ISSN No:-2456-2165 https://doi.org/10.5281/zenodo.14565229

Electric Field Potential and Methodology of its Theoretical Calculation

Xolboyev Yunusali Xasan o‘g‘li, Tursunmetov Kamiljan Axmetovich
National University of Uzbekistan, University St. 4, Tashkent 100174, Uzbekistan

Abstract:- For the first time, a method for the theoretical calculation of equipotential electric field lines created by a system of two-point charges has been developed, and a family of equipotential lines for a system of two-point charges has been created.

Keywords:- *Electric Field, Charge, Field Strength, Potential, Equipotential Lines, Equation, Ferrari Method.*

I. INTRODUCTION

The interaction of electric charges was studied by Charles Coulomb in his torsion balance at the end of the 18th century (in 1785). At this point, the question arises as to how the force of interaction of electric charges appears. The presence of electric interaction is the presence of an electric field around electric charges. The field formed around stationary electric charges or charged systems is an electrostatic field [1-3]. The electrostatic field can be characterized in terms of quantity and energy. To quantitatively characterize the electric field, a physical vector quantity called the electric field strength is introduced. If we introduce a test charge q_0 into the electric field, according to Coulomb's law, a force $\vec{F}$ is exerted on the charge $q_0$by the field. The force exerted by this field on a unit test charge q_0 introduced into an electric field is called electric field strength and is expressed as follows:

$$\vec{E} = \frac{\vec{F}}{q_0} \qquad (1)$$

To describe the electric field graphically, the intensity vector at each point of the field must be given. It is convenient to use power lines to do this lines of force or vector lines of field strength are such lines drawn in the electric field that the direction of the effort applied to any point of this line coincides with the direction of the vector E (Fig. 1) [1].

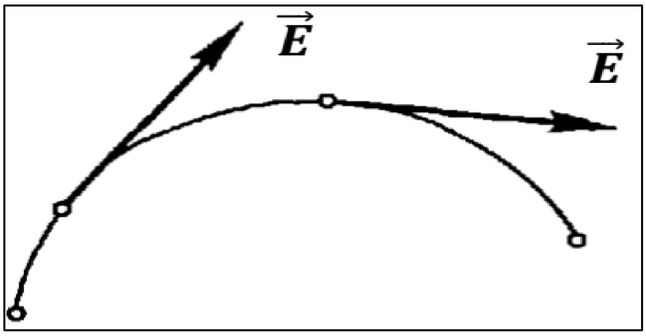

Fig 1: Electrical Field

With the help of lines of force, you can get information about the amount of tension in different parts of the field and how it changes in space. Electric field lines of force are conventionally assumed to be outgoing from a positive charge and incoming to a negative charge (Fig. 2).

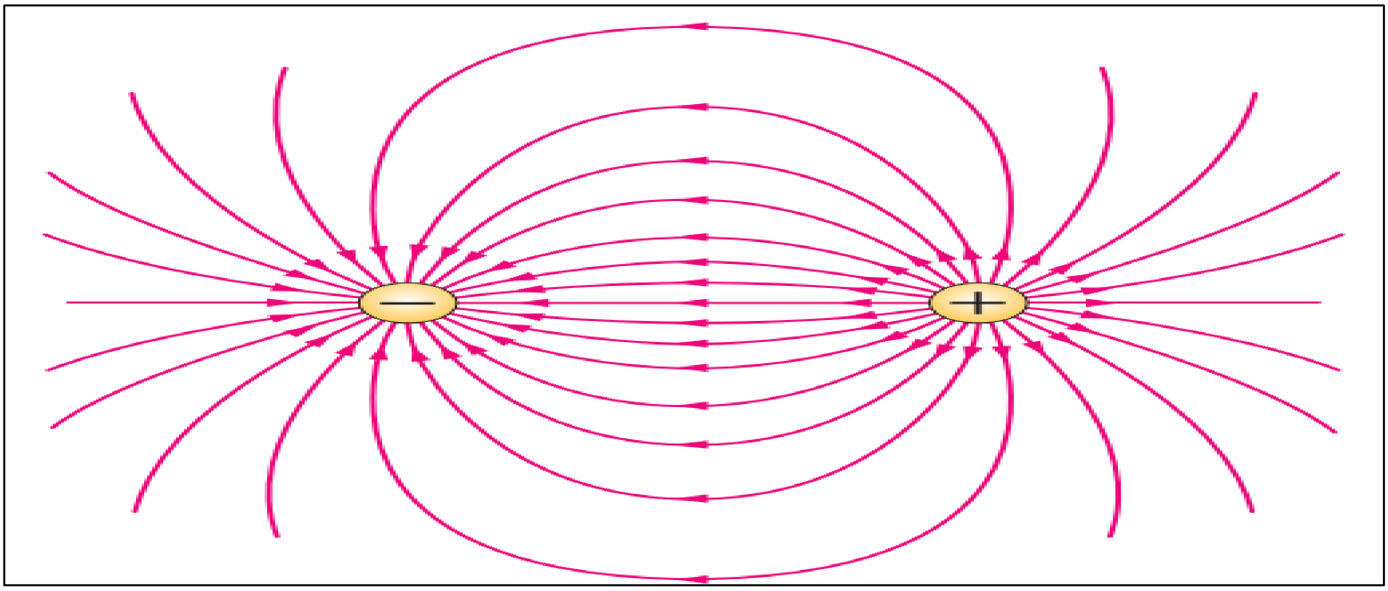
Fig 2: Electric Field Lines

ISSN No:-2456-2165 https://doi.org/10.5281/zenodo.14565229

The voltage lines shown in Figure 2 are appropriate for charges with equal modulus and a small distance between the charges. If the values of the charges are different, the appearance of the electric field lines of force will change.

Let a point charge with charge $-q$ be placed at a sufficiently large distance from a point charge with charge $+2q$. Half of the voltage lines from the positive charge terminate at the negative charge, and the other half terminate at infinity (Figure 3). The lines of force are symmetrical and are like the lines of force of an electric field produced by a single positive charge [2].

