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Theory and Design of Audio Rooms: Physical Formulation

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Abstract:- The time-dependent statistical chains of transition matrix B are successfully used to calculate the reverberation time TR and sound energy density field in audio rooms.

This is a real breakthrough in the search for a statistical derivation of Sabines' imperial theory which has never been rigorously proven since 1898.

Additionally, we provide proper design and calculations of sound wave energy density in audio rooms via two cuboid room examples.

We provide proper design and calculations of sound wave energy density in audio rooms via two cuboid room examples. We also propose some general statistical rules that can be applied to statistical problems of physical phenomena in general, such as the thermal diffusion equation and the theory of sound waves in audio rooms.

We also give a description of the theory and design of what we call comfortable sound rooms based on the purpose of their use via well-defined measurables.

Finally, we provide a consistent derivation of Sabines' theory without using Ls=4V/A or any other statistical assumptions.

I. INTRODUCTION

In fact, the listener in an audio room hears the sound coming from the source either directly or through reflections from the walls, floor and ceiling.

These reflections are called reverberations and shown in Fig.1.

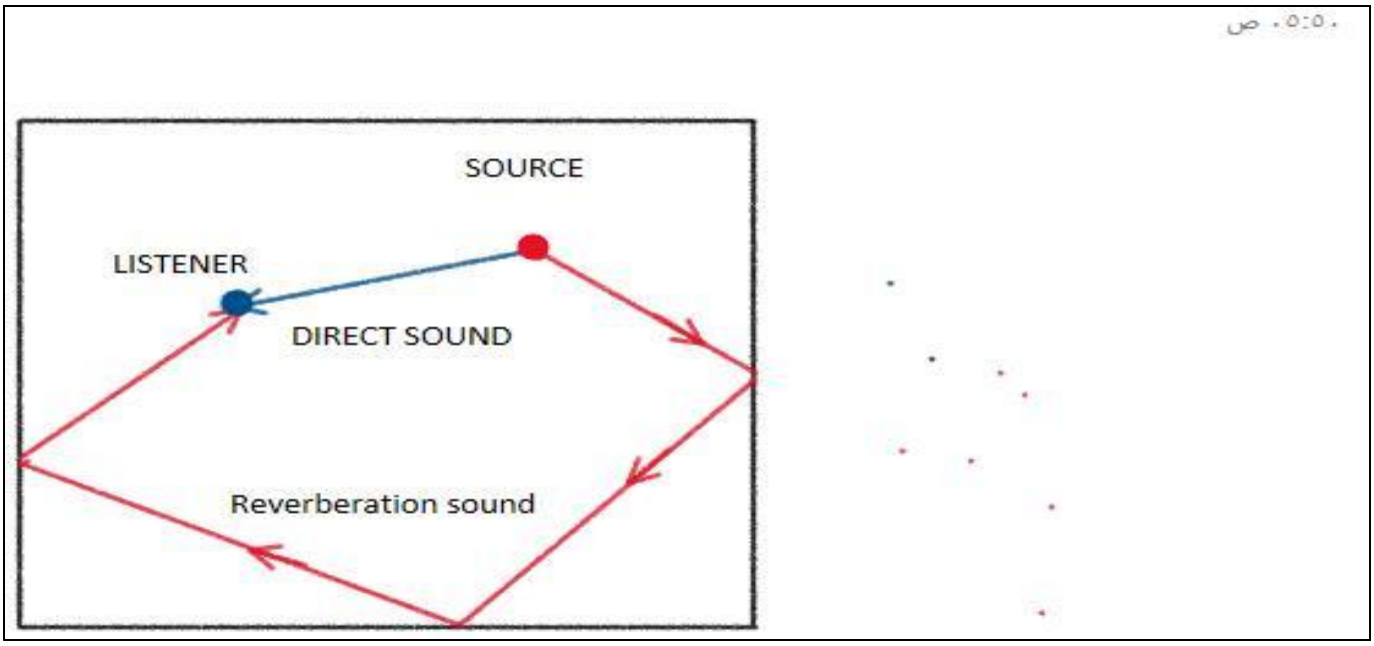

Fig 1: Direct Sound and Reverberation Sound in a Rectangular Audio Room

➢ *The Quality of Audible Sound in Audio Rooms is Determined by Five Main Factors:*

- An Appropriate TR Reverberation Time. The reverberation time TR is defined as the time elapsed until the sound intensity drops to one millionth of its initial intensity when the source suddenly stops.
- The intensity of audible sound in practice is between the hearing threshold of 10^-12 watt/m^2 and the pain threshold of 1 watt/m^2.

