A Study of Algorithms for the pth Root of Matrix

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Abstract:- Some results for Pth root of square matrix are revived. It shows that matrix sign function and Wiener-Hopf factorization plays important role in Pth root of matrix. Some new algorithms for computing Pth root numerically can design by these results. We can analyze Stability properties of iterative methods for convergence.

Keywords:- *Pth Root Of Matrix, Matrix Sign Function, Newton's Method, Cyclic Reduction, Wiener – Hopf Factorization, Graeffe- Iteration, Laurent Polynomial.*

I. INTRODUCTION

A is nth ordered square matrix with real or complex entries having all positive eigenvalues (Positive definite matrix) are non-negative real number. p Ɛ N the set of positive integer. We get matrix uniquely having.

- $X^p = A$.
- λ the eigenvalue of X such that $\lambda \in \left\{z: -\frac{\pi}{p} < \arg(z) < \frac{\pi}{p}\right\}$. where $X = A^{\frac{1}{p}}$.

➢ *Applications:*

- **Application1**. Matrix logarithm, $\log(A) = p * \log(A)^{1/P}$. for, well approximated p by polynomial or function of rationales.

- **Application2**. Function from matrix sector $sect_p(A^p)^{(-1/p)} * A$. Function from matrix sector P_2 is matrix sign function.

- **Application3**. Hoskins and Walton iteration method discussed in [18]. By Newton's formula of iteration,

$$X_{k+1} = \left(\frac{1}{p}\right) * \left[(p-1) * X_k + A * X_k^{1-p}\right]. \quad \text{And} \quad X_0 = A. \tag{1.1}$$

A symmetric positive definite. (X_k converges is $A^{1/p}$ but convergence is not definite. by Smith [27].

- **Application 4**. Benner et al. [1] has columns of $U = [U_1^*, ., ., ., ., U_p^*] \in C^{pnxn}$ this spans invariant subspace of

$$U = \begin{bmatrix} 0 & 1 & - & - & - & - \\ - & 0 & 1 & - & - & - \\ - & - & 0 & 1 & - & - \\ - & - & - & 0 & 1 & - \\ - & - & - & - & 0 & 1 \\ A & - & - & - & - & 0 \end{bmatrix} \in \text{Cpn x pn} \tag{1}$$

Where, $C * U = U * Y$, such that, $Y \in C^{nxn}, and\ |U_1| \neq 0$.

then $X = U_2 * U_1^{-1} = A^{1/p}$.

- **Application5**. Algorithm for computing $A^{1/p}$ by Shiel et al. [26].

$A^{1/p} contain\ X_k = G^k * [I_n, 0,0,0]^T$.

For $G = C + I$. and matrix C like (1.2).

That is Compute $\lim_{k\to\infty} Xk(1{:}n,\ :)\, Xk(n+1 : 2n,\ :))\big)^{-1}$

- **Application6**. For Tsay et al [29]. It gives other way to find $A^{1/p}$ square matrix. It is like General continued fractions of block for Toeplitz matrix. Method have O(n^5) flops and O(n^3) storage. It is in [28]. Some time it is unstable.

- **Application7**. Tsay et al [28]. It is iterative method convergence proof is not available, but it is Numerically stable like

$G_{K+1} = G_k * [(2 * I + (P-2) * G_k) * (I + (P-1) * G_k)^{-1}]^p$,

Where $G_0 = A$.

$R_{K+1} = R_k * (2 * I + (P-2) * R_k)^{-1} * (I + (P-1) * G_k)$,

Where $R_0 = I$.

Here $G_k \to I$ *and* $R_k \to A^{1/p}$.

It'^{s} proof of convergence not in [28].

Analysis of perturbation in [15] as follows,

$$A^{\left(\frac{1}{p}\right)} = \frac{p * \sin\left(\frac{\pi}{p}\right)}{\pi} * A * \int_0^\infty (x^p * I + A)^{-1} dx \tag{2}$$

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It shows iterations are numerically stable in complex computing p[th] root of triangular matrix, by Hasan et al. [13].

