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Understanding and Differentating General and Special Relativity

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ABSTRACT

This article stresses the importance of comprehending projectile motion for its extensive applications in both science and real-world contexts. This understanding not only enhances our grasp of physics and mathematics but also demonstrates how straightforward motions can embody complex scientific concepts.

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TABLE OF CONTENT

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CHAPTER ONE
INTRODUCTION

Projectile motion is a fundamental concept in physics that describes the motion of an object or particle projected near the surface of a planet, with a focus on Earth in this discussion. When only influenced by gravitational forces and neglecting other factors such as air resistance, the object follows a specific curved path known as its trajectory.

This motion can be analyzed by breaking it down into two independent components: horizontal motion and vertical motion. The horizontal component of motion is characterized by uniform velocity throughout the trajectory. In other words, if there are no external forces acting on the object in the horizontal direction, its speed and direction remain constant. This is often referred to as horizontal uniform motion.

On the other hand, the vertical component of motion is influenced by the force of gravity, resulting in accelerated motion. This means that the object experiences a change in velocity due to gravitational acceleration, causing it to speed up or slowdown in the vertical direction. The acceleration due to gravity, denoted by 'g', is a constant value near the surface of the planet. As a result, the vertical motion of the projectile depends on this constant acceleration (in this article I will use the Earth´s gravity for all calculations, the gravitational acceleration of the Earth is $9{,}81\, {}^{m}/_{s^2}$.

When an object is projected with an initial velocity at an angle relative to the horizontal, the motion can be further analyzed by considering the horizontal and vertical components separately. The horizontal component of the initial velocity remains unaffected by gravity and can be calculated using trigonometric functions. The vertical component of the initial velocity can also be determined using trigonometry, considering the launch angle and the magnitude of the initial velocity.

By understanding the mechanics of projectile motion, we can gain insights into a wide range of practical scenarios. For example, when throwing a baseball, the trajectory of the ball can be analyzed by considering its initial velocity, the angle of projection, and the force of gravity. Similarly, shooting an arrow involves understanding how its initial velocity and launch angle determine its trajectory. Even more complex scenarios, such as launching rockets into space, require a thorough understanding of projectile motion and the additional factors impacting the motion, such as atmospheric conditions and the presence of other forces.

In summary, projectile motion describes the curved path followed by an object or particle projected near the surface of a planet, considering only the influence of gravitational forces. By breaking down the motion into horizontal- and vertical components, we can analyze the trajectory and behavior of the object. Studying the mathematical principles underlying projectile motion allows us to better comprehend various everyday activities and more complex scenarios like sending rockets into space, enabling us to make accurate predictions and calculations.

A. What is Projectile Motion

- *What is Projectile Motion?*

Projectile motion refers to the motion of an object that is thrown, or projected, into the air and subject only to gravitational forces and air resistance. This can include many objects like throwing a rock into a like, or a rocket being launched into space.

However, one key characteristic of projectile motion is that the only force action upon the object, in an ideal scenario (ideal scenario means no air-resistance i.e,. vacuum) is gravity.

- *Trajectory of Projectile Motion*

The trajectory of the projectile´s motion is the path it follows through space. Due to the force of gravity pulling the object downwards and the initial forwards velocity, the trajectory of a projectile is a parabolic path. If you were to track the path of an object in projectile motion, you´d find that it forms a shape like the letter "U", just upside-down.

- *Components of Projectile Motion*

In a projectile motion there is to independent components of projectile motion: The horizontal component and the vertical component. While these two motions occur simultaneously, they do not affect each other.

- *The Horizontal Component.*

In the absence of air resistance, horizontal motion is uniform, meaning the object moves at a constant velocity in the horizontal direction (to keep things easy, we normally do our calculations like this in air-resistance as-well) Along the horizontal direction (x-axis) there is no acceleration, therefore there is no change in its horizontal velocity, in other words, the velocity would stay the same during the entire motion.

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B. The Vertical Component.

The vertical motion is unlike the horizontal motion a uniformly *accelerated* motion, meaning its vertical velocity changes at a constant rate due to the acceleration caused by gravity (The average gravitational acceleration on earth is 9,81 $^{m}/_{s^2}$). Gravity accelerates the object downwards, making it move faster and faster for every second that passes in the downward direction.

One thing to remember that the total trajectory of the object is not just the horizontal or the vertical motion but a combination of these two. This results in the curved, parabolic path.

➢ *Role of Projectile Motion in Real-Life Situations*

- Projectile motion literally shapes the world around us. Its principles come into play every time an object is thrown, dropped, or launched.
- In sports, the understanding of projectile motion can help you improve technique or devise strategies in sport like basketball, soccer, or golf.
- In the field of space- and rocket science, projectile motion is both fundamental and- instrumental. Engineers need to understand it to calculate the correct launch angle and velocities necessary to put something into orbit.

Thus, understanding the mathematics and physics of projectile motions allows for deeper insights into a multitude of physical phenomena. However, it´s important to consider that real-life scenarios are much more complex and are involving external forces such as air resistance and non-uniformed gravitational fields.