  Let's choose something like 10^-4 to 10^-7 watt/m^2 to suit a normal, healthy ear.

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This corresponds to a range of 40 to 70 dB, which is the recommended sound level in audio rooms to be completely audible and most comfortable to the human ear.

- The sound intensity must be uniform throughout the sound room with a practical distribution of audible sound throughout the sound room.
- Low noise-to-signal ratio.
- The value of direct sound intensity compared to reverberated sound intensity is minimal.

The most important factor among the five above is the reverberation time TR, which must be calculated when designing audio rooms.

TR differs significantly in audio rooms for different usage purposes.

The following is a guideline for TR in different sound rooms:

- The appropriate or recommended TR for large cathedrals and mosques last between 2 and 2.5 seconds.
- TR = 2 seconds is an optimal reverberation time for a concert speech and musical entry.
- TR = One second is an optimal reverberation time for a conference room in amphitheater.
- TR = 0.3 to 0.5 seconds is the standard for recording studios.
- TR Below 0.3 seconds there is an acoustically dead zone while
- TR above 2.5 seconds is a boring echogenic piece.

It is worth mentioning that sound intensity is conveniently measured in decibel units since the human ear is created to hear according to a logarithmic function.

Is (db) = 10 Log (base10) Is / Is (0)
Is(0) is conveniently chosen as the hearing threshold for a normal human ear of a healthy person.
Is(0)= 10^-12 watt/m^2 or zero decibel.

And the pain threshold that harms humans ears is at 1 watt/m^2, or 120 db.

II. THEORY

We begin by explaining Sabine's original theory of reverberation time in audio rooms.

The quantifiable theory of room acoustics was established based on Wallace Sabine's theory of reverberation in 1898.

W. Sabin discovered that assuming that the speed of sound in air at normal temperature and pressure C, is fixed at 330 m/s, then an empirical formula for TR (60 db) simply denoted TR, appears as [1, 2],

TR=53.13 V/ASC. . second . . . . (1)

And by replacing C with 330 m/s, equation 1 reduces to,

TR = 0.161 V/A S . . second. . . . . (1*)

For empty rooms..

Sabine's formula for the reverberation time of sound waves TR in audio rooms, founded over a century ago, is experimental and has never been proven since.

where V is the volume of the room in m^3, A is its total interior surface in m^2 and S is the average sound absorption defined as, S (av) = (A1 S1 + A2 S2 +.. An Sn) / (A1 + A2 + ...An) . . (2)

We propose to reformulate equation 1 and adapt it when the number of humans N is populated inside an audio room.

For this purpose, we assume that *the body sound absorption of an average human individual is between 0.2 and 0.4 Sabine.*

Thus for soundproof rooms populated by N humans, we propose that the denominator of equation 2 be simply replaced by,
[AS+ N (humans) * 0.3] in the Sabine units.

In other words, equation 1 becomes:

TR = 0.161 V/(AS + N(h) *0.3). . second. . . ……………. (3)

Giant scientist W. Sabine presented his precise imperial formula of reverberation time TR as a result of precise experimental measurements in different sizes and shapes of audio rooms in 1898,

TR = 0.161 V/A S . . second . . . (1*)

Moreover, he intuitively founded a sort of intelligent derivation of his formula that makes it a kind of theory.

For this purpose, Sabines defined a statistical length Ls as the average statistical distance between two reverberations or successive sound reflections on the walls, floor and ceiling of the audio room (See Fig. 1) as follows:

Ls = 4 V / S . . ………………………………………(4)

In fact, Equation 4 forms a satanic trap or explosive minefield, not because it is inaccurate but because no scientist has been able to understand its interpretation or pilot it mathematically for over a century.

Since the time of W. Sabine, many physicists and mathematicians have proposed an improvement or reformulation of Sabine’s reverberation formula, but they have fallen into the satanic trap of using equation 4 without rigorous proof [3].