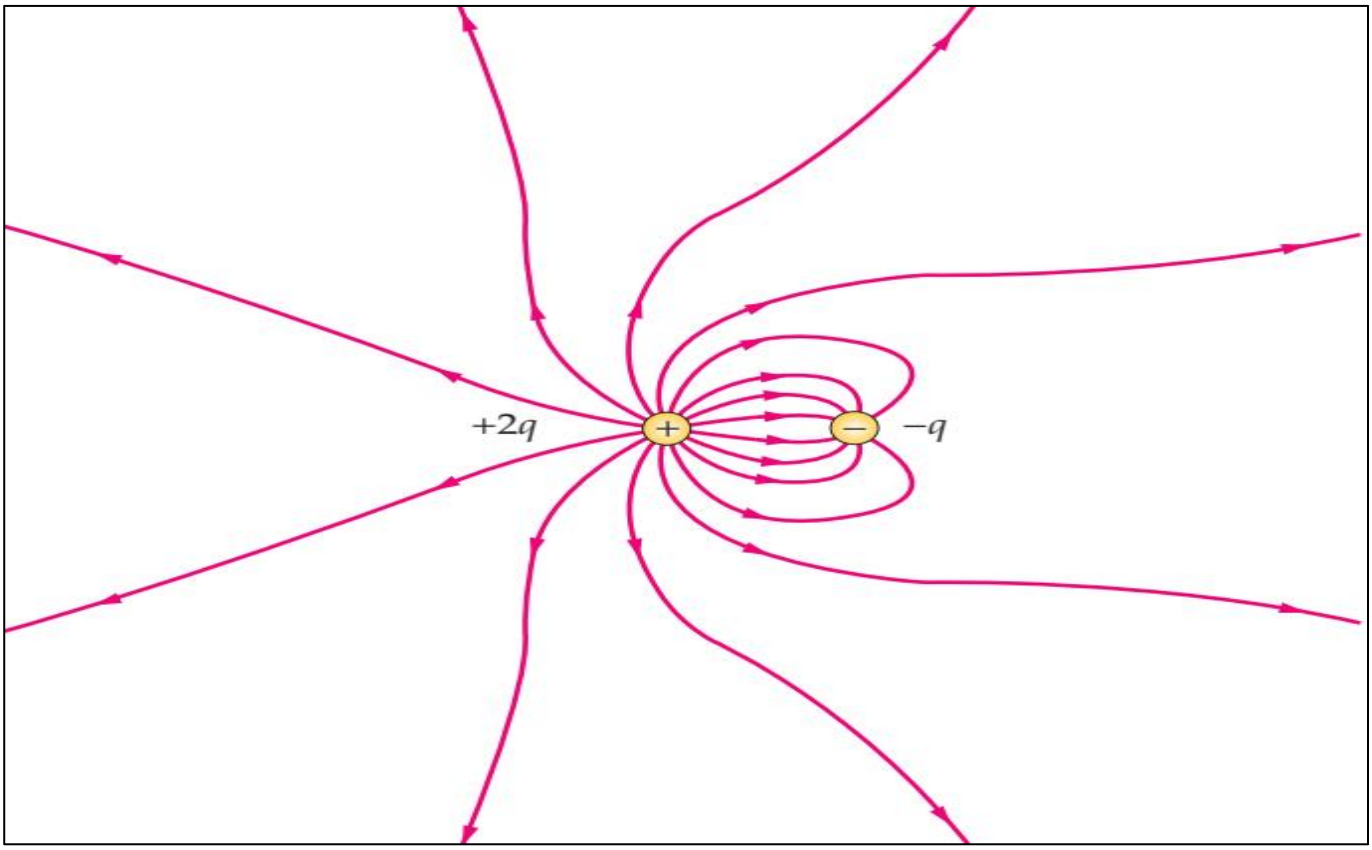

Fig 3: Non Symmetrical Lines of Force

In the study of the electric field, only the strength of the electric field and the electric field lines of force does not provide complete information about the electric field. It is important to characterize the electric field in terms of energy. The quantity that characterizes the electric field in terms of energy is the electric field potential. The work done in moving the electric charge q_0 standing at a point of the electric field from this point to infinity is called the potential of the electric field, or the potential energy of a positive charge placed in the electric field is called the potential of the field at that point and it is expressed as follows [1-3]:

$$\varphi = \frac{A_{l\to\infty}}{q_0} \quad \text{yoki} \quad \varphi = \frac{W}{q_0} \tag{2}$$

So, it is the energetic characteristic of the field, which allows to calculate the work done in moving the charge.

In some cases, the work done when moving a charge in an electric field is zero. In this case, the charge is transferred between points of the same potential, that is, equipotential. A set of points, all of which have the same potential, is called an equipotential surface or equipotential lines. The equipotential surface (lines) is perpendicular to the electric field lines at any point (Fig. 4).

ISSN No:-2456-2165 https://doi.org/10.5281/zenodo.14565229

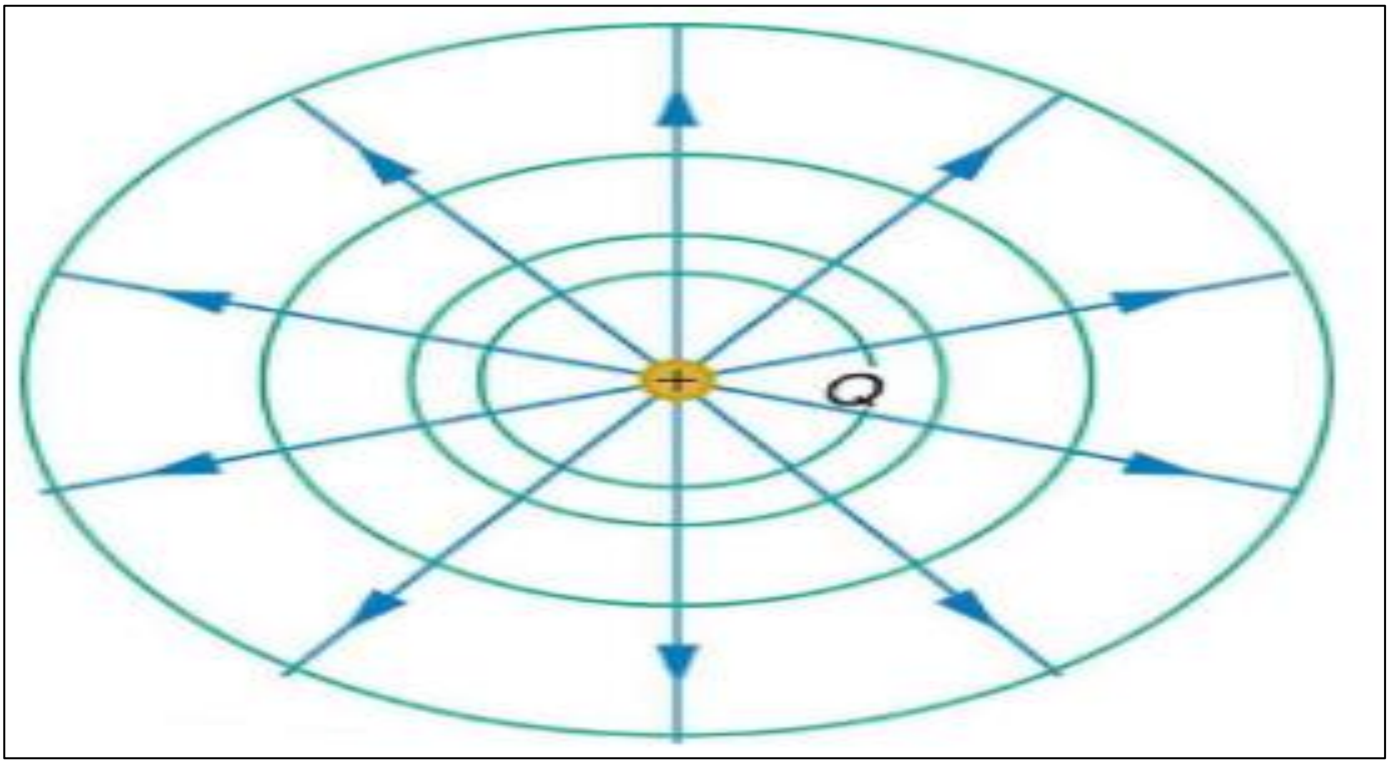

Fig 4: Equipotential Lines

This property helps to find electric field lines when the equipotential surface (lines) are known.