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CHAPTER TWO
UNDERSTANDING THE MATH BEHIND PROJECTILE MOTION

➢ *Initial Velocity v_0*

The initial velocity v_0 is the velocity at which the object is launched or thrown. It is the magnitude of the initial velocity and is always positive. The launch angle (θ) are also crucial to determining the projectile´s trajectory, maximum height, and range. If the projectile´s position along the x- and y-axis are known, as well as the launch angle, the initial velocity (V_0) can be calculated with the following expression:

$$v_0 = \sqrt{\frac{(x)^2 g}{x \sin 2\theta - 2y cos^2\theta}}.$$

Velocity is as we know a vector, therefore the initial velocity is a vector quantity, and it can be resolved into two perpendicular components according to the launch angle- a horizontal component (V_0x) and a vertical component v_{0y}. These two components are crucial in understanding the independent horizontal and vertical motions of the projectile.

➢ *Horizontal Initial Velocity Calculation and Explaining*

To determine the horizontal initial velocity v_{0x} of a projectile, we can utilize the known initial velocity v_0 and the launch angle φ. The horizontal initial velocity can be calculated using the following expression:

$$v_{0x} = v_0 * \cos\varphi = v_{0x} \, {}^{m}/_{s}.$$

It is essential to note that this calculation assumes no air resistance is present. Therefore, we assume that the velocity of the object will remain constant throughout its entire flight.

➢ *Were*

- v_{0x}= Represents the horizontal initial velocity.
- v_0= Represents the initial velocity.
- $\cos\varphi$= Represents the launch angle.

By considering the horizontal initial velocity, we can analyze the projectile's motion solely in the horizontal direction, disregarding any vertical effects. However, it is important to acknowledge that in the real world, factors such as air resistance may affect the horizontal velocity of the projectile as well.

In summary, the calculation of the horizontal initial velocity involves applying trigonometric principles to the given initial velocity and launch angle. The resulting horizontal velocity allows us to focus on the projectile's horizontal motion while assuming a constant velocity throughout the projectile's flight, disregarding the influence of air resistance.

We have the initial velocity V_0 and we know the launch angle. To find the horizontal initial velocity one can use the following expression.

$$v_{0x} = v_0 * \cos\varphi = v_{0x} \, {}^{m}/_{s}.$$

➢ *Vertical Initial Velocity Calculation and Explaining*

The vertical component of velocity can be calculated by using the initial velocity v_o and the launch angle by using the following expression.

$$v_{oy} = v_o * \sin\theta = v_{0y} \, {}^{m}/_{s}.$$

Unlike the horizontal component, the vertical component of velocity does not remain constant throughout the projectile's motion. However, the acceleration experienced by the object remains constant. As the object ascends, its vertical velocity gradually decreases until it reaches its maximum height. Subsequently, it accelerates downward due to the influence of gravity. It, is important to note that the object's velocity changes continuously in the vertical direction due to the gravitational force acting upon it. This change in velocity results in variations in the object's vertical position and trajectory. In, summary, the initial velocity and launch angle of a projectile determine both the horizontal and vertical velocities. These velocities, in turn, define the projectile's trajectory, range, and maximum height during its motion. It is worth mentioning that calculations involving factors such as air resistance or wind may introduce additional complexities to the analysis of projectile motion.

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Unlike the horizontal component, the vertical component does not remain constant, however, its acceleration will remain constant. Its velocity will slowly decrease as the object reaches its maximum height and then accelerates as the object falls towards earth due to gravity.

To summarize, an object´s initial velocity and launch angle will determine both the horizontal- and the vertical velocities, which then will define its trajectory, range, and maximum height during its projectile motion. Calculations involving elements such as air resistance and wind.

➢ *The Displacement or Position Equation*

The displacement also known as the position equation is given by the following expression.

$$d = v_{0y}t + 0{,}5gt^2.$$

➢ *Were*

- d= Represents the the height reached by the object above its starting point at any time t.
- v_{0y}= Represents the initial vertical velocity of the object, which mentioned earlier, is given by the initial velocity.
- t= Represents the time, which begins counting from the moment when the object is launched.
- g= Represents the acceleration due to gravity. On the surface of the earth, its approximate value is 9,81 $^{m}/_{s}{}^{2}$
- $0{,}5gt^2$= Represents the vertical distance that the object has fallen from the peak of its trajectory due to gravity. The closer the object is to the ground, the greater its fall and hence the greater h, given a certain t.

➢ *The Velocity Equation*

The velocity along the y-axis can be calculated at any time during the flight with the following expression.

$$v = v_{0y} - gt.$$

➢ *Were*:

- v_y= Represents the vertical velocity of the object at any time (t) during its flight.
- $-gt$= Represent the gravity´s effect on slowing down (when the object is moving upwards) or having a positive acceleration then it falls downwards, the object´s vertical motion. The longer the object is in the air (That means the longer time the flight takes), the lager this term becomes.

The combination of these two calculations, for both displacement and velocity, allows you to describe the motion of an object in vertical movement accurately, given its initial conditions. It´s important to note that these equations don't take air resistance into account, and they´re idealized scenarios. In a real-world scenario this- and other factors would complicate things.

➢ *The Horizontal Movement and its Mathematics*

In physics, horizontal movement refers to the movement of an object along the horizontal axis. When considering projectile motion, the trajectory of the object is observed in two dimensions: horizontal $x - axis$ and vertical $y - axis$ respectively. The motion is influenced by gravitational forces which cause acceleration solely in the vertical direction $y - axis$, while the movement along the horizontal axis remains unaffected $(x - axis)$.

➢ *The Horizontal Distance is Given by the Following Expression.*

$$Distance_X = v_{0x} * t = m.$$

➢ *Were*

- $Distance_x$= Represents the distance traveled along the$x - axis$.
- v_{0x}= Represents the initial velocity in the horizontal direction.
- t= Represents the time in seconds.
- m= Represents the distance in meters.