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This means that they added nothing to the theory of absorption of sound energy by reverberation.

Sabine reverberation theory is based on two deep equations or corner pillars, namely equations 1 and 4. To arrive at equation 1, one must first prove equation 4, which is almost impossible.

The reason is that equation 4,

Ls = 4 V / S . . . . . (4)

is statistical in nature based on a deep probabilistic scheme which is missing in current mathematics and theoretical physics [4,5].

The proper derivation of equation 4 should instead be a physical statistical derivation based on the correct definition of transition probability such as the numerical theory of Cairo techniques and, therefore, the derivation of equations 1 and 4 is contained in the strings numerical statistics of transition matrices B. [1, 2,5,6].

Contrary to Google search and Wikipedia, we can say that probability and statistics belong to modern physics (classical physics plus numerical statistical theory called Cairo techniques [4,5]) rather than mathematics.

In other words, the only correct verification of sound intensity and reverberation time, which forms the basis of the theory and design of audio rooms, can only be considered in the Cairo theory of techniques or any other adequate theory.

We remind again that equation 4 is of a statistical nature and expressed as follows,

Ls = 4 V/S. . . . (4)

For all 3D geometric shapes.

Note that equation 4, which has never been proven mathematically, is conditional but it leads to the following rule which we call the AB rule:

- *The AB Rule: 3D bodies of Different Shapes cannot have the Same Volume-to-Surface Ratio Unless they have Exactly the Same Volume and Surface Area.*

Note again that rule AB is only true under certain conditions.

Returning to Sabine's original theory, he assumed that the energy density of sound waves decreases according to the geometric sequence resulting from absorption or successive reverberations, as shown in Figure 1.

U(t)= (1-S)^N. U(0) . …………………………………..(5)

Equation 5 represents the discrete time dependence of the decrease in sound energy density U in audio rooms.

Where N is the number of reflections corresponding to the same number of repetitions or iterations in numerical simulation.

And the vector U(0) are the initial conditions or the sound intensity at time t=0 when the sound source is suddenly cut off.

However, equation 5 which is a discrete geometric sequence reduces to an exponential function for a continuous distribution, namely:

U(t)=U(0) Exp(- λ* t) . . …………………………………..(6)

Equation 6 represents the continuous time dependence of the decrease in sound energy density U in audio rooms.

Note that Equation 6 conforms with the AB rule.

*Equation 6, which has also never been explicitly proven theoretically, is another profound theorem in itself.

The existence of the exponent λ as an exponential coefficient independent of time shows that hypothesis 6 resulting from the fundamental hypothesis dU/dt =-Const * U is true.*

Our starting point will now be the dependence of λ* on RO (the input elements of the main diagonal of matrix B), which we find surprising and worth examining in detail.

The dependence of λ* on RO will reveal or derive new relationships and connections between theoretical physical quantities and mathematical axioms never before discussed.

This is the topic of the next section Numerical Results III, where we will focus on uncovering this relationship by examining the particular configuration of two 3D cubic audio rooms without loss of generality.

III. NUMERICAL RESULTS

- *In Figure 2 We Show the Simplest 3D Geometry of a Sound Room:*

A cube side of length l and the cube is discretized into 8 equidistant free nodes.

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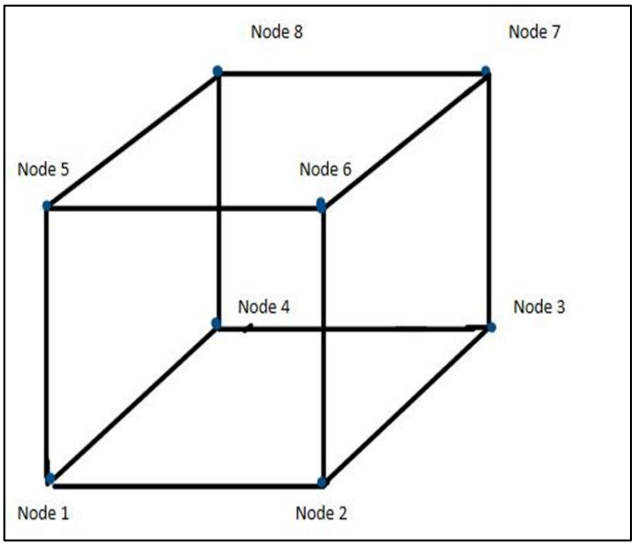

Fig 2: 3D Cubical Sound Room of Side Length L Divided into 8 Equidistant Free Nodes

In Figure 3 we present a slightly more complicated geometric shape:

A cube side of length l and the cube is discretized into 27 equidistant free nodes.

