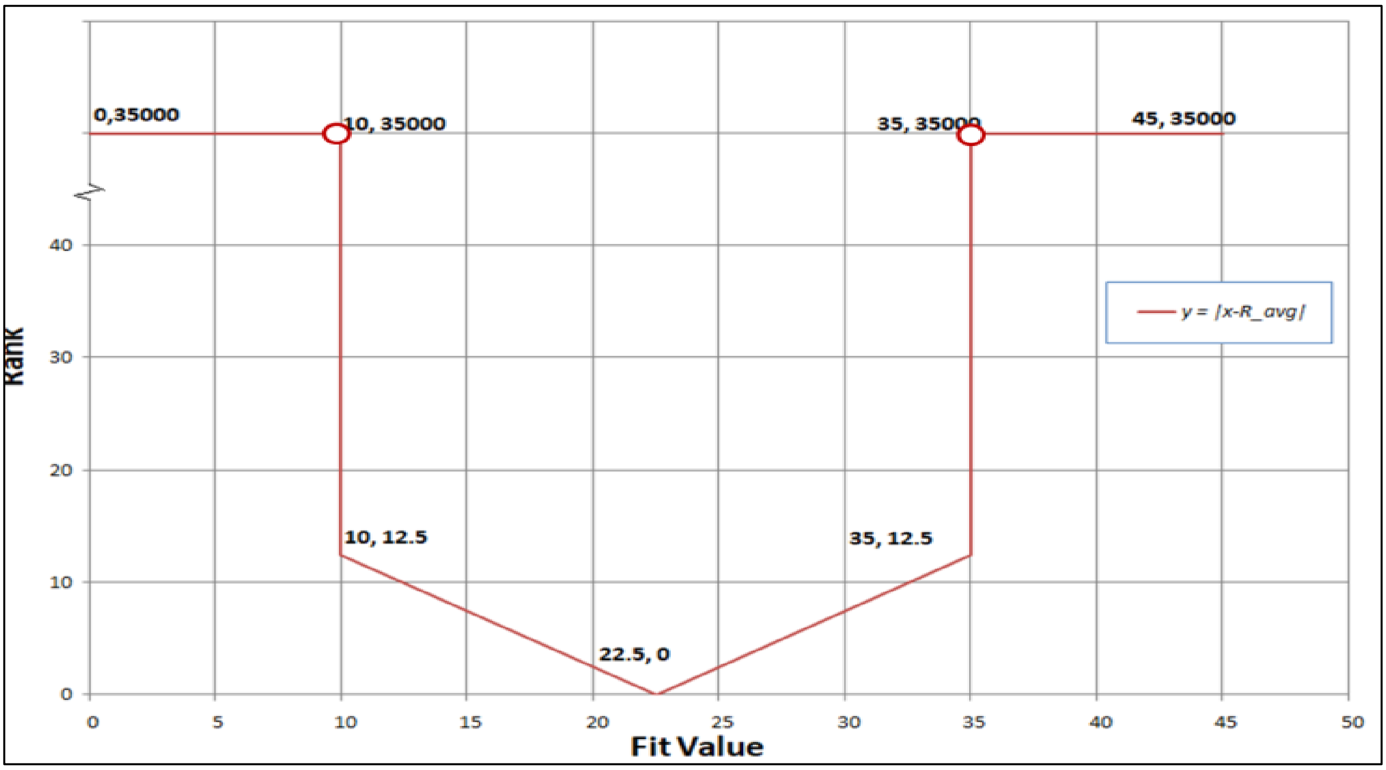

Fig 5 Minimization Mapping using Ranking Function

Table 4 Matrix M1

	Q1	Q2	Q3	Q4	Q5
P1	2	12	8	6	5
P2	1	11	7	5	4
P3	2	9	5	3	2
P4	1	10	6	4	3
P5	3	8	4	2	1

Table 5 Matrix M2

	Q1	Q2	Q3	Q4	Q5
P1	10	5	12	11	13
P2	11	6	13	12	35000
P3	12	7	35000	13	35000
P4	13	8	35000	35000	35000
P5	35000	9	35000	35000	35000

These matrices are added to give the net ranks. The elements greater than $1000 * \text{Max}_{\text{limit}}$ are changed to $1000 * \text{Max}_{\text{limit}}$ itself to avoid unnecessary iterations in computation as they all represent unacceptable pairs and are hence similar. The final M matrix in Table 6 gives the rank of fits against all the combinations.