In projectile motion, the horizontal motion is considered uniform, implying that the horizontal velocity remains constant throughout the object´s flight. This is because there is no acceleration in the horizontal direction (If we disregard all other forces such as air-resistance) As a result of this, the object will land with the same velocity as the initial velocity. One can therefore denote the velocities along the horizontal direction as following $v_x = v_{0x}$, this signifies that the velocities along the $(x - axis)$ will remain equal to it initial horizontal velocity v_{0x}, throughout its trajectory.

The time of flight is naturally the total duration of the flight for which the projectile remains airborne. It can be calculated by considering the time from the launch of the projectile until it lands in its final position.

In summary, considering horizontal movement in projectile motion involves understanding the horizontal distance traveled, which is determined by the initial horizontal velocity and time.

- *Deriving of the Equation*

At the highest point in its path, the projectile´s velocity will naturally become 0 m/s. According to the velocity equation, (Equation 1.4), we can find the time it takes to reach the highest point. We will call the highest point for (t_1). So, when v = 0, we get the following expression:

$$0 = v_{oy} - g * t_1 =\longrightarrow t_1 = \frac{V_{0y}}{g}.$$

- *This is the time it would reach the object to reach its maximum height.*

Since a projectile motion path is symmetric, the time it take an object to go up is naturally equal to the time takes for the object to go down. So, the total time of flight will therefore be twice the time taken to reach the maximum height. The total time can be calculated with the following expression, and let's call that time "T"

$$\mathrm{t} = 2 * t_1 = 2 * \left(\frac{V_{0y}{}^{m}/_{s}}{g}\right) = \frac{(2 * V_{0y}{}^{m}/_{s})}{g}.$$

- *Therefore, it is the last part of equation 1.8 that will us the total time of the flight.*

$$\mathrm{t} = 2 * \frac{(V_{0y}{}^{m}/_{s})}{g}.$$

- *Time can also be calculated with the initial velocity V_0 with the following expression:*

$$t = 2 * \frac{V_0}{g} \sin \alpha.$$

If the object is launched from some elevation, i.e., not from ground, calculation of total flight-time will get more calculated, and we need to obtain a quadratic equation to solve.

- The time can be calculated with the following expression:

$$t = \frac{V_0 * \sin\alpha + \sqrt{(V_0 * \sin\alpha)^2 + 2gh}}{g}.$$

- *The Maximum Height*

The maximum height refers to the highest vertical position attained by an object when thrown upward or launched into the air. It marks the point in the motion when the object ceases its upward movement and begins descending. When the object reaches its maximum height, its velocity will naturally reach zero, and the object is therefore forced to commence its descent.

- Substituting t = t_1 into the velocity equation, we get the following expression:

$$h = V_0 \, {}^{m}/_{s} * \left(\frac{V_0 \, {}^{m}/_{s}}{g}\right) - 0{,}5 * g * \left(\frac{V_0 \, {}^{m}/_{s}}{g}\right)^2.$$

- *Simplifying, we get the following expression:*

$$h = \left(\frac{V_0{}^2 \, {}^{m}/_{s}}{g}\right) - 0{,}5 * \left(\frac{V_0{}^2 \, {}^{m}/_{s}}{g}\right).$$

- *Combining similar terms, will give us h in the simplest term, in the following expression:*

$$h = \left(\frac{V_0{}^2}{2g}\right).$$

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- *The maximum height can also be calculated with the following expression:*

$$H_{max} = v^2 * \frac{sin^2(\theta)}{2g}.$$

- Again, things are getting more complicated for objects that are thrown from an initial elevation differing from 0.

- The range of the projectile with an initial elevation differing from 0, can be calculated with either of the two following expressions:

$$v_0 \cos\theta * \left(\frac{V_0 \sin\theta + \sqrt{(V_0 \sin\theta)^2 + 2gh}}{g}\right).$$

This equation therefore provides the maximum height that the projectile will reach given certain initial launch conditions. The range of a projectile, also known as the horizontal displacement, is deeply intertwined with the concepts of initial velocity, launch angle, and gravitational acceleration, which I will discuss individually.

➢ *Proof of the Maximum Height*

We can use trigonometry to prove the maximum height, because as previously mentioned the velocity in the highest point of the projectile motion path the velocity is 0 $^{m}/_{s}$.

- *This can be proven mathematically with the following expression:*

$$0\,^{m}/_{s} = v_{0y} - gt\,.$$

The relation between the maximum range and displacement along the horizontal plane (x-axis) and the maximum height h (y-axis) can be proved with the following expression:

$$h = \frac{d \tan\theta}{4}.$$

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CHAPTER THREE
THE ANGLE OF REACH

- *Maximum Distance of a Projectile*

In an ideal environment where air resistance is negligible, the optimal angle to launch a projectile to reach the greatest horizontal distance is 45°.

The angle of reach is often represented as theta (θ), and is as previously mentioned a crucial parameter in the domain of projectile motion. It delineates the initial angle at which the object or projectile is launched into the atmosphere, relative to the horizontal axis.

- *The angle of reach can be calculated with either of the two following expressions:*

$$R = v^2 * \frac{\sin(2\theta)}{g}.$$

$$R = v_{0^2} * \frac{\sin(2\theta)}{g}.$$

- *Conclusion Regarding Angle of Reach*

In the study of projectile motion, the interplay of the angle of reach and maximum distance showcases intriguing aspects of physics. While this analysis assumes ideal conditions without considering factors such as air resistance or wind speed, it provides a starting point for understanding the principles governing projectile motion. These principles have widespread applications in many fields, including engineering, sports science, the military, and even space exploration, making their study an important aspect of physics.