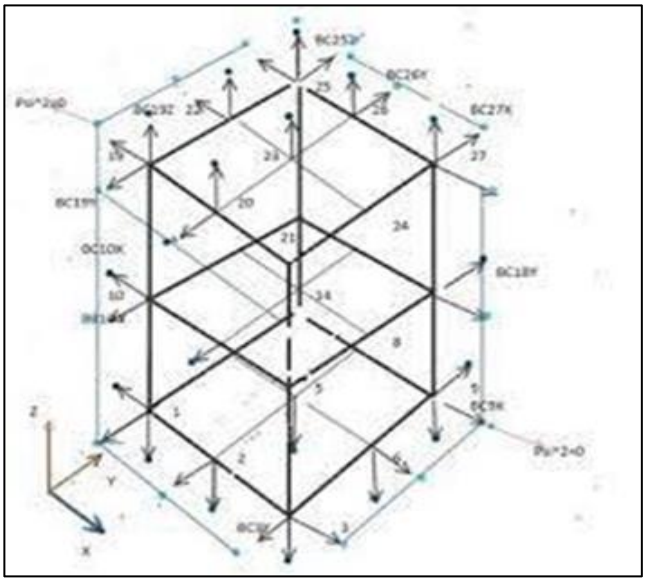

Fig 3: 3D Cubical Sound Room of Side Length L Divided into 27 Equidistant Free Nodes

We use the two particular 8x8 matrix chains B in Figure 2 and the 27x27 matrix chains B in Figure 3 to show that they are consistent with formulas 5 and 6 without loss of generality.

In other words, the mechanics of B-matrix chains with specific values of RO can be the statistical equivalence of Sabines theory.

Let us recall again that the element RO of [0,1] is the input element of the main diagonal of the statistical transition matrix B.

Note that in the 4D unit space of the strings of the matrix B, the real time t in the Sabines formula is replaced by N dt and the dimensionless time is simply N.

Furthermore, the matrix chain B which is a product of the statistical theory of Cairo techniques can find a computational expression for the exponential decay coefficient λ*.

Here, the statistical transition matrix **B** 8x8 for audio rooms shown in Fig.2 is given by [3,4,5],

RO 1/6-RO/6 0 1/6-RO/6 1/6-RO/6 0 0 0
1/6-RO/6 RO 1/6-RO/6 0 0 1/6-RO/6 0 0
0 1/6-RO/6 RO 1/6-RO/6 0 0 1/6-RO/6 0
1/6-RO/6 0 1/6-RO/6 RO 0 0 0 1/6-RO/6
1/6-RO/6 0 0 0 RO 1/6-RO/6 0 1/6-RO/6
0 1/6-RO/6 0 0 1/6-RO/6 RO 1/6-RO/6 0
0 0 1/6-RO/6 0 0 1/6-RO/6 RO 1/6-RO/6
0 0 0 1/6-RO/6 1/6-RO/6 0 1/6-RO/6 RO

In fact, time-dependent solution given by equations 5,6

Is the same as that given by the B-matrix chains for the initial value problem [1,2,4,5,7], namely,

U(t)=B^N . U(0) . . . ……………………………………... (7)

U(0) is the initial distribution vector of the sound energy density at t=0.

The numerical solution of the matrix B^N shows that,

B^N. U(0)=r^N U(0) ………………………………… . . (8)

with r less than 1, which means that equations 7,8 are a geometric sequence corresponding to that of W. Sabine with r^N in place of Sabines (1-S)^ N of Fig. 1.

Note that Sav of the limits of the sound room is equal to RO (the storage of the energy density at the free nodes considered).

