

# Examining the Mathematical Difficulties of Students in Quadratic Equations: An Analytical Approach

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**Abstract:-** This study examines the mathematical difficulties faced by senior secondary school students in solving quadratic equations, employing a mix of quantitative and qualitative research methodologies. Key themes of difficulty were identified, including conceptual understanding, factorization, completing squares, and applying the quadratic formula. A sample of 200 students participated in a structured assessment, whose responses allowed for a detailed analysis of learning challenges. Chi-square tests were utilized to investigate significant differences in difficulties based on demographic variables, revealing notable trends among various school environments and student genders. The study highlights the complexity of teaching quadratic equations and suggests targeted interventions to improve student understanding and performance in this critical area of mathematics education.

## I. INTRODUCTION

The study of quadratic equations is a foundational component of algebra, playing a critical role in the mathematical education of students. These equations not only serve as a bridge to more advanced mathematical concepts but also underpin various real-world applications, from physics to finance (Kaput, 1999). Despite their importance, many students encounter significant difficulties when grappling with quadratic equations, which can impede their overall mathematical proficiency and confidence (Lin & Hsieh, 2001). Understanding the nature of these challenges is essential for educators seeking to improve instructional strategies and enhance student outcomes.

This analytical approach aims to systematically examine the specific mathematical difficulties that students face with quadratic equations. By exploring both cognitive and instructional factors, the research seeks to identify common misconceptions, gaps in foundational knowledge, and the impact of pedagogical methods on student understanding (Ketterer & Jansen, 2012). Previous studies have highlighted that many students struggle with key concepts such as factoring, the application of the quadratic formula, and interpreting the graphical representations of quadratic functions (Holt, 2006).

Through a comprehensive analysis of existing literature and the integration of qualitative and quantitative data, this study endeavors to provide a deeper insight into the roots of these difficulties. Ultimately, the findings aim to inform best practices in teaching quadratic equations, fostering more effective educational strategies that can lead to improved student engagement and achievement in mathematics (NMAP, 2008).

### ➤ *Background to the Study*

Mathematics serves as the foundation for many academic and practical applications, yet a significant number of students struggle with its concepts, particularly in algebra. Among the various topics covered within algebra, quadratic equations represent one of the most challenging areas for learners. Quadratic equations, typically expressed in the form  $ax^2 + bx + c = 0$ , require a strong understanding of underlying mathematical principles for successful problem-solving. However, many students exhibit profound difficulties when attempting to grasp these concepts, leading to poor performance in algebra.

Research suggests that these difficulties stem from a combination of factors, including conceptual misunderstandings, lack of foundational skills, and careless study habits (Kershner & O'Connor, 2021; K estimable & Hersh, 2020). These obstacles complicate the learning process, creating a barrier to comprehending not only quadratic equations but also broader mathematical concepts. The challenges students face with quadratic equations highlight the need for targeted analysis to unveil the intricacies of these learning difficulties. This study seeks to analyze and interpret the common difficulties experienced by students in solving quadratic equations. The qualitative and quantitative data collected from students in Senior Secondary School (SSS) 2 and 3 reveal patterns of struggle in areas such as conceptualization, factorization, completing the square, application of the quadratic formula, and the overall solving process. The sample included 200 students whose answer sheets were thoroughly evaluated to identify prevalent errors. From this pool, five students were selected for in-depth interviews based on the specific mistakes they made, allowing for a nuanced understanding of their learning challenges.

As the analysis unfolds, it is crucial to understand how variables like age and sex may influence students' difficulties in quadratic problem-solving. The study employs Chi-square tests to explore potential differences in performance and understanding between different demographic groups. The results aim to inform educators about the specific areas of difficulty and the impact of various factors on students' abilities to tackle quadratic equations.

#### ➤ *Statement of the Problem*

The study aims to investigate the learning difficulties associated with solving quadratic equations among Senior Secondary School students. Quadratic equations are fundamental to algebra and mathematics as a whole; however, many students face significant obstacles in understanding and applying the principals involved. Observations indicate that students often engage in problem-solving without a clear understanding of the underlying concepts, leading to widespread struggles in this area. This research seeks to identify specific themes underlying these difficulties, including conceptual misinterpretations, issues with factorization, challenges in completing the square, confusion surrounding the quadratic formula, and the overall problem-solving process. By exploring these learning barriers, the study seeks to uncover the factors contributing to students' poor performance in algebra and their consequent inability to achieve high grades in mathematics.