Figure 5 shows the electric field and equipotential lines for two equal and opposite charges [5]. On the contrary, by calculating the equipotential lines, the electric field lines can be drawn perpendicular to the equipotentials as shown in Figure 6.

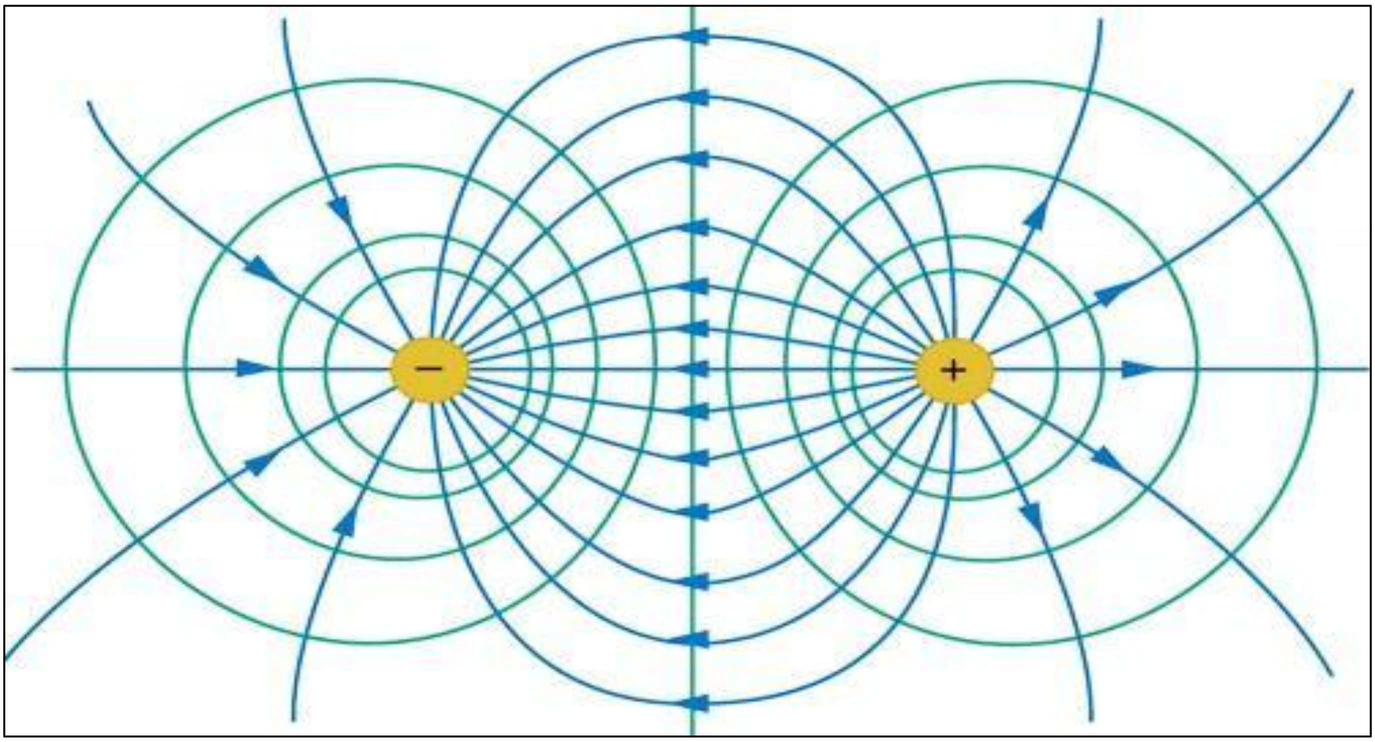

Fig 5: An Electric Field Produced by Two Point Charges of Opposite Direction

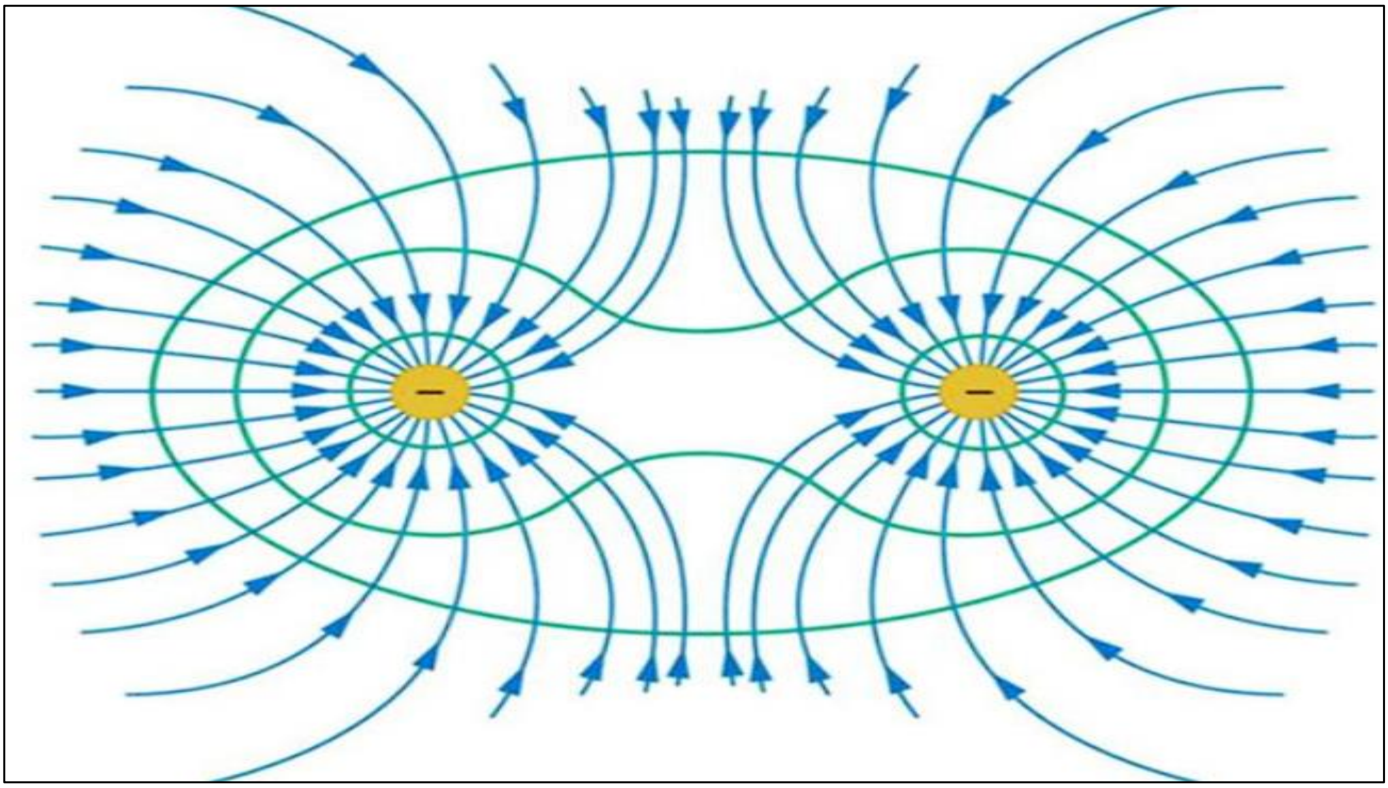

Fig 6: Equipotential Surfaces Formed by Two Point Charges of Opposite Sign

II. RESEARCH METHODS

Just as an electric field can be graphically represented by lines of force, it can also be represented graphically by equipotential lines . Several scientists have studied research on electric field potential and equipotential surface and graphical calculation of equipotential surfaces [6-16]. However, the theoretical determination of equipotential lines with exact value is still a problem. Therefore, we will see the method of calculating the potential for the simplest case. The expression for calculating the potential of a system of 2 positive point charges, the distance between which is equal to 2d and the amount of charge is equal to each other, is expressed as follows [3]:

$$\varphi(x,y) = \frac{q}{4\pi\varepsilon\varepsilon_0}\left(\frac{1}{\sqrt{(x-d)^2+y^2}} + \frac{1}{\sqrt{(x+d)^2+y^2}}\right) \tag{3}$$

In order to find equipotential points using this expression, it is necessary to find the relationship between 2 variables. For simplicity, we take $\varphi(x,y) = c$ and $kq = 1$ that is, since $k = 9 \cdot 10^9 \frac{N \cdot m^2}{C^2}$ Assuming $q_1 = q_2 = \frac{1}{9} nC$, equation (3) looks like this:

$$\frac{1}{\sqrt{(x-d)^2+y^2}} + \frac{1}{\sqrt{(x+d)^2+y^2}} = c \tag{4}$$

For the given conditions c- for the given value of the potential d and from this equation we determine $y = f(x)$. To do this, let's replace:

$$\frac{1}{\sqrt{(x-d)^2+y^2}} = c \cdot sin^2\alpha, \qquad \frac{1}{\sqrt{(x+d)^2+y^2}} = c \cdot cos^2\alpha, \tag{5}$$

It can be seen from this

$$(x-d)^2 + y^2 = \frac{1}{c^2 \cdot sin^4\alpha}, \qquad (x+d)^2 + y^2 = \frac{1}{c^2 \cdot cos^4\alpha},$$

$$x = \frac{(x-d)^2 - (x+d)^2}{4d} \quad => \quad x = \frac{1}{4dc^2}\left(\frac{1}{cos^4\alpha} - \frac{1}{sin^4\alpha}\right),$$

ISSN No:-2456-2165 https://doi.org/10.5281/zenodo.14565229

When we add the above expressions (5) one by one, we get the following expression (5):