The above statement is somewhat equivalent to the mathematical theorem of Stoke and G. Green**. Bring the border inward.**

In other words, Stoke and Green's theorem of taking the total or average around the boundary and relating it to what's happening inside is in some ways the same thing contained in the techniques physics and the statistical theory of matrix chains B.

This means that Eq 7 offers the solution for the reverberation time in audio rooms without the need for equation 4 or any other statistical assumptions.

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It also predicts and proves an exponential decay of sound energy density in audio rooms as Eq 6,

$$U(t)=U(0)\ Exp(-\lambda^* t)$$

Same takes place for the transition-matrix B 27x27.

Note that the geometric sequences are the discrete version of exponential functions, which are continuous.

Note also that while the numerical values of RO are elements of the closed interval [0,1] but the calculations prove that the solution diverges for RO .GE. 0.5.

It can be shown that [1,2,9],

B^N. IC = (0.5+0.5 RO)^N . IC. .. . . ………………..(9)

IC is the initial conditions vector assumed uniform for simplicity.

It follows from equation 9 that,

r= (0.5+0.5 RO) . . . . . . ………………………………(10)

It is worth mentioning that equation 10 is confirmed by equation 8. Both equations give the same result for the basis of the geometric sequence r.

And,

λ^* = - Log (0.5 + 0.5 RO)

Table I shows the numerical results of r and λ^* vs RO for the transition matrix B 8x8.

RO	0	0,1	0.2	0-3	0.4	0.5	---
r	0.5	0.55	0.6	0.65	0.7	0.75	---
Sav walls	0.0	0.1	0.2	0.3	0.4	0.5	----
λ^* (exp decay coefficient)=log (r) to the base e.	0.693	0.598	0.511	0.431	0.357	0.287	-----

Table I. the numerical results of r and λ^* vs RO for the transition matrix B 8x8.

A striking rule emerges, the statistical theory of Cairo techniques proposes an expression to evaluate λ^* in terms of the average absorption S.

It also shows that λ being minimum for a maximum S = 1. This minimum proposed by the chains of matrix B should be Log(2)=0.693

This means that λ cannot be zero and that energetic decay by diffusion has a maximum rate. It is surprising that W. Sabine noticed and experimentally measured this minimum with almost the same value mentioned above a century ago.

This also shows that, the B-matrix strings provide the first rigorous proof of Sabine's reverberation theory as follows:

U(t) = U(0). Exp -t [Constant. Area / C . Volume]. . . (8)

Again, C is the speed of sound in the audio room.

Equation 8 applies to sound energy in audio rooms and the thermal diffusion equation with Dirichlet boundary conditions.

The exponential dependence of equation 8 can be described from the chains of matrix B, equations 5,6.

A simple algebraic manipulation of equation 8 reduces equation 8 to,

I(t)/I(0)=-t V / C . A.S = 10^-6 , when t = TR.

Therefore,
TR = 6 .[3^2]V / C A S
TR= 54/ C A S

Which is Sabines formula 1. By replacing the speed of sound C by 330 m/s, we obtain,
RT= 0.1636/ A S

Which is Sabines formula 1*.

Showing that Sabines formula is fairly accurate.

The term 3^2 =9 is the square of number of intervals h in side length (Fig.1) which is (n+1)^2.

Obviously this term would be 4^2=16 for the case of Fig.2 and we arrive at the same result for RT[1].

It is worth mention that the factor (n+1)^2 is a consequence of Greens theory.

Moreover, it is also an approximate basis for calculation of sound intensity Is W/m^2 in audio rooms.

If **Is** is assumed to be uniform.

The reverberation time TR seconds for an empty room, as given by Sabines formula, can be expressed as follows:

TR = 53.46 V/ C A S. . second. . . . . . (10)

Assuming the speed of sound in air C, at NPT is 330

m/s, then,

TR = 0.162 V/A S,. . second. . . . . (11)

For empty rooms.

where V is the volume of the room in m^3, A is its tot

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The reverberation time TR seconds for an empty room, as given by Sabines formula, can be expressed as follows: TR = 53.46 V/ C A S. . second. . . . . . (12)

Cairo techniques theory predicts the intensity of sound in audio rooms Is expressed as,

Is = Σ (sigma) or sum of sound power sources (P1+P2+..+Pn)in watts / (A S + N (humans* 0.25) . . . Watts/m^2. . . . (13),

Eq 13 can be expressed in terms of reverberation time TR as,

Is = sum of sound power sources in watts * TR / 0.161 V .. . Watt/meter ^ 2 .... (14)

Equations 13, 14 can be used effectively in audio room theory and design.