Table 6 M1+M2=M

	Q1	Q2	Q3	Q4	Q5
P1	12	17	20	17	18
P2	12	17	20	17	35000
P3	14	16	35000	16	35000
P4	14	18	35000	35000	35000
P5	35000	17	35000	35000	35000

Further employing Hungarian algorithm as explicated in Section 5, the M matrix is solved and is depicted in Table 7.

Table 7 Optimised Solution

	Q1	Q2	Q3	Q4	Q5
P1	0	5	0	3	***0***
P2	0	5	***0***	3	34982
P3	0	2	34978	***0***	34980
P4	***0***	4	34978	34984	34980
P5	34983	***0***	34975	34981	34977

Hence, the SA identified is tabulated in Table 8 with minimum and maximum interference value.

Table 8 Selective Assembly of the Sample Problem

Sl. No.	Part 1	Part 2	Min. Int.	Max. Int.
1	P1	Q5	18	35
2	P2	Q3	16	35
3	P3	Q4	20	35
4	P4	Q1	23	35
5	P5	Q2	15	31

VIII. CONCLUSIONS

In this study, the two-part suiting problem within an assembly is transformed into a minimization assignment problem by assigning a rank to the design fit value. Subsequently, the problem is resolved utilizing the Hungarian algorithm. The key observations derived from the implemented method of suiting are outlined below:

- The developed method demonstrates a capacity to yield optimal solutions, encompassing a maximal number of viable suit pairs and achieving optimal matches.
- The computational formulation of the entire suiting problem, transitioning from manual processes to computational algorithms, notably diminishes the requisite time compared to traditional manual suiting methodologies.
- The method allows in experimenting with the fit values giving an ability to be selectively stringent or lenient with the tolerance values as demanded by the application.
- Designs with micron level tolerance fit, say 2µ to 3µ as in fuel pump component parts of automobiles is achieved by suitably machining with the achieved dimensions of its mating part. Introduction of this suiting software liberates such production processes from the tedious in-situ suiting methods to cost-effective batch production processes.
- The mapping function used for the acceptable tolerance range is similar to Taguchi's Loss function, $L = k(y-m)^2$. Thus the presented method ensures SA is of tolerance fit with minimum quality loss, ie. maximum quality and maximum suiting pairs compared to existing mathematical algorithms implemented.
- The devised formulation delivers solutions through the utilization of the Hungarian algorithm. Given the polynomial time complexity of the algorithm it is imperative to acknowledge that the computational time escalates proportionally with the number of parts slated for pairing [12].

FUTURE SCOPE

The suiting problem addressed in this study, on mating pairs, holds the potential for extension to encompass an entire assembly, beginning with a minimum of three parts. As the 2-part suiting problem unfolds into a 2-dimensional matrix assignment problem, scaling up to an n-part selection problem introduces an assignment problem encapsulated within an n-dimensional matrix. The formulation of solutions and the algorithms implicated in this expanded context become subjects of heightened interest.

The prospect of devising a computational method tailored to handle assembly challenges involving multiple parts holds significant promise. With comprehensive inspection data available for all groups of parts, the envisioned methodology allows for the optimal solution and assembly of any given pairs in a singular execution. Such a computational approach not only ensures optimization but also facilitates the refinement and perfection of assembly procedures. As elaborated by Tan and Wu, 2012[7] Fixed Bin Selective Assembly of n parts, is an axial multi-index transportation problem requiring further research.

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