$$(x-d)^2+y^2+(x+d)^2+y^2=2x^2+2d^2+2y^2=\frac{1}{c^2}\left(\frac{1}{cos^4\alpha}+\frac{1}{sin^4\alpha}\right) \quad (6)$$

$$x=\frac{1}{4dc^2}\left(\frac{1}{cos^4\alpha}-\frac{1}{sin^4\alpha}\right)=-\frac{4}{dc^2}\frac{cos2\alpha}{sin^4 2\alpha} \quad (7)$$

For convenience $N=-\frac{dc^2}{4}$ and $sin^2 2 = t$

We change the appearance of the equation by entering notations:

$$-\frac{dc^2}{4}x=\frac{\sqrt{1-sin^2 2\alpha}}{sin^4 2\alpha} \quad => \quad N^2x^2=\frac{1-t}{t^4}$$

$$t^4+\frac{1}{N^2x^2}t-\frac{1}{N^2x^2}=0 \quad (8)$$

Adding $\frac{1}{N^2x^2}=M$ to this equation, we make the equation look like this:

$$t^4+Mt-M=0 \quad (9)$$

We can see that this equation is a quadratic equation with respect to t. To solve this equation, we use the Ferrari method of solving equations of the 4th degree [4]. The essence of the Ferrari method is as follows:

We are given a quadratic equation of the following form:

$$Ax^4+Bx^3+Cx^2+Dx+E=0 \quad (10)$$

To solve this equation, we first calculate the following expressions:

$$\alpha=-\frac{3B^2}{8A^2}+\frac{C}{A}, \qquad \beta=\frac{B^3}{8A^3}-\frac{BC}{2A^2}+\frac{D}{A}, \qquad \gamma=-\frac{3B^4}{256A^4}+\frac{CB^2}{16A^3}-\frac{BD}{4A^2}+\frac{E}{A}.$$

$$P=-\frac{\alpha^2}{12}-\gamma, \qquad Q=-\frac{\alpha^3}{108}+\frac{\alpha\gamma}{3}-\frac{\beta^2}{8}, \qquad R=-\frac{Q}{2}\pm\sqrt{\frac{Q^2}{4}+\frac{P^3}{27}}$$

$$U=\sqrt[3]{R}, \qquad z=-\frac{5}{6}\alpha+U-\frac{P}{3U}, \qquad W=\sqrt{\alpha+2z}$$

Then, using the obtained expressions, we have the following solution:

$$x=-\frac{B}{4A}+\frac{\pm W\mp\sqrt{-\left(3\alpha+2z\pm\frac{2\beta}{W}\right)}}{2} \quad (11)$$

Now, we find the solution of equation (9) using the above-mentioned Ferrari method. It is known that according to equation (9) $A=1, B=0, C=0, D=M$ and $E=-M$. Using the above, we find the solution to the equation:

$$\alpha=0, \quad \beta=M, \quad \gamma=-M, \quad P=M, \quad Q=-\frac{M^2}{8}, \quad R=\frac{M^2}{16}\pm\sqrt{\frac{M^4}{256}+\frac{M^3}{27}},$$

ISSN No:-2456-2165 https://doi.org/10.5281/zenodo.14565229

$$U = \sqrt[3]{\frac{M^2}{16} \pm \sqrt{\frac{M^4}{256} + \frac{M^3}{27}}}, \qquad z = \sqrt[3]{\frac{M^2}{16} \pm \sqrt{\frac{M^4}{256} + \frac{M^3}{27}}} - \frac{M}{3\sqrt[3]{\frac{M^2}{16} \pm \sqrt{\frac{M^4}{256} + \frac{M^3}{27}}}},$$

$$W = \sqrt{2\left(\sqrt[3]{\frac{M^2}{16} \pm \sqrt{\frac{M^4}{256} + \frac{M^3}{27}}} - \frac{M}{3\sqrt[3]{\frac{M^2}{16} \pm \sqrt{\frac{M^4}{256} + \frac{M^3}{27}}}}\right)},$$