- *The Paper is Divided in to Two Parts:*

- Section 2 to 6 theoretical properties.
- Section 7 to 10 algorithmic results.

- ***Part: 1.***

- **Section 2:**

- ✓ Represent A(1/p) in integral analytic function with unit complex circle in plane of complex.
- ✓ A1/p approximated by numerical Fourier integration point for error r2*N, N is Fourier point number and r < 1 depends on p as well as A.

- **Section 3:**

- ✓ Using sign function of matrix prove $A^{(1/p)}$ is multiple of (2.1).
- ✓ Constant of multiplication can known can explicitly.

- **Section4:**

- ✓ $F(z) = z^{-\frac{p}{2}}((1+z)^p A - (1-z)^p I)$ is Matrix factorization by Wiener-Hopf.
- ✓ Key tool is Cayley transformation $X \to z = \frac{(1-x)}{(1+x)}$ it gives mapping function to imaginary axis to unit circle.
- ✓ It relates $X^p - A$ with imaginary axis to F (z) with respective to unitary circle.

- **Section5:**

- ✓ $A^{\frac{1}{p}}$ relates with central coefficients $H_0, ., ., ., ., ., ., H_{\left(\frac{p}{2}\right)-1}$.
- ✓ $A^{\frac{1}{p}}$ is related to central coefficient H_0, …., $H_{p/2-1}$of matrix Laurent series
- ✓ $H(z) = H_0 + \sum_{i=1}^{+\infty}(z^i + z^{-i})H_i$ where $H(z) * F(z) = I$.

- **Section6:**

- ✓ Observe $A^{\frac{1}{p}}$as an inverse of fix point function $\left(\frac{1}{p}\right) *$ $[(1+p) * X - X^{p-1} * A]$ by iterative $X^{-p} - A = 0$.
- ✓ It gives sufficiency to convergence and stability.

- **Section7:**

- ✓ Obtain Algorithm for inverting $A_{np \times np}$ matrix to n x n blocks.
- ✓ Polynomial interpretation of A - circulate matrix for finding $A^{1/p}$.
- ✓ block companion matrix C (1.2) forms A – circulate matrix.

- **Section8:**

- ✓ Form some iterations F(z) initially evaluate process and then use on Graeffe'[s] iteration.

- **Section9:**

- ✓ Two algorithms on F(z).
- ✓ Apply to reduce F(z) and on poly Laurent inverse matrix.

- **Section10:**

- ✓ Analyze results of preliminary numerical experiments.

II. RESULTS

- As p = 2q there exist at least one real root and remaining complex roots.
- There exist exactly equal positive real and negative real roots.

- *Sections in Detail:*

- ***Section 2:*** *Define Matrix Polynomial*

$\Psi(z) = (1+z)^p * A - (1-z)^p * I = \sum_{j=0}^{p} z^j * \binom{p}{j} * (A + (-1)^{j+1} * I)$.. (2.1)

$\Psi(z)$ is non singular in $|z| = 1$.

$\Psi(z)$ and its inverse is analytic in

$$A = \left\{z \in C : \acute{\rho} < |z| < \frac{1}{\acute{\rho}}\right\} \quad (3)$$

Where $\acute{\rho} = \max\{z \in C : \det\Psi(z) = 0, |z| < 0\}$

Proposition proved, Proposition2.1 P[th] root of A becomes

$$X = \frac{p * \sin\left(\frac{\pi}{p}\right)}{i * \pi} * A * \int_{|z|=1}((1+z)^{p-2} * \Psi(z)^{-1})\, dz$$
(2.3)

Moreover,

$$X = \frac{2*p*\sin\left(\frac{\pi}{p}\right)}{N} * A * \sum_{i=0}^{N-1}\left(A - \left(\frac{1-\omega_N{}^i}{1+\omega_N{}^i}\right)^p I\right)^{-1} * \frac{\omega_N{}^i}{(1+\omega_N{}^i)^2} + O(r^{2N}) \quad (4)$$