#### ➤ *Objectives of the Study*

The primary objectives of the study are as follows:

- To explore and categorize the specific difficulties students encounter when learning quadratic equations in algebra.
- To analyze students' responses and mistakes in solving quadratic equations and identify common themes corresponding to these difficulties.
- To examine the influence of contextual factors, such as the school environment, age, and gender, on students' understanding and performance in solving quadratic equations.
- 4.To provide insights and recommendations for educators to develop targeted instructional strategies that address the identified difficulties in teaching quadratic equations.

#### ➤ *Significance of the Study*

This study is significant in various ways. First, it aims to enhance understanding of the specific difficulties students face when learning quadratic equations, potentially informing educators and curriculum developers about necessary changes in instructional strategies. By identifying precise areas of misunderstanding, targeted interventions can be implemented to improve students' comprehension and performance. Furthermore, this research can contribute to the broader discourse on mathematics education by identifying common issues that, if addressed, could enhance student engagement and success in mathematics. Lastly, findings from this study

may serve as a resource for future research aimed at improving educational practices in the field of algebra.

#### ➤ *Hypothesis of the study*

Null Hypothesis (H<sub>0</sub>): There is no significant difference in the difficulties in solving quadratic equations using different methods based on students' schools, age groups, and sex.

#### ➤ *Alternative Hypotheses (H<sub>A</sub>):*

- For School Differences: There is a significant difference in the difficulties in solving quadratic equations using different methods between Kissy Municipal Secondary School and Salam Islamic Secondary School.
- For Age Differences: There is a significant difference in the difficulties in solving quadratic equations using different methods across different age groups.
- For Sex Differences: There is a significant difference in the difficulties in solving quadratic equations using different methods based on students' sex.

#### ➤ *Research Questions*

- What are the specific learning difficulties that students face when solving quadratic equations in algebra?
- Is there a significant difference in the difficulties of solving quadratic equations among students from different secondary schools?
- How do age differences affect students' abilities in solving quadratic equations?
- Is there a significant difference in the difficulties faced by male and female students when solving quadratic equations?

## II. LITERATURE REVIEW

#### ➤ *Analysis and Interpretation of Data on Learning Difficulties in Quadratic Equations*

The ability to solve quadratic equations is a fundamental mathematical skill, essential for advanced algebra and various real-world applications. However, students often experience significant challenges in mastering this topic. This literature review synthesizes research findings relevant to the themes developed during the analysis of data regarding learning difficulties in quadratic equations, particularly among senior secondary school students.

#### ➤ *Conceptual Difficulties*

Research indicates that conceptual understanding plays a crucial role in a student's ability to engage with mathematical problems, including quadratic equations. Many students struggle to grasp the underlying concepts associated with quadratic equations, leading them to approach problems mechanically rather than understanding the principals involved (Zbiek et al., 2018). This lack of conceptual knowledge often results in misconceptions that impede performance (NCTM, 2018). Students may attempt to solve

problems without a solid understanding of what the solutions represent, hindering their ability to achieve mastery (Smith & Thompson, 2020).

➤ *Difficulties in Factorization*

Factorization is a critical step in solving quadratic equations, yet numerous students exhibit substantial difficulties with this process. Studies have shown that many students demonstrate insufficient fluency in recognizing when and how to apply various factorization techniques (Baker et al., 2019). These challenges in factorization are often attributed to a weak algebraic foundation, leading to errors that can distort their understanding of quadratic applications further (Wang & Hsieh, 2021).

➤ *Difficulties in Completing the Square*

Completing the square is another essential method for solving quadratic equations that students frequently find challenging. Research suggests that many learners view this technique as a rote procedure rather than as a tool for deeper understanding (Ellis & Grant, 2020). This perception can prevent students from appreciating the broader implications of this technique in the context of quadratic functions and conic sections. Such difficulties indicate a need for better instructional strategies to clarify the conceptual underpinnings of the completing-the-square method (Newell & Morrison, 2022).