Making simplifications, we get the following expression for t:

$$t = \sqrt{\frac{3\sqrt[3]{\left(\frac{M^2}{16} \pm \sqrt{\frac{M^4}{256} + \frac{M^3}{27}}\right)^2} - M}{6\sqrt[3]{\frac{M^2}{16} \pm \sqrt{\frac{M^4}{256} + \frac{M^3}{27}}}}} - \sqrt{\frac{-3\sqrt[3]{\left(\frac{M^2}{16} \pm \sqrt{\frac{M^4}{256} + \frac{M^3}{27}}\right)^2} + M}{6\sqrt[3]{\frac{M^2}{16} \pm \sqrt{\frac{M^4}{256} + \frac{M^3}{27}}}} + \frac{M}{\sqrt{\frac{24\sqrt[3]{\left(\frac{M^2}{16} \pm \sqrt{\frac{M^4}{256} + \frac{M^3}{27}}\right)^2} - 8M}{3\sqrt[3]{\frac{M^2}{16} \pm \sqrt{\frac{M^4}{256} + \frac{M^3}{27}}}}}}} =$$

Here $M = \frac{16}{x^2 d^2 c^4}$. Now using expression (6),

$$y = \sqrt{\frac{4}{c^2}\frac{2-t}{t^2} - x^2 - d^2} \qquad (12)$$

We form the expression. Here, c is the field potential and d is the distance from the charges to the coordinate origin. Above, t was taken as a variable, and c and d were taken as constant numbers. Equipotential lines can be obtained by drawing the graph of the function $y = f(x)$ using the expression (12).

III. RESULTS

One can obtain the resulting lines for different values of $d = 1$ and c. Equipotential lines obtained for different values of $d = 1$ and c are presented in Fig. 7. By drawing perpendicular lines to the equipotential lines depicted in Fig. 7, it is possible to get an image of the field created by the system of charges.

ISSN No:-2456-2165 https://doi.org/10.5281/zenodo.14565229

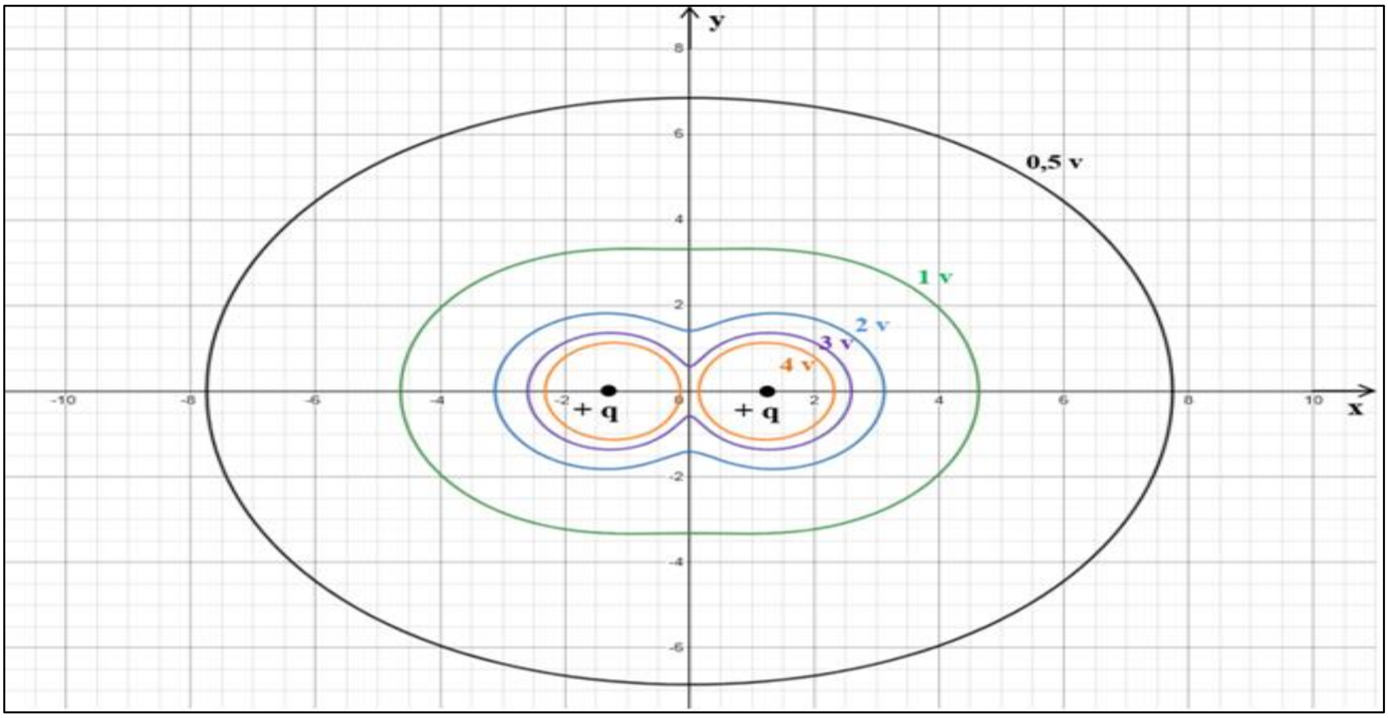

Fig 7: Equipotential Surfaces Formed by Two Systems of Point Charges with the Same Positive Sign

This figure shows a family of equipotential lines with potentials of 4 V, 3 V, 2 V, 1 V and 0,5 V for a system of two point identical positive charges, and these connections correspond to the connections obtained experimentally. If the equation is solved for a system of two point identical negative charges and equipotential lines are drawn, the same figure as Fig. 7 is obtained. But the potentials corresponding to the resulting equipotential lines are -4 V, -3 V, -2 V, -1 V and -0,5 V.