Where p <r < 1

- ✓ ***Algorithm 2.1***

- For obb p put $P = 2 * P$ and $A = A = A^2$.
- For p multiple 4 then repeat $P = \frac{p}{2}$.
- And $A = \sqrt{A}$, till odd p/2.
- Set $N = N_0$.
- Where $X_N = \frac{2p\sin\left(\frac{\pi}{p}\right)}{N} A \sum_{i=0}^{N-1}\left(A - \left(\frac{1-\omega_N{}^i}{1+\omega_N{}^i}\right)^p I\right)^{-1} \frac{\omega_N{}^i}{(1+\omega_N{}^i)^2}$
- If $\|A - X_N{}^p\| > \varepsilon$ set $N = 2 * N$ and repeat step 3, otherwise output $X_N \to X$.

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- *Section3: Reduction for Matrix Sign Computation.*

- **Proposition3.1.** Let $'\Omega = w_p^{ij}, i, j = 0\ to\ p-1$.

Let $X = A^{(1/p)}$ define block diagonal matrix

$D = diag\ (I, X, X^2, X^3, \ldots\ldots, X^{(p-1)})$.

Then C = $\frac{1}{p} * (\ D(\Omega \otimes \text{I}) S('\Omega \otimes I) D^{-1}$.

Where S = diag(I, w_p^1 X, w_p^2 x², w_p^3 X……… w_p^{p-1}X^{p-1}).

And ⊗ denotes kronecker product. Consequence of this is,

sign (C) = $\frac{1}{p}\ D(\Omega * \otimes * \text{ I})\ \text{sign}(S)\ ('\Omega * \otimes * I) * D^{-1}$. (5)

Where $sign\ (S) = diag\left(sign(X), w_p^1\ sign(X), w_p^2\ sign(X), \ldots\ldots, w_p^{p-1} sign(X)\right)$.

- **Proposition 3.2** If $p = 2 * q$ where p is odd, then the first block column of the matrix sign(C)

is given by $V = \frac{1}{p}\begin{bmatrix} y_0 X^0 \\ y_1 X^1 \\ y_2 X^2 \\ \vdots \\ y_{p-1} X^{p-1} \end{bmatrix}$,

where $X = A^p$ and $\gamma_i = \sum_{j=0}^{p-1} w_p^{ij}\ \theta_j$. And

$i = 0\ to: p-1, for\ j = \left[\frac{q}{2}\right] + 1: \left[\frac{q}{2}\right] + q, \qquad \theta_j = 1\ otherwise.$

✓ **Algorithm 3.1** (Matrix p^th root by Matrix sign function).

- **Input:** For p, n are integers, $A\ \mathcal{E} C^{nxn}$
- **Output:** X, the p^th root of A.

For obb p put $p = 2 * p\ and\ A = A^2$.

For p multiple 4 then repeat $P = \frac{p}{2}$.

And A = $\sqrt{A}$, till odd p/2.

❖ Compute $sign(C)\ and\ let\ V = (V_i)\ for\ i = 0\ to\ p - 1$, be first block column.

❖ Compute $X = (p/(2 * \alpha)) * V_1$.

Where α = $1 + 2 * \sum_{j=1}^{[q/2]} \cos\left(\frac{2\pi}{p}\right), and\ q = p/2$.

❖ It gives generalization of (3.2) as follows.

- sect_p(C) = $\begin{bmatrix} 0 & X^{-1} & \ldots & 0 \\ \vdots & 0 & \ddots & \vdots \\ 0 & \ddots & \ddots & X^{-1} \\ AX^{-1} & 0 & \ldots & 0 \end{bmatrix}$.

- *Section4: Reduction to Weiner Hopf Factorization.*

- **Proposition4.1** Let $S(z)$ is any polynomial of matrix like $F(z) = U(z) * U(z^{-1})$.

Where $U(z) = zp * S(z^{-1})$.

Then p^th root X of A is.

$X = -\sigma^{-1} * (q * I + 2 * S(-1) * S(-1)^{-1})$.