➤ *Difficulties in Using the Quadratic Formula*

The quadratic formula provides a straightforward path to finding solutions for quadratic equations; however, many students struggle with both applying and comprehending the formula (Foster & Dindyal, 2021). Misunderstanding the components of the formula, particularly the significance of the discriminant, can lead to erroneous interpretations of the number and nature of solutions (Loveless et al., 2020). This finding underscores the importance of targeted pedagogical approaches that emphasize both procedural and conceptual knowledge in relation to the quadratic formula.

➤ *Solving Process Difficulties*

Finally, the process of solving quadratic equations can pose significant challenges for students. Many learners face difficulties in organizing their work, leading to mistakes in calculations and problem-solving approaches (Bishop et al., 2019). A lack of effective problem-solving strategies often results in systematic errors and incomplete solutions, further compounding students' challenges in succeeding in algebra (Holland & Noyce, 2021). Educators are urged to instill robust problem-solving techniques that encourage students to reflect on their reasoning processes.

### III. MODEL DESCRIPTION

The chi-squared test checks the difference between the observed value and the expected value. Chi-Square shows or in a way check the relationship between two categorical variables which can be calculated by using the given observed frequency and expected frequency.

➤ *Chi-square formula*

$$\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i}$$

where

- $O_i$  = observed value (actual value)
- $E_i$  = expected value.

### IV. ANALYSIS AND INTERPRETATION OF DATA

Different themes were developed in the process of data analysis with reference to learning difficulties in quadratic equation in algebra. Learning in quadratic equation problems have been an obstacle and a challenge for many students. Students could not understand. It seems that they just try to solve without understanding. This indicates that due to understanding in quadratic equations in algebra, students are not able to solve and are poor in algebra quadratic. Equations as well as learning and achieving high grades in mathematics. The final themes are conceptual difficulties, difficulties in factorization, difficulties in completing the square, difficulties in the quadratic formula, and solving process difficulties. The analysis and interpretation of data based on these themes are presented in the chapter. Many students could not understand the conceptual meaning of factorization, completing square problem, some students were careless in their study and examination. In algebra, a quadratic problem, the researcher found so many difficult areas: conceptual difficulties, difficulties in factorization, difficulties in completing square, difficulties in the quadratic formula and solving process difficulties. There were 200 students in SSS 2 & 3. First the researcher gave the exam/test to all students in their school. And the researcher checked the answer sheets and selected highly difficult occurred in the answer sheet. From these answer sheets the researcher selected five students for the interview. They were selected on the basis of mistakes. They committed in their answer sheets.

**A. Difficulties in Learning Quadratic Equation Problem of Senior Secondary Schools Students**

Quadratic equation problem has been an obstacle and challenge for many students. Lack of many mathematical skills caused difficulties in solving quadratic equation problem. Yet we know that many students fail in the mathematics. In this research, found that the students were poor in quadratic problem in learning mathematics. Many students did not understand the conceptual meaning of the problem. Some students were careless in their study. They do not know the conceptual meaning of quadratic equation. In quadratic problem, the researcher found so many difficult areas;

**B. Chi Square Test and Distribution of Difficulties in Solving Quadratic Equations Between Kissy Municipal Secondary School and Salam Islamic School**

Null Hypothesis (H<sub>0</sub>): There is no significant difference in the difficulties in solving quadratic equations using different methods between Kissy Municipal Secondary School and Salam Islamic Secondary School

Alternative Hypothesis (H<sub>A</sub>): There is a significant difference in the difficulties in solving quadratic equations using different methods between Kissy Municipal and Salam Islamic Secondary School.

**Table 1: Observed Values**

School/Method	Kissy Municipal Secondary School	Salam Islamic Secondary school	Total
Conceptual Method	20	35	55
Factoring Method	15	40	55
Completing Square	48	40	88
Formula Method	22	20	42
Solving Process	20	30	50
Total	125	165	290

This comparison helps to understand the strengths and weaknesses of each school in various mathematical problem-solving techniques.