The expression for calculating the potential of a system of point charges, the distance between them is equal to $2d$ and the modules of the charge quantities are equal to each other, one of them is positive q_1 and the other is negative q_2, is expressed as follows:

$$\varphi(x,y)=\frac{1}{4\pi\varepsilon\varepsilon_0}\left(\frac{q_1}{\sqrt{(x-d)^2+y^2}}+\frac{q_2}{\sqrt{(x+d)^2+y^2}}\right) \tag{13}$$

For simplicity, we take $\varphi(x,y)=c$ and $kq_1=1$ and $kq_2=-1$, that is, since $k=9\cdot10^9\frac{N\cdot m^2}{C^2}$, assuming that $|q_1|=|q_2|=\frac{1}{9}nC$ nC, equation (3) becomes:

$$\frac{1}{\sqrt{(x-d)^2+y^2}}-\frac{1}{\sqrt{(x+d)^2+y^2}}=c \tag{14}$$

For the given conditions c - for the given value of the potential d and from this equation we determine $y=f(x)$. To do this, let's replace:

$$\frac{1}{\sqrt{(x-d)^2+y^2}}=c\cdot ch^2\alpha, \qquad \frac{1}{\sqrt{(x+d)^2+y^2}}=c\cdot sh^2\alpha, \tag{15}$$

It can be seen from this:

$$(x-d)^2+y^2=\frac{1}{c^2\cdot ch^4\alpha}, \qquad (x+d)^2+y^2=\frac{1}{c^2\cdot sh^4\alpha},$$

$$x=\frac{(x-d)^2-(x+d)^2}{4d} \quad => \quad x=\frac{1}{4dc^2}\left(\frac{1}{sh^4\alpha}-\frac{1}{ch^4\alpha}\right),$$

When we add the above expressions (5) one by one, we get the following expression (5):

$$(x-d)^2+y^2+(x+d)^2+y^2=2x^2+2d^2+2y^2=\frac{1}{c^2}\left(\frac{1}{ch^4\alpha}+\frac{1}{sh^4\alpha}\right) \tag{16}$$

ISSN No:-2456-2165 https://doi.org/10.5281/zenodo.14565229

$$x = \frac{1}{4dc^2}\left(\frac{1}{sh^4\alpha} - \frac{1}{ch^4\alpha}\right) = -\frac{4}{dc^2}\frac{ch2\alpha}{sh^4 2\alpha} \qquad (17)$$

For convenience, we change the equation to look like this by entering the notations

$$N = -\frac{dc^2}{4} \quad \text{va} \quad sh^2 2\alpha = t:$$

$$-\frac{dc^2}{4}x = \frac{\sqrt{1 - sh^2 2\alpha}}{sh^4 2\alpha} \quad => \quad N^2x^2 = \frac{1+t}{t^4}$$

$$t^4 - \frac{1}{N^2x^2}t - \frac{1}{N^2x^2} = 0 \qquad (18)$$

Adding $\frac{1}{N^2x^2} = M$ to this equation, we make the equation look like this:

$$t^4 - Mt - M = 0 \qquad (19)$$

We can see that this equation is a quadratic equation with respect to t. To solve this equation, we use the Ferrari method of solving equations of the 4th degree [4].

It is known that according to equation (9) $A = 1, B = 0, C = 0, D = -M$ and $E = -M$. Using the above, we find the solution to the equation:

$$\alpha = 0, \quad \beta = -M, \quad \gamma = -M, \quad P = M, \quad Q = -\frac{M^2}{8}, \quad R = \frac{M^2}{16} \pm \sqrt{\frac{M^4}{256} + \frac{M^3}{27}},$$

$$U = \sqrt[3]{\frac{M^2}{16} \pm \sqrt{\frac{M^4}{256} + \frac{M^3}{27}}}, \qquad z = \sqrt[3]{\frac{M^2}{16} \pm \sqrt{\frac{M^4}{256} + \frac{M^3}{27}}} - \frac{M}{3\sqrt[3]{\frac{M^2}{16} \pm \sqrt{\frac{M^4}{256} + \frac{M^3}{27}}}},$$

$$W = \sqrt{2\left(\sqrt[3]{\frac{M^2}{16} \pm \sqrt{\frac{M^4}{256} + \frac{M^3}{27}}} - \frac{M}{3\sqrt[3]{\frac{M^2}{16} \pm \sqrt{\frac{M^4}{256} + \frac{M^3}{27}}}}\right)},$$

Making simplifications, we get the following expression for t:

$$t = \sqrt{\frac{3\sqrt[3]{\left(\frac{M^2}{16} \pm \sqrt{\frac{M^4}{256} + \frac{M^3}{27}}\right)^2} - M}{6\sqrt[3]{\frac{M^2}{16} \pm \sqrt{\frac{M^4}{256} + \frac{M^3}{27}}}}} - $$

$$+ \sqrt{\frac{-3\sqrt[3]{\left(\frac{M^2}{16} \pm \sqrt{\frac{M^4}{256} + \frac{M^3}{27}}\right)^2} + M}{6\sqrt[3]{\frac{M^2}{16} \pm \sqrt{\frac{M^4}{256} + \frac{M^3}{27}}}} + \frac{M}{\sqrt{\frac{24\sqrt[3]{\left(\frac{M^2}{16} \pm \sqrt{\frac{M^4}{256} + \frac{M^3}{27}}\right)^2} - 8M}{3\sqrt[3]{\frac{M^2}{16} \pm \sqrt{\frac{M^4}{256} + \frac{M^3}{27}}}}}}} =$$

ISSN No:-2456-2165 https://doi.org/10.5281/zenodo.14565229

Here $M = \frac{16}{x^2 d^2 c^4}$. Now using expression (16),

$$y = \sqrt{\frac{4}{c^2}\frac{2+t}{t^2} - x^2 - d^2} \qquad (20)$$

We form the expression. Here, c is the field potential and d is the distance between the charges. Above, t was taken as a variable, and c and d were taken as constant numbers. Equipotential lines can be obtained by drawing the graph of the function $y = f(x)$ using the expression (12).

One can obtain the resulting lines for different values of $d = 1$ and c. Equipotential lines obtained for different values of $d = 1$ and c are presented in Fig. 8. By drawing perpendicular lines to the equipotential lines depicted in Fig. 8, it is possible to get an image of the field created by the system of charges.