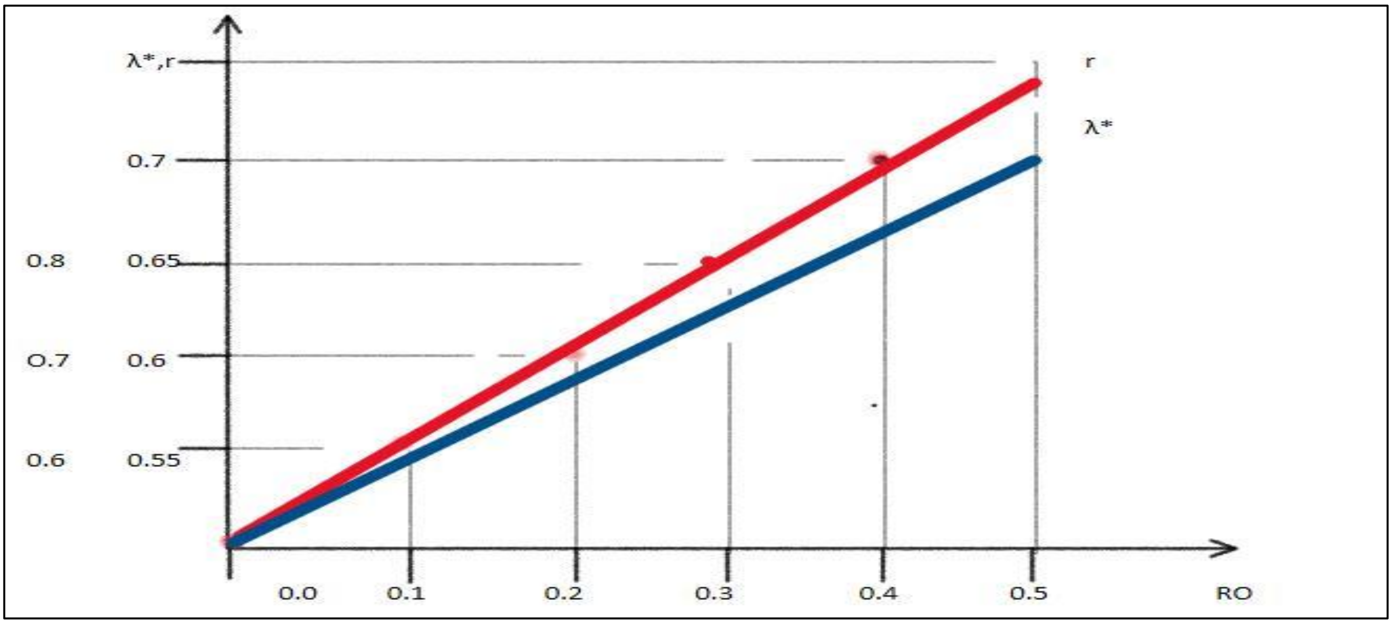

Fig 4: numerical results for r and λ* vs RO presented in table I.

Figure 4 shows that r and λ* are linear functions of RO as predicted by the time-dependent solution given by equation 4.

- Note that reverberation time TR is inversely proportional to Sav or RO.

IV. CONCLUSION

Sabine's semi-imperial formula, proposed a century ago, is the most accurate accepted formula for calculating reverberation time in audio rooms (RT), and allows approximately estimating the volume of uniform sound .

Sabines' theory is based on the statistical hypothesis according to which the average length Ls between two successive reflections or reverberations is given by Ls = 4V/A.

This statistical hypothesis has never been proven mathematically since Sabine's time, but transition matrix statistical chains provide the required derivation.

Finally, we provide a consistent derivation of Sabines' theory without using Ls=4V/A or any other statistical assumptions.

The author uses his own double precision algorithm [10,11].

Python or MATLAB library is not required.

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