Where $\sigma = \sum_{j=-\left[\frac{q}{2}\right]}^{\left[\frac{q}{2}\right]} w_p^j = 1 + 2 * \sum_{j=-[-1]}^{\left[\frac{q}{2}\right]} \cos\left(\frac{2\pi}{p}\right)$.

✓ **Algorithm 4**: p^th root of A by Weiner-Hopf factorization).

- **Input:** p and n are integers, $A\ \varepsilon\ C^{\ nxn}$.
- **Output:** Approximation of p^th root X of A.

❖ For obb p put $p = 2 * p$ and $A = A^2$.

For p multiple 4 then repeat $P = \frac{p}{2}$.

And $A = \sqrt{A}$, till odd p/2.

❖ Compute $F(z) = U(z) * U(z^{-1})$

Where $F(z) = z^{-q} * \Psi(z)$ and set $S(z) = z^p * U(z^{-1})$.

❖ Find $X = -\sigma^{-1} * (q * I + 2 * S(-1) * S(-1)^{-1})$,

Where $\sigma = \sum_{j=-[q/2]}^{[q/2]} w_p^j = 1 + 2 * \sum_{j=-[-1]}^{[q/2]} \cos\left(\frac{2\pi}{p}\right)$.

- *Section 5: Reduce to Matrix Laurent Polynomial Inversion.*

- **Proposition5.1** Principal p^th root X of A can be written as,

$$X = 4 * p * \sin\left(\frac{\pi}{p}\right) * A * \prod_{j=0}^{q-1} \propto_j * H_j. \qquad (6)$$

Here H(z) is Laurent series.

$H(z) = F(z)^{-1} = \sum_{j=-\infty}^{\infty} z^j H_j = H_0 + \sum_{j=1}^{\infty} (z^j + z^{-j})\ H_j$ and

$\propto_0 = \frac{1}{2}\begin{pmatrix} p-2 \\ q-1 \end{pmatrix}, \propto_{0j} = \begin{pmatrix} p-2 \\ q-j-1 \end{pmatrix}, j = 1: q-1.$

✓ **Algorithm: 5.1 (p^th root of matrix using Laurent polynomial).**

- **Input:** p and n ∈ $N., A \in C^{nxn}$.
- **Out put:** Approximation of p^th root X of A.

❖ For obb p put $p = 2 * p\ and\ A = A = A^2$.

For p multiple 4 then repeat $P = \frac{p}{2}$.

And A = $\sqrt{A}$, till odd p/2.

- Find coefficients $H_0, \ldots, H_{q-1}$ of inverse $H_j = H_0 + \sum_{j=1}^{\infty}(z^j + z^{-j}) * H_j$ of $F(z) = z^{-q} * \Psi(z)$.

Find $X = 4\,p \sin\left(\frac{\pi}{p}\right) A \prod_{j=0}^{q-1} \propto_j H_j$,

Where $\propto_0 = \frac{1}{2}\binom{p-2}{q-1}$, $\propto_j = \binom{p-2}{q-j-1}$, $j = 1{:}\, q-1$.

- *Section 6: (Newton's Iteration for p^{th} Root).*

- **Proposition6.1: R**esiduals $R_k = I - X_k^p A$.

for $X_{k+1} = (1/p) * [\,(p-1) * X_k - A * X_k^{1+p}]$, and $X_n = I$ obeys X_0,

$R_{k+1} = \sum_{i=2}^{p+1} a_i * R_k^i$, where a_i >0.and $\sum_{i=2}^{p+1} a_i = 1$

That is for $\|R_0\| < 1$, $\{\|R_0\|\} \rightarrow 0$ as $k \rightarrow \infty$.

- **Proposition 6.2:** For every eigen values ƛ >0 of matrix A. Iteration

$X_{k+1} = (1/p) * [\,(p+1) * X_k - X_k^{1+p} * A]$, $X_0 = I$, accelerates to $A^{-1/p}$.

if $p(A) = p + 1$. the value not accelerates to inverse p^{th} root of A.