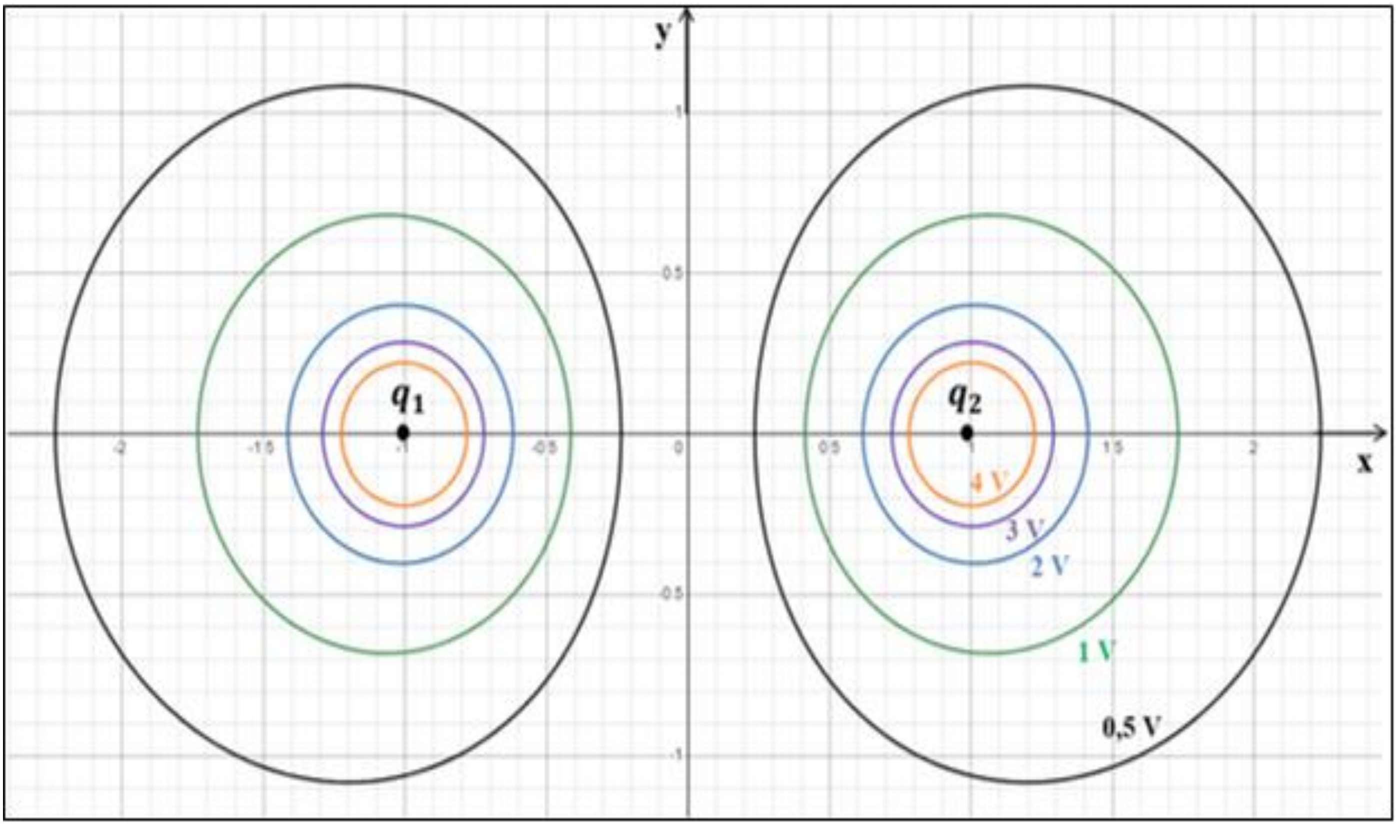

Fig 8: Equipotential Surfaces Formed by a System of Point Charges with Two Charges of the Same Opposite Direction

This figure shows a family of equipotential lines with potentials of 4 V, 3 V, 2 V, 1 V and 0.5 V for a system of 2 positive and negative point charges of equal charge quantities, and these connections are experimentally corresponding to the resulting bindings.

The problem can also be solved for more complex cases. The equation can also be solved using the Ferrari method for a system of charges with a distance of $2d$ and a charge ratio of a ($a>0$), i.e. 2 positive or 2 negative point charges. Together with this, it is possible to obtain lines obtained for different values of $a = {}^{q_1}/_{q_2} = 3$, $d = 1$ and c. Equipotential lines obtained for different values of d=1 and c are presented in Fig. 9. By passing perpendicular lines to the equipotential lines depicted in Fig. 9, it is possible to get an image of the field formed by the system of charges.

ISSN No:-2456-2165 https://doi.org/10.5281/zenodo.14565229

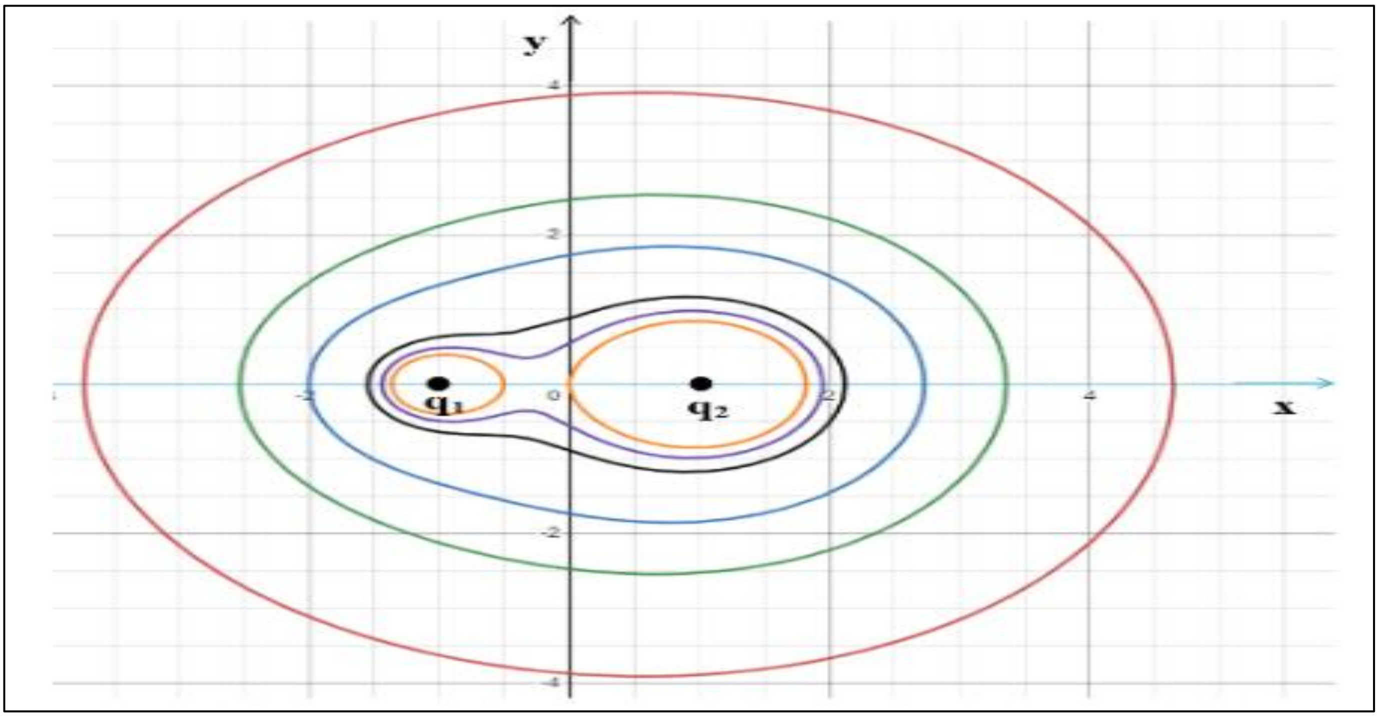

Fig 9: Equipotential Surfaces Formed by a System of Point Charges with a Ratio of Two Charges Equal to 3.