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CHAPTER FOUR
CALCULATION EXAMPLES PROJECTILE MOTION

➢ *Calculation Assignment*

Let´s imagine that we throw a ball. The ball has an initial velocity of $60\, {}^{m}/_{s}$. The velocity forms an angle of 50°with the horizontal plan, and ball leaves hand of the person who throws the ball 1,80m above the ground.

- Calculate the velocity of the ball at its highest point in its trajectory.
- Calculate how long it will take before the ball reaches its point, and how high above the this is.
- Calculate how far the ball will reach.
- Calculate the velocity the ball will have when it hits the ground.
- Calculate the angle the ball will hit the ground with by using the tan-theorem.
- Calculating the start velocity in the x-direction
- $v_x = 60\, {}^{m}/_{s} * \cos 50° = 38{,}56\, {}^{m}/_{s}$.
- Calculating the start velocity in the y-direction
- $v_y = 60\, {}^{m}/_{s} * \sin 50° = 46{,}00\, {}^{m}/_{s}$.

➢ *Calculating the Velocity of the Ball at its Highest Point in its Trajectory*

At the highest point in the path, the velocity vector $\underset{v}{\rightarrow}$ is horizontal. This is because the tangent is horizontal. Then we can conclude the following $v = v_x = v_{0x}$. in other words, the velocity in the highest point is $\underset{v}{\rightarrow}$ =38,56 ${}^{m}/_{s}$.

- *Calculating how long it will take the ball to reach its highest point, and how high above the ground it will reach.*
- The time t can be calculated like this.

$$v_y = v_{0y} + g_y t = 0 \Longrightarrow t = \frac{v_{0y}}{g}.$$

$$t = \frac{46\, {}^{m}/_{s}}{9{,}81\, {}^{m}/_{s^2}} = 4{,}69\ s$$

- The highest point in the path or trajectory can be calculated like this:

$$H = h + (v_{0y}t + 0{,}5(-g_y)(t)^{\ 2}$$

$$H = 1{,}80m + 46\, {}^{m}/_{s} * (4{,}69s) + 0{,}5 * \left(-9{,}81\, {}^{m}/_{s^2}\right) * (4{,}69s)^{\ 2} = 109{,}65m$$

- *Proving that 109,65m is the Highest Point in the Trajectory with the Following Expression:*

$$0\, {}^{m}/_{s} = v_{0y} - gt \implies 46{,}00\, {}^{m}/_{s} - 9{,}81\, {}^{m}/_{s^2} * 4{,}69s = 0{,}00\, {}^{m}/_{s}\ .$$

As we see in the previously calculation the answer became $\mathbf{0{,}00}\, {}^{m}/_{s}$, naturally in the top point of the trajectory the velocity must be 0. So, this is a good way to control that you have calculated the correct height.

➢ *Calculating how Far the Ball will Reach.*

To be able to calculate how far the ball would reach we must first calculate the time, t. Since the ball is thrown from an elevated area this must be calculated with the quadratic formula.

$$t = \frac{-b - \sqrt{b^2 - 2ac}}{g}.$$

$$t = \frac{-46\, {}^{m}/_{s} - \sqrt{(46\, {}^{m}/_{s})^{\ 2} - 2\left(-9{,}81\, {}^{m}/_{s^2}\right) * 1{,}80m}}{-9{,}81\, {}^{m}/_{s^2}} = 9{,}38s$$

A quadratic equation has always two answers, one negative, and one positive, but of course in this case we are only interested in the positive answer. If we wanted the negative answer of this equation, we could simply just remove the change from -46${}^{m}/_{s}$ in the start of the equation to positive 46${}^{m}/_{s}$.

Now that we have calculated the time it will take the ball to travel the distance along the x-axis, we can also calculate how far the ball would reach by using the following equation:

$$x = v_{0x}t.$$

As we know the velocity along the x-axis is constant (that is, if we disregard all other external forces such as for example air-resistance among others)

- $Distance_{x-axis} = 38{,}56\, {}^{m}/_{s} * 9{,}38s = 360m$, the ball would reach 360m before it would hit the ground.
- Calculating the velocity, the ball has when it hits the ground.

To be able to calculate the velocity the ball has when it hits the ground, we first need to find v_x and v_y, this can be done in the following way:

$$v_x = v_{0x} = 38{,}56\, {}^{m}/_{s}.$$

$$v_y = v_{0y} + (-g_y)t.$$

$$v_y = 46\, {}^{m}/_{s} + \left(-9{,}81\, {}^{m}/_{s^2}\right) * 4{,}69s = -0{,}107\, {}^{m}/_{s}.$$

We can consider this as a triangle. By using other words, the velocity the ball has when it hits the ground can be considered as the hypotonus in this triangle.

$$v = \sqrt{(v_x)^2 + (v_y)^2} \quad .$$

$$v = \sqrt{(38{,}56\, {}^{m}/_{s})^2 + (-0{,}107\, {}^{m}/_{s})^2} = 4.12\, {}^{m}/_{s}.$$

- Calculating the angle, the ball has when it hits the ground.
- The angle can be found by using trigonometry.

$$\tan^{-1} = \left(\frac{v_y}{v_x}\right)$$

$$\tan^{-1} = \left(\frac{4{,}12\, {}^{m}/_{s}}{38{,}52\, {}^{m}/_{s}}\right) = 6{,}10°$$