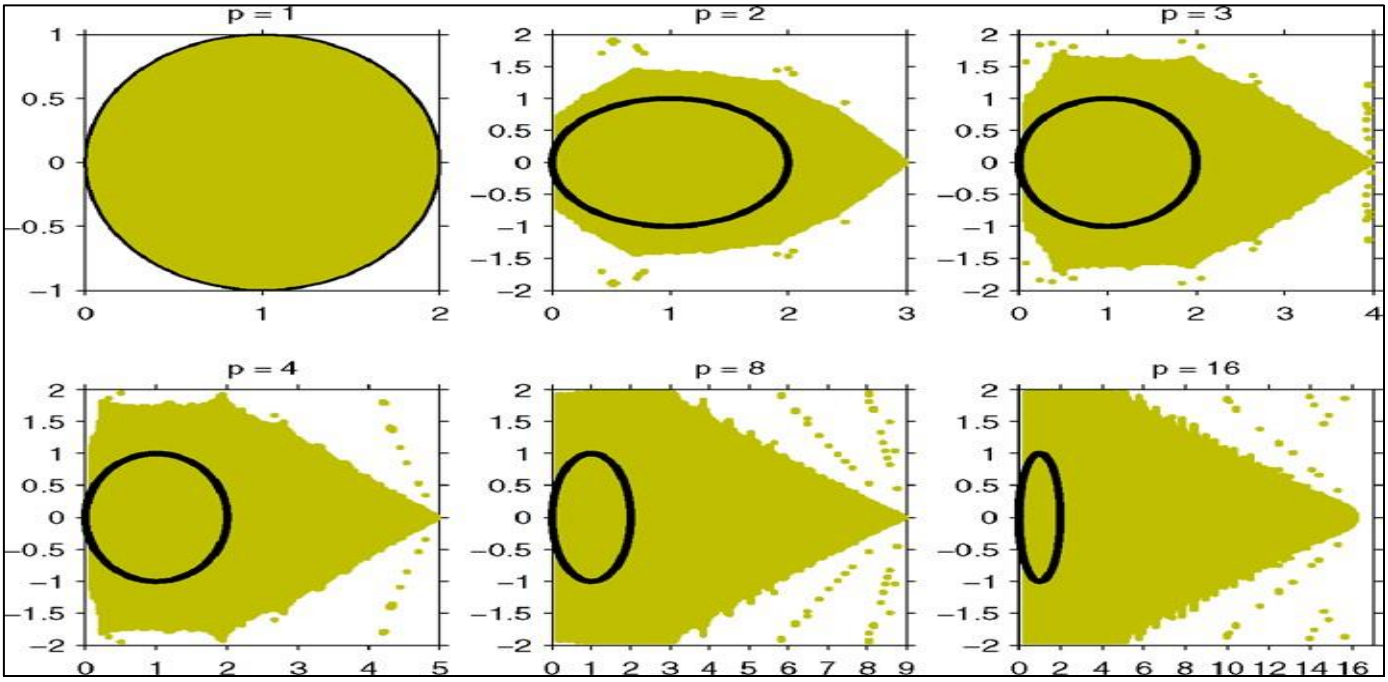

Fig 1: Convergence Regions (Shaded) in Circle for Iteration (6.5), together with Unit Circle
Note: Differing Axis Limits

- *Section 7. Inversion of A-Circulenes Matrix.*

✓ **Algorithm 7.1:**

- **Input:** p, n ∈ N and $A \in C^{nxn}$ and commuting matrices

$W_0 \ldots W_{p-1} \in C^{nxn}$defining matrix p = $\begin{bmatrix} W_0 & W_1 & \cdots & W_{p-1} \\ AW_{p-1} & W_0 & \ddots & \vdots \\ \vdots & \ddots & \ddots & W_1 \\ AW_1 & \cdots & AW_{p-1} & W_0 \end{bmatrix}$

- **Out put:** Initial column block of p^{-1}.
  - Leads $(p-1) \in N$ as $p - 1 = \sum_{i=0}^{d+1} 2^{m_i}$.
  - $Set\ B0 = D\text{-}1 * P * D$.
  - For i = 0, find $B_i = B_{i-1} * (D^{-1} * B^{i-1} * D^i)$.