**Table 2: Chi - Square (X<sup>2</sup>) Table of Expected Values**

School/Method	Rising Academy	Salam Islamic
Conceptual	23.7	31.3
Factoring	23.7	31.3
Completing Square	37.9	50.1
Formula	18.1	23.9
Solving Process	21.6	28.4

**Table 3: Calculation of Chi - Square (X<sup>2</sup>)**

Observed Values (O)	Expected Values(E)	( O - E)	( O - E) <sup>2</sup>	( O - E) <sup>2</sup> /E
20	23.7	- 3.7	13.69	0.5776
35	31.3	3.7	13.69	0.4374
15	23.7	- 8.7	75.69	3.1937
40	31.3	8.7	75.69	2.4182
48	37.9	10.1	102.01	2.6916
40	50.1	- 10.1	102.01	2.0361
22	18.1	3.9	15.21	0.8403
20	23.9	- 3.9	15.21	0.8403
20	21.6	- 1.6	2.56	0.1185
30	28.4	1.6	2.56	0.0901
Total				13.2438

Compare the calculated chi-square value with the tabulated chi-square value: The tabulated chi-square value can be found in a chi-square distribution table with The degrees of freedom = (number of rows - 1) × (number of columns - 1). In this case, the degrees of freedom (df) = (2-1) × (5-1) = 4. Looking up the tabulated chi-square value with 4 degrees of freedom and a significance level of 5% (0.05), we find that the tabulated chi-square value is 9.49. Making a decision (Analysis): Since the calculated chi-square value (X<sup>2</sup>) = 13.2438 is greater than the tabulated chi-square value (9.49), we reject the null hypothesis. This means that there is a significant difference in the difficulties in solving quadratic equations using different methods between Kissy Municipal and Salam Islamic. In conclusion, based on the chi-square test, there is evidence to suggest that there is a significant difference in the difficulties.

in solving quadratic equations using different methods between Kissy Municipal Secondary School and Salam Islamic Secondary School

**Table 4: Observed Values of Age**

Age limit/Method	13-16yrs	17-20	Total
Conceptual	20	30	50
Factoring	25	30	55
Completing The Square	54	34	88
Formula	20	22	42
Solving Process	26	24	50
Total	145	140	285

**Table 5: Expected Values of Age**

Age Limits/ Method	13-16yrs	17-20yrs
Conceptual	25.4	24.6
Factoring	28.0	27.0
Completing The Square	44.8	43.2
Formula	21.4	20.6
Solving Process	25.4	24.6

**Table 6: Calculation of the Chi Square of Age**

OBSERVED VALUES (O)	EXPECTED VALUES (E)	(O- E)	(O-E) <sup>2</sup>	(O-E) <sup>2</sup> /E
20	25.4	-5.4	29.16	1.1480
30	24.6	5.4	29.16	1.1854
25	28.0	-3.0	9.00	0.3214
30	27.0	3.0	9.00	0.3333
54	44.8	9.2	84.64	1.8893
34	43.2	-9.2	84.64	1.9593
20	21.4	-1.4	1.96	0.0916
22	20.6	1.4	1.96	0.0951
26	25.4	0.6	0.36	0.0142
24	24.6	-0.6	0.36	0.0142
Total				7.0522

➤ *Null Hypothesis (H<sub>0</sub>):*

There is no significant difference in the difficulties in solving quadratic equations using different methods with Age.

➤ *Alternative Hypothesis (H<sub>a</sub>):*

There is a significant difference in the difficulties in solving quadratic equations using different methods with age limits

*C. Compare The Calculated Chi-Square Value With The Tabulated Chi-Square Value:*

The tabulated chi-square value can be found in a chi-square distribution table with The degrees of freedom = (number of rows - 1) × (number of columns - 1). In In this case, the degrees of freedom (df) = (2-1) × (5-1) = 4. Looking up the tabulated chi-square value with 4 degrees of freedom and a significance level of (5%) = 0.05, we find that the tabulated chi-square value is 9.49. The calculated Chi-square value  $X^2 = 7.0522$ , which is less than the tabulated chi-square value  $X^2 = 9.49$ . Since The calculated chi-square value (7.0522) is less than the tabulated chi-square. Value (9.49), we fail to reject the null hypothesis (H<sub>0</sub>). Hand concludes that there is no significant difference in the difficulties in solving quadratic equations using different methods with age limits.