➢ *Illustration of the Projectile Motion Calculation.*

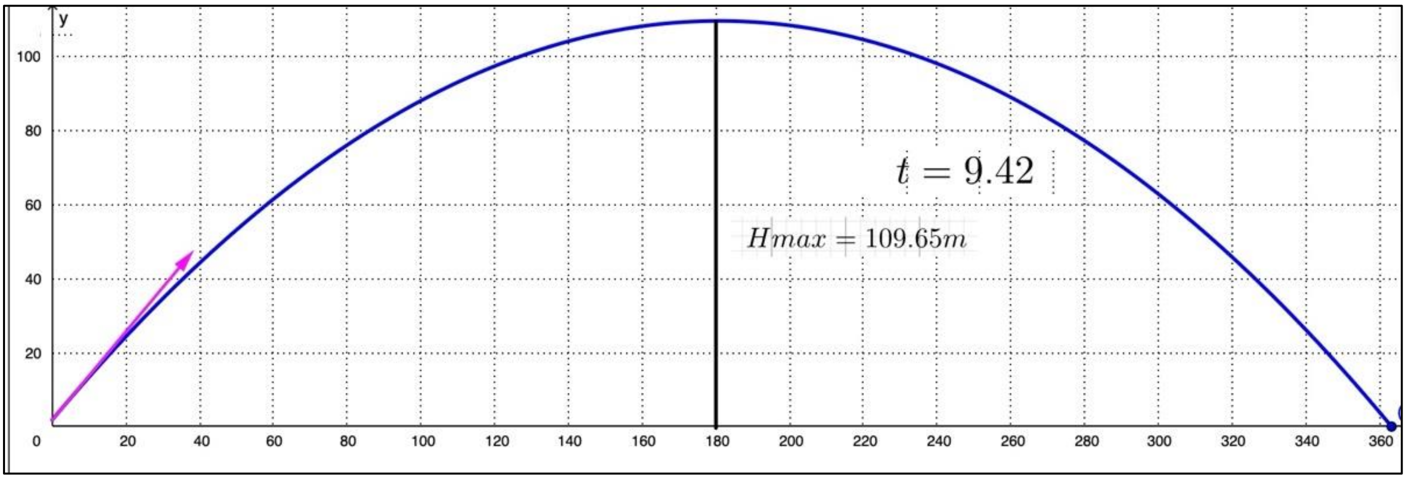

Fig 1 Projectile Motion of the Calculation in Chapter 4

$$v_0 = 60\, {}^{m}/_{s}, Initial_{angle} = 50°;\ v_x = 38{,}56\, {}^{m}/_{s};\ v_y = 46{,}00\, {}^{m}/_{s}$$

$$h_{max} = 109{,}65m; Max_{distance} = 360m; t_{calculated} = 9{,}38s;\ t_{Geogebra} = 9{,}42s$$

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CHAPTER FIVE
PROJECTILE MOTION FROM AN ORBITING BODY

- *Introduction*

Projectile motion is a fundamental physical phenomenon that occurs when an objects is influenced by the forces of gravity- and if applicable, air resistance. In the context of celestial bodies (star, planets and moons), the concept of projectile motion extends to encompass the first- and second cosmic velocities, commonly known as escape velocities.

There are as previously mentioned two velocities that are crucial for a successful launch of any object that are supposed to go into orbit. The first- and second cosmic velocity, commonly known as the circular orbital velocity- and the escape velocity (These two velocities can be read about further down in the article)

To comprehend the intricate mathematics behind these concepts, various scientific principles and equations are employed. To provide the foundation for understanding of the forces involved Newtons laws of motion- and particularly Newtons law of universal gravitation is used in mathematical calculation to determine precise values of the first- and second cosmic velocities for different celestial bodies.

By developing a comprehensive understanding of these two velocities, engineers- and physicists can effectively plan- and execute various types of space missions. This knowledge enables the successful deployment of satellites, exploration of distant planets.

- *The First Cosmic Velocity (Circular Orbital Velocity)*

The first cosmic velocity represents the minimum speed required for a celestial body in orbit to counteract the gravitational force exerted by the object it orbits around. When the first cosmic velocity is achieved the orbiting body will maintain in a stable orbit around the central object without the need for any additional propulsion.

When the projectile or an object is maintaining this velocity, it experiences a delicate balance between the gravitational attraction- and the centrifugal force. One can therefore conclude that at "The first cosmic velocity" these two forces (gravitation- and centrifugal force) are in equilibrium.

The first cosmic velocity is crucial for a successful launch, as this velocity allows the object or projectile to maintain its desired orbital path.

- *The First Cosmic Velocity v_1, can be Calculated with the Following Expression:*

$$v_1 = \sqrt{\frac{GM}{r}}.$$

Any object travelling at this velocity will remain in a constant circular orbit around the main body, providing no other forces interfere with the object. The first cosmic velocity is also known as the orbital velocity.

- *Second Cosmic Velocity (Escape Velocity)*

The second cosmic velocity also known as the escape velocity. This is the minimum velocity needed for an object to completely escape the gravitational force of the planet it is being launched from. When the escape velocity is surpassed, the object can by other words break free from the gravitational influence and proceed on an independent trajectory.

Escape velocity serves as a benchmark for space missions, as it is necessary to travel at or above this velocity to overcome the gravitational attraction from the planet the object or rocket are being launched from. For example, when travelling to the moon achieving escape velocity is crucial for a successful departure.