- Compute
- $V = [I, 0,0, \ldots 0]S = [I, 0,0, \ldots 0]B_{m_0}D^{-2^{m_0}}B_{m_1}D^{-2^{m_1}} \ldots . B_{m_{d-1}}D^{p-1-2^{m(d-1)}}$
- Output $V * (I * \otimes * k^{-1})$

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- *Section 8: (Inverting Matrix Laurent Polynomial).*

- 8.1 Evaluation / Interpolation:

✓ **Algorithm8.1:**

- **Input:** Multiples $F_0, \dots, \cdot, \cdot, F_p\ of\ F(z)$;

Here $h = \sum_{i>h} \|H_i\|_\infty\ is$ negociabel.

- **Output:** Guise of H_i for i = 0, q-1 of Laurent series matrix $H(Z) = F(Z)^{-1}$.

Find integer $N = 2^v\ for\ N > 2 * h + 1$,.

w_N^i the n^th root of unity for i = 0 to N-1.

Where $w_N = \cos(\frac{2\pi}{N}) + i * \sin(\frac{2\pi}{N})$.

❖ Find $w_i = F(w_n^i),\ i = 0\ to\ (N-1)$.

❖ Find $V_i = (w_i^{-1})\ for\ i = 0\ to\ (N-1)$.

❖ Repeat value of V_i to recover $K_i\ of\ K(z) = \sum_{i=-h}^{h} z^i K_i$,

that interpolate H(z) at root of unity.

❖ Results $K_i to\ H_i$. for i = 0 to q -1.

➢ *(Graeffe Iteration):*

- **Proposition 8.1:** Assume that P(z) = $\sum_{i=-q}^{q} z^i * P_i$ has P(z) = U(z) * V(z^-1),

For $U(Z) = (z * I - X_1) * (z * I - X_2) \dots . . (z * I - X_q)$,

$V(Z) = (z * I - Y_1) * (z * I - Y_2) \dots \dots (z * I - X_q)$,

And matrices X_j, Y_j are of the form,

$\|X_j\|, \|Y_j\| \leq \sigma \leq 1, j = 1{:}q, for\ suitable\ operator\ \|.\|$. Moreover AB = BA

for any $A, B \in \{X_1, \cdots, X_q\} \cup \{Y_1, \cdots, Y_q\}$.

Then the sequence generated by $P^{(i)}(z) = \sum_{j=-q}^{q} z^j P_j^{(i)}$ is such that,

$\|P_0^{(i)} - I\| \leq q^2 \sigma^{2.2^i}$, $\|P_j^{(i)}\| < \binom{q}{j} \sigma^{j^{2^i}} + O(\binom{q}{j+1}) \sigma^{(j+2)^{2^i}})$.

✓ **Algorithm 8.2**: (**Inversion by Graeffe iteration**).

- **Input:**
 The coefficients F0, … , . , . Fq of F(z); an error tolerance $\varepsilon > 0$.
- **Output:** Approximation of $H_i = 0\ to q - 1$, of matrix Laurent series,

H(z) = F(z) − 1.

❖ Find coefficients $Q_{-q}^{(i)} \dots . Q_q^{(i)}\ of\ matrix\ polinomial\ Q^{(0)}(z) = F(z)$,

$Q^{(i+1)}(z) = G_i^{-1} Q^{(i)}(-z) Q^{(i)}(z), i \geq 0$. For i = 0,1,…,h-1, together with matrix G_i, until $\|Q^{(i)}(z) - I\|_\infty \leq \varepsilon$.

❖ *Find* $2 * q - 1$ central coefficient of $L^{(i)}(z) = L^{(j-1)} Z^2 * G_{i-j}^{-1} * Q^{(i-1)} * (-Z)$. and $L^{(i)}(z) = G_{i-j}^{-1} * Q^{(i-1)} * (-Z)$. for j = 2 ∗ i.

❖ Result is $L^{(h)}(Z)$.