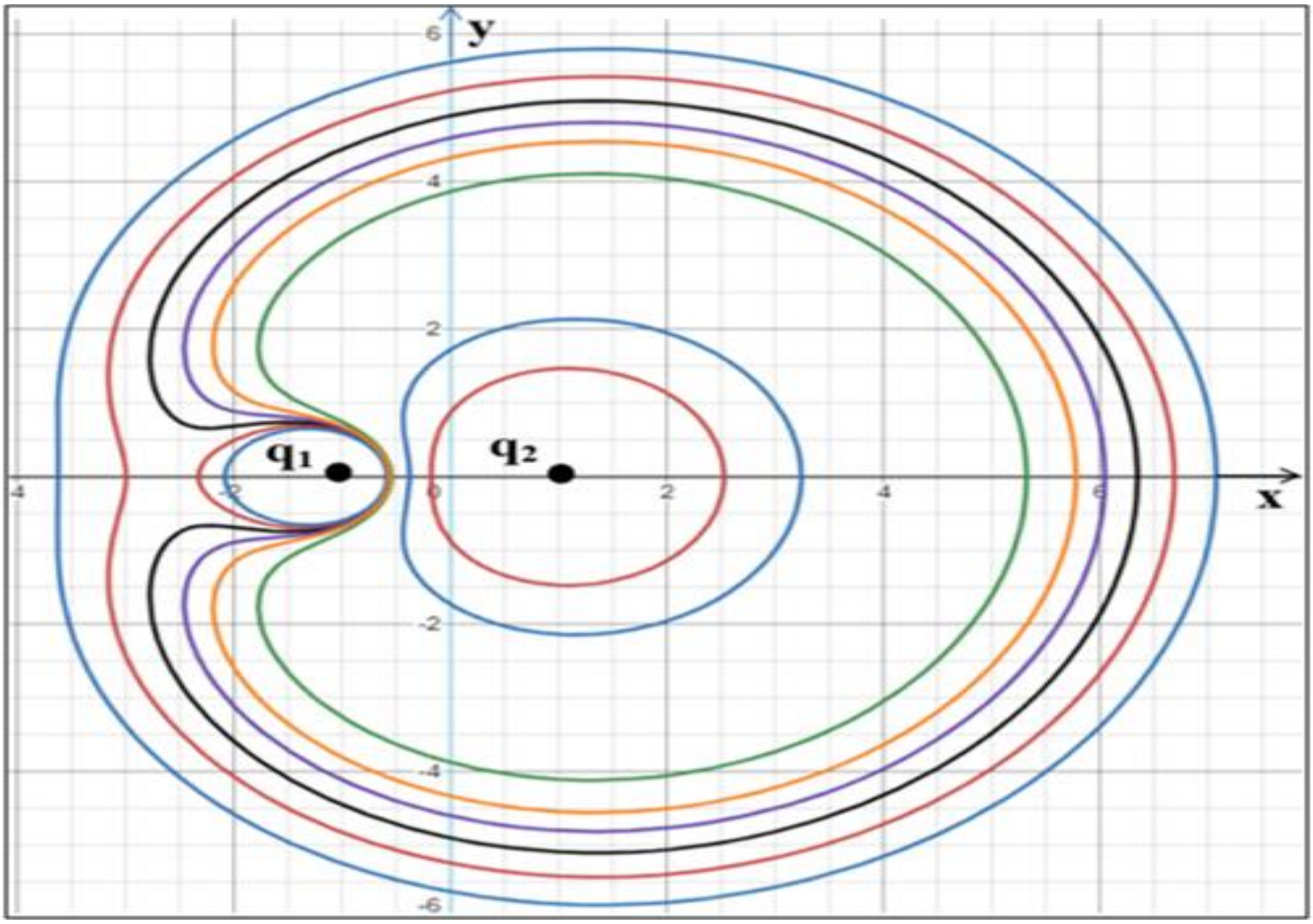

Fig 10: Equipotential Surfaces Formed by a System of Point Charges with a Ratio of Two Charges Equal to -8

The equation can also be solved using the Ferrari method for a system of point charges with a distance of 2d and a ratio of charges equal to a (a<0) , that is, one positive and the other negative point charges. Together with this, it is possible to obtain lines obtained for different values of $a =$

ISSN No:-2456-2165 https://doi.org/10.5281/zenodo.14565229

$q_1/q_2 = -8$, $d = 1$ and c. Equipotential lines obtained for different values of d=1 and c are presented in Fig. 10. By drawing perpendicular lines to the equipotential lines depicted in Fig. 10, it is possible to create an image of the field created by the system of charges.

IV. SUMMARY

The points belonging to the equipotential lines created by the system of two point charges were made by solving the equation created on the basis of the formula for calculating the total potential created by this system of charges. The Ferrari method of solving fourth-order equations was used to solve the resulting equation. By solving the equation, the connection function between the ordinate and the abscissa of the points belonging to the equipotential lines was obtained. Equipotential lines generated by the system of charges were obtained using the obtained function using the Desmos graphic calculator. A numerical array consisting of values was created in a graphic calculator, and then graphs were created based on the results. The obtained equipotential lines are very close to the results reported in the literature. Based on this proposed methodology, we can obtain equipotential lines for a system of different charges, that is, we can study the electric field of a system of point charges both quantitatively and its configuration.

REFERENCES

[1]. S.G. Kalashnikov. Elektr. Universitet. Toshkent-2022.

[2]. Paul A. Tipler. Physics for scientists and engineers. New York-2008.

[3]. А.Н. Матвеев. Электричество и магнетизм. Высшая школа. Москва-1983.

[4]. Ákos Tóth. Solving multiple quartic equations on the GPU using Ferrari's method. Sándor Szénasi, Budapest, Hungary-2018. https://phys.libretexts.org/

[5]. Ian Cooper. Electric field and electric potential due to various charge distributions School of Physics, University of Sydney.

[6]. Sadiku Mathew n. o, Elements of electromagnetic, India, oxford, 4th edition, 2007.

[7]. Sadiku Mathew n.o., Numerical techniques in electromagnetic, USA, CRC press, 1st edition, 2007.

[8]. Balandina L.I., Vasyukov V.I., Podguzov G.V. Study of the electrostatic field with the help of an electrolytic bath: study guide for universities / edited by M. B. Chelnokova. M.: BMSTU 1992. 10 p.

[9]. Odintsov A. A., Ryabokon M. S., Voloki A. V. Modeling the electrostatic field using stationary electric fields // Student Scientific Bulletin. October 21 - 24, 2011. pp. 160 - 168.

[10]. Ryazanov G.A. Experiments and modeling in the study of the electromagnetic field. - M .: Science, 1966. - 208 p.

[11]. Tamm I.E. Fundamentals of the theory of electricity. - M .: Fizmatlit, 2003. - 616 p.

[12]. Martinson L. K., Morozov A. N., Smirnov E. V. The Electromagnetic Field: a textbook for universities. M.: BMSTU. 2013. 422 p.

[13]. A. N. Tikhonov, A. A. Samara. Equations of Mathematical Physics. - M .: Science, 1977. – 735p .

[14]. A device for determining the coordinates and electric potential // Patent of the Russian Federation for useful model No. 178350. 06/20/2017 / Skuibin B.G., Ryabokon M.S., Safarov M.A. [and etc.].

[15]. V. V. Krivolapov. "Development of a method for calculating the force lines of a vector field" BMSTU, Moscow, 2016. 42 p.