- *The Escape Velocity, v_2, can be Calculated with the Following Expression:*

$$v_2 = \sqrt{\frac{2GM}{r}}.$$

If we look at these formulas one can see that the formula for calculating the first cosmic velocity are is derived from using the central force and gravitational force. The second formula, "The second cosmic velocity" is also derived from using the central force- as well as the gravitational constant, but this time it is multiplied with two.

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- ✓ V_1= Represents the First cosmic velocity, (Circular orbital velocity.)
- ✓ v_2= Represents the Second cosmic velocity, (Escape velocity.)
- ✓ G= Represents the gravitational constant $6.67 * 10^{-11}\, {Nm^2}/{kg^2}$.
- ✓ M= Represents the mass of the body/planet being orbited.
- ✓ r= Represents the indicated the radius at which the objects orbits.

- *Applying Projectile Motion Principles in Orbit*

In a projectile motion from an orbiting body in an environment devoid of air resistance, a critical factor is the angle at which an object is projected. This calculation is quite complex as it incorporates not only trigonometric functions but differential equations, as well.

- *For Analyzing the Initial Velocity and Angle, for both Horizontal and Vertical Directions*

- *See Equation 1.1 and 1.2.*

- *The Horizontal Distance can be Calculated with the Following Expression:*

$$Distance_x = v * \cos(\theta) * t.$$

- *The Vertical Distance can be Calculated with the Following Expression:*

$$Distance_y = h + v * \sin(\theta) - 0{,}5 * g * t^2.$$

- *Conclusion, Projectile Motion from Orbiting Body.*

First and second cosmic velocities* offer fascinating insights into the complexities of our universe. Understanding these concepts is crucial in space exploration.

By incorporating the gravitational principles of physics, as well as using higher mathematics as a tool, we can both predict- and guide the movement of celestial bodies with a precision which is incredibly accurate. By studying this field further, we will continue to enhance our understanding- and our ability to navigate thru cosmic.

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CHAPTER SIX
SOME MATHEMATICAL FORMULAS USED WITHIN PROJECTILE MOTION CALCULATION

- *Initial Velocity x-Component*

$$v_x = v_0 * \cos(\theta) = {}^{m}/_{s}$$

- *Initial Velocity y-Component*

$$v_y = v_0 * \sin(\theta) = {}^{m}/_{s}$$

- *Displacement / Position Equation*

$$d = V_{0y} * t + 0{,}5 * g * t^2$$

- *Velocity Equation along the y-Component (at any Time during the Flight)*

$$V = V_{0y} - g * t$$

- *The Horizontal Distance along the x-Component*

$$d_x = v_0{}_x * t = m$$

- *To Hit a Target at Range "x" and Altitude "y" when Fired from 0,0 and with Initial Velocity v the Required Angles of Launch Theta are:*

$$\theta = \arctan\left(\frac{v^2 \pm \sqrt{v^4 - g(gx^2 + 2yv^2)}}{gx}\right)$$

- *The Vertical Distance can be Calculated with the Following Expression:*

$$d_y = h_{max} + v_{y0}t - \left(\frac{gt^2}{2}\right) = m$$

- *Time of Flight for the Projectile can be Calculated with the Following Expressions.*

$$t_{flight} = 2 * \left(\frac{v_y}{g}\right) or\ 2 * \left(\frac{v_0 \sin\theta_0}{g}\right)$$

*Notice that we can calculate this on several ways, we can use the initial velocity V_0, or we can use the velocity V_y. If we use V_y we don't have to calculate

- *If the Projectile is Thrown from an Elevated Area Time can be Calculated with either of the two Following Expressions.*

$$t = \frac{v_y + \sqrt{(v_y)^2 + 2gh}}{g}.$$

$$t = \frac{v_0 \sin\theta + \sqrt{(v_0 \sin\theta)^2 + 2gh}}{g}.$$

- *Range of the Projectile can be Calculated with either of the two Following Expressions.*

$$R = v_0 \cos\theta \left(\frac{v_0 \sin\theta + \sqrt{(v_0 \sin\theta)^2 + 2gh}}{g}\right) = m.$$

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- *Maximum Height Projectile Motion*

$$H_{max} = \frac{v_0 sin^2\theta}{2g}.$$

$$H = h + (v_{0y}t + 0{,}5a_y t)^{\,2}.$$

- *Not to be Confused with "Thrown from a ledge" h Represents Height of a Person.*

➢ *The First Cosmic Velocity*

$$v_1 = \sqrt{\frac{GM}{r}}.$$

➢ *The Second Cosmic Velocity (Escape Velocity)*

$$v_2 = \sqrt{\frac{2GM}{r}}.$$

➢ *Horizontal Distance Orbiting Body*

$$Distance_x = v * \cos(\theta) * t.$$

➢ *Vertical Distance Orbiting Body*

$$Distance_y = h + v * \sin(\theta) - 0{,}5 * g * \ t^2.$$

➢ *Were*

- v_x= Represents the velocity among the x-axis.
- v_y= Represents the velocity among the y-axis.
- v = Initial velocity combined
- $v_{0\,(y\ or\ x)}$ = Represents the start velocity along the y- or x-axis.
- d = Represent the displacement or distance in meter
- d_x= Represents the horizontal distance (Distance along the x-axis)
- d_y= Represents the vertical distance (Distance along the y-axis)
- g= Represents the gravitational acceleration which on Earth is $9{,}81\, {}^{m}/_{s^2}$
- t = Represents the time.
- h= Represent the height of the of the projectile at any given time.
- h_{max}= Represent the maximum height of the projectiles path.
- t_{flight}= Represents the total time of flight for the projectile.
- v_1= Represents the first Cosmic velocity of a projectile that is being launched from a rotating body.
- v_2= Represents the second Cosmic velocity of a projectile that is being launched from a rotating body, this formula is the same formula used for calculating the escape velocity.
- M= Represents the mass of the body that the projectile is being launched from.
- G= Represents the gravitational constant $6{,}67 * 10^{-11}\, {}^{N_m^2}/_{k_g^2}$.
- r= Represents the radius of the body that the projectile is being launched from.