**Table 7: Observed Values of Sex: Girls vs. Boys**

Sex/ Method	Girls	Boys	Total
Conceptual	20	35	55
Factoring	25	30	55
Completing the square	47	41	88
Formula	10	42	52
Solving Process	18	32	50
Total	120	180	300

**Table 8: Expected Values of Sex**

Sex/method	Girls	Boys
Conceptual	22.0	33.0
Factoring	22.0	33.0
Completing The Square	35.2	52.8
Formula	20.8	31.2
Solving Process	20.0	30.0

**Table 9: Calculation of the Chi Square Values of Sex**

Observed Values (O)	EXPECTED VALUES (E)	(O-E)	(O-E) <sup>2</sup>	(O-E) <sup>2</sup> /E
20	22.0	-2.0	4.00	0.1818
35	33.0	2.0	4.00	0.1212
25	22.0	3.0	9.00	0.4091
30	33.0	-3.0	9.00	0.2727
47	35.2	11.8	139.24	3.9557
41	52.8	-11.8	139.24	2.6371
10	20.8	-10.8	116.64	5.6077
42	31.2	10.8	116.64	3.7385
18	20.0	-2.0	4.00	0.2000
32	30.0	2.0	4.00	0.1333
Total				17.2571

➤ *Null Hypothesis (Ho) :*

There is no significant difference in the difficulties in solving quadratic equations using different methods with Sex

➤ *Alternative Hypothesis (Ha):*

There is a significant difference in the difficulties in solving quadratic equations using different methods with sex.

*D. Compare The Calculated Chi-Square Value With The Tabulated Chi-Square Value:*

The tabulated chi-square value can be found in a chi-square distribution table with The degrees of freedom = (number of rows - 1) × (number of columns - 1). In In this case, the degrees of freedom (df) = (2-1) × (5-1) = 4. Looking up the tabulated chi-square value with 4 degrees of freedom and a significance level of (5%) = 0.05, we find that the tabulated chi-square value is 9.49. The calculated Chi-square value  $X^2 = 17.2571$ , which is less than the tabulated chi-square value = 9.49 We reject the Null Hypothesis and accept the alternative hypothesis because the calculated chi-square value (17.2571) is greater than the tabulated chi-square value (9.49) at a significance level of 0.05. This means that there is a significant difference in the difficulties in solving quadratic equations using different methods with sex. In other words,

the difficulty level of solving quadratic equations using different methods varies significantly between males and Females.

**V. SUMMARY**

This study investigated the significant learning difficulties that students face when solving quadratic equations in algebra, focusing on conceptual understanding and various solving methods. Through a structured approach involving a test administered to 200 senior secondary school students, key themes such as conceptual difficulties, difficulties in factorization, completing the square, the use of the quadratic formula, and problem-solving processes emerged. Data were analyzed using a Chi-square test to evaluate differences in difficulties across demographic subsets, including different schools, age groups, and sex.

## VI. FINDINGS

The findings revealed that:

- **Conceptual Difficulties:** Many students struggled to grasp the conceptual underpinnings of quadratic equations.
- **Factoring and Completing the Square:** There were significant misconceptions surrounding factorization and completing the square, leading to incorrect solutions.
- **3.Quadratic Formula:** Students exhibited frequent errors when employing the quadratic formula.
- **Solving Process:** Carelessness in the problem-solving process further complicated students' ability to arrive at the correct solutions.
- **The Chi-square test** indicated significant differences in the difficulties experienced by students from different schools and between genders, supporting the alternative hypotheses that challenges in learning quadratic equations vary by educational context and demographic variables.

## VII. CONCLUSION

The study concluded that understanding quadratic equations remains a substantial obstacle for many secondary school students. The technical challenges encompass both conceptual understanding and practical application in solving problems. Variations in performance related to school, age, and gender suggest that tailored interventions are required to address these difficulties effectively.

## RECOMMENDATIONS

In light of the findings, the following recommendations are proposed:

- **Enhanced Instructional Strategies:** Develop and implement teaching strategies that clarify the conceptual foundations of quadratic equations.
- **Targeted Workshops:** Facilitate workshops focused on specific areas of difficulty, such as factorization and application of the quadratic formula.
- **Regular Assessments:** Conduct regular assessments to identify and address gaps in understanding before advancing to more complex concepts.
- **Peer Tutoring:** Implement peer tutoring programs where stronger students can assist those struggling with quadratic equations, creating a supportive learning environment.
- **Further Research:** Continue research to explore the underlying causes of these difficulties, with a focus on developing adaptive teaching methods based on learner needs.

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