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CHAPTER SEVEN
CONCLUSION

Projectile motion study is an important area of research that involves examining the trajectories of objects launched into the air. It is a complex phenomenon that combines both vertical and horizontal movements, and its understanding is based on mathematical principles derived from two-dimensional kinematics. By using parametric equations of motion, engineers- and physicists are able to calculate and analyze the intricate paths followed by projectiles.

One of the key factors that significantly affects the trajectory and displacement of a projectile is its initial velocity. The initial velocity are denoted as v_0.

v_0 can be broken down into two components: $v_x = v_0 * \cos\varphi$, for the movement along the horizontal direction, often denoted as the x_{-axis} and $v_y = v_0 * \sin\theta$ for the movement along the vertical direction, often denoted as the y_{-axis}.

The maximum height of the projectile is heavily impacted by the initial velocity v_0 and the initial launch angle.

The higher the initial velocity- and launch angel is, the higher the projectile will reach (see illustration below $v_0 = 75\ {}^{m}/_{s}\ Initial_{angel} = 65°$ *illustration 1.2.*)

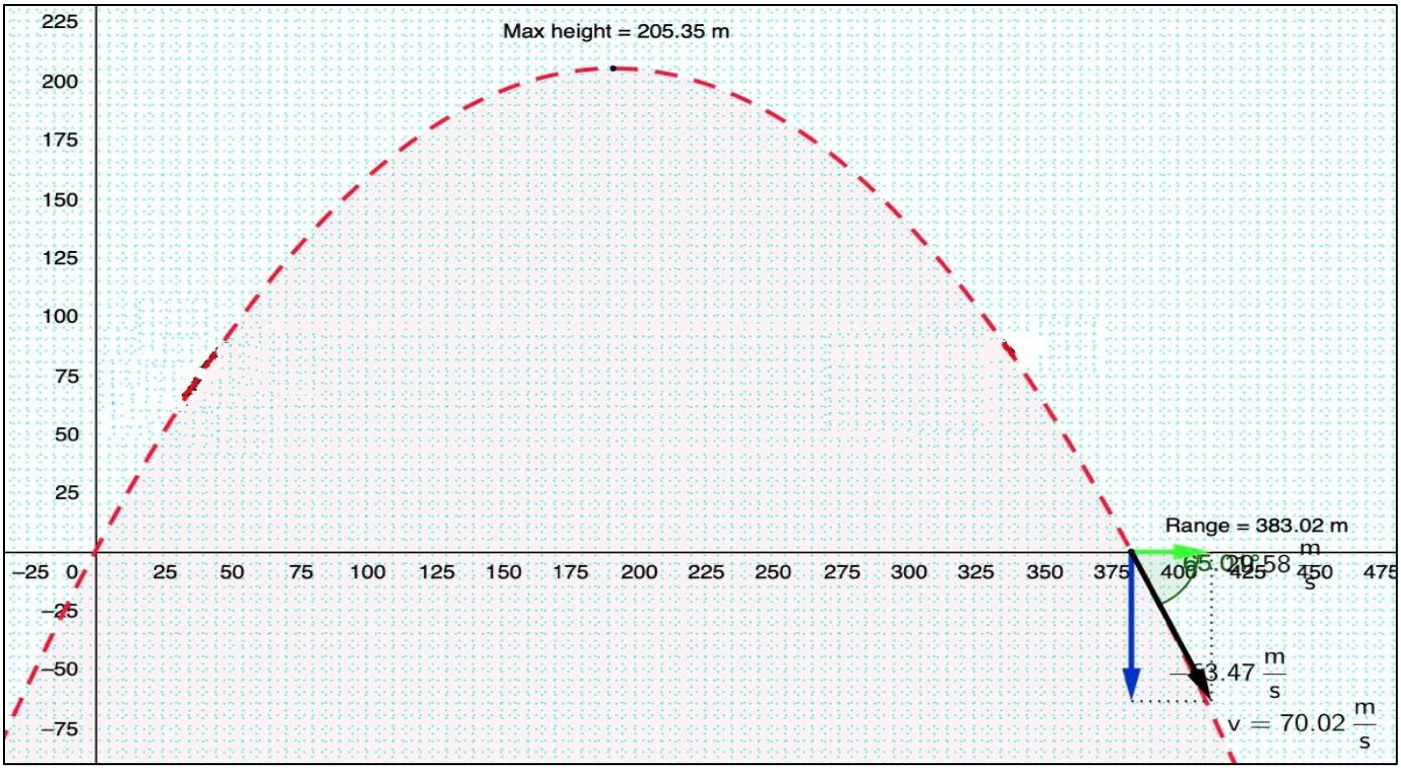

Fig 2 The Higher the Initial Velocity

➢ *Illustration 1.2*

From illustration 1.2 we can observe that the projectile will have a maximum height of 205,35m but in return only a maximum reach of 383m and a total flight of 12,95s

For a projectile to its longest possible flight time is the best initial launch angle 45°.

This is because the horizontal- and the vertical components of the velocity vector then becomes equal. Since these two components then becomes equal this will allow the projectile to cover the maximum possible horizontal distance, and thus achieving the longest possible range.

At a 45° launch angle the projectiles motion are shaped like a symmetrical parabolic shape- and the projectiles maximum height will be half of the maximum range of the reached by the projectile (see illustration below $v_0 = 75\ {}^{m}/_{s}\ Initial_{angel} = 45°\ v_x = 53{,}03\ {}^{m}/_{s}\ ;\ v_y = 53{,}03\ {}^{m}/_{s}$ *Illustration 1.3.*)

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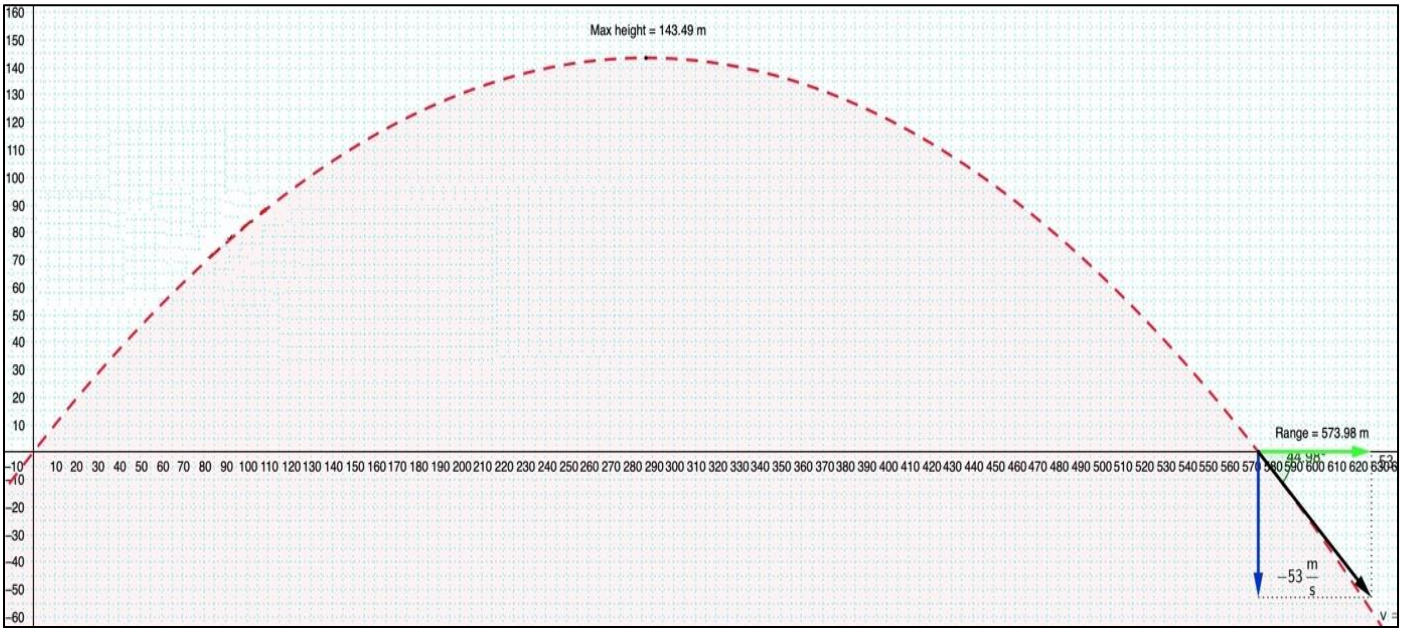

Fig 3 At a 45° Launch Angle

➢ *Illustration 1.3*

From illustration 1.3 we can observe that the projectile will only reach a maximum 143,49m (y-axis) and 286,99 (x-axis / displacement) but in return it will have a maximum reach of 573,98 or 574m.

Displacement, on the other hand, refers to the overall change in position from the point of origin to the final point in the trajectory, regardless of the path taken by the object. In projectile motion, the calculation of displacement involves breaking down the motion into its vertical and horizontal components. The vertical displacement is affected by gravitational forces, whereas the horizontal displacement remains constant as it is unaffected by gravity.

The horizontal motion of a projectile remains uniform throughout its flight since gravity does not exert any direct influence on it. This uniform motion continues until the projectile encounters a barrier or reaches the ground. The horizontal displacement, denoted as d, can be calculated using the equation $d = v_0 * \cos\varphi * t$.

When considering the motion of objects in orbit around a central body, such as planets or satellites, the effects of gravity become even more pronounced. Gravity transforms what would have been a linear path into a circular or elliptical orbit, making the object being in a constant state of free fall around the planet that it is orbiting. The movements of orbiting objects can be predicted using scientific principles, such as Kepler's laws of planetary motion and Newton's laws.

The study of projectile motion involves understanding concepts such as initial velocity, displacement, horizontal motion, launch angle, and the effects of gravity on orbiting bodies. These elements, when integrated and analyzed, provide a comprehensive understanding of the behavior of projectiles. The knowledge gained from studying projectile motion has wide-ranging applications, from improving sports performance and designing efficient transportation systems to exploring celestial bodies and developing military tactics. By continually enhancing our understanding of projectile motion, scientists and researchers can unlock invaluable insights into the laws of physics and advance various fields of study.

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