- *Section9. For Computing Wiener-Hopf. Factorization.*

- **Proposition 9.1:** Consider $F(Z) = Z^{-p}\Psi(z) of\ F(Z)$.

$F(Z) = Z^{-p}\Psi(z) = S(Z^{-1}) * S(Z) = S(Z) * S(Z^{-1})$.

And let $H(Z) = F(Z)^{-1} = \sum_{i=-\infty}^{+\infty} z^j H_i$.H(z).

$F(Z) = S(Z^{-1})$, and it is by $(m+1) * (m-1)$ Toplitz system

$$T_m \begin{bmatrix} X_0 \\ X_1 \\ \vdots \\ X_m \end{bmatrix} = \begin{bmatrix} I \\ 0 \\ \vdots \\ 0 \end{bmatrix}, T_m = \begin{bmatrix} H_0 & H_1 & \cdots & H_m \\ H_1 & H_0 & \ddots & \vdots \\ \vdots & \ddots & \ddots & H_1 \\ H_m & \cdots & H_1 & H_0 \end{bmatrix} \text{where } m \geq q,$$

and by series $\hat{S}(z) = \sum_{j=0}^{q} z^{q-j} * X_j$. Moreover $x_j = 0$ for j = q+1:m.

- **Proposition 9.2:** Define U, V, W the (q + 1) ∗ (q + 1) block Toeplitz matrises.

$$U = \left(\binom{p}{q+i-j} * (A + (-1)^{i-j} * I) \right) \quad where\ i, j = 1{:}q+1.$$

$$V = \left(\binom{p}{q+i-j+1} * (A + (-1)^{i-j} * I) \right) \quad where\ i, \quad j = 1{:}q+1$$

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$$W = \left(\binom{p}{i-j-1} * (A + (-1)^{i-j} * I) \right) \qquad where\ \ i,j = 1{:}\, q+1\ .$$

Where $\binom{p}{m} = 0, if m < 0\ or\ m > p.$

Define the sequences,

$U_{k+1} = U_k - V_k * U_k^{-1} * W_k - W_k * U_k^{-1} * U_{k+1},$

$V_{k+1} = V_k * U_k^{-1} * V_k,$

$W_{k+1} = -W_k * U_k^{-1} * W_k,$

For k = 0, 1, 2, ... and U_0 = U, W0 = W, U_k is non singular for any k.

Then the limit $U^* = \lim_{k\to\infty} U_k$ exist and $U^* = T_q^{-1}$.

Where T_q is the Toeplitz matrix defined as,

$$T_m \begin{bmatrix} X_0 \\ X_1 \\ \vdots \\ X_m \end{bmatrix} = \begin{bmatrix} I \\ 0 \\ \vdots \\ 0 \end{bmatrix}, T_m = \begin{bmatrix} H_0 & H_1 & \cdots & H_m \\ H_1 & H_0 & \ddots & \vdots \\ \vdots & \ddots & \ddots & H_1 \\ H_m & \cdots & H_1 & H_0 \end{bmatrix} \text{where m} \geq q.$$

Moreover, the convergence of U_k to U^* is quadratic.

- ***Section 10:*** *Conclusion Results.*

- **Input:** Algorithms 2.1 based on 2.4.

Sign: Algorithm 3.1.

Li-ei: Algorithm 5.1 for Algorithm 8.1.

Consider two test problems:

✓ **Test 1:** For $unit\ \varepsilon$ –circulene Matrix A i.e. polynomial $X^n \rightarrow \varepsilon$.

Where n = 5 and $\varepsilon = 10^{-8}$.

Eigenvalues of A are fifth root of unity multiplied by $\varepsilon\frac{1}{5}$.

Matrix is normal and limit → $\varepsilon \to 0$ has no p^{th} root.

✓ **Test2:** A is matrix of order 5x5 associate with polynomial $\prod_{i=5}^{5}(x-i)$.

Clearly eigenvalues are 1, 2, 3, 4, 5 and matrix is not normal.

Chart of infinity norm of residual error A - X^p for several values of p.

Then $X \longrightarrow A^{1/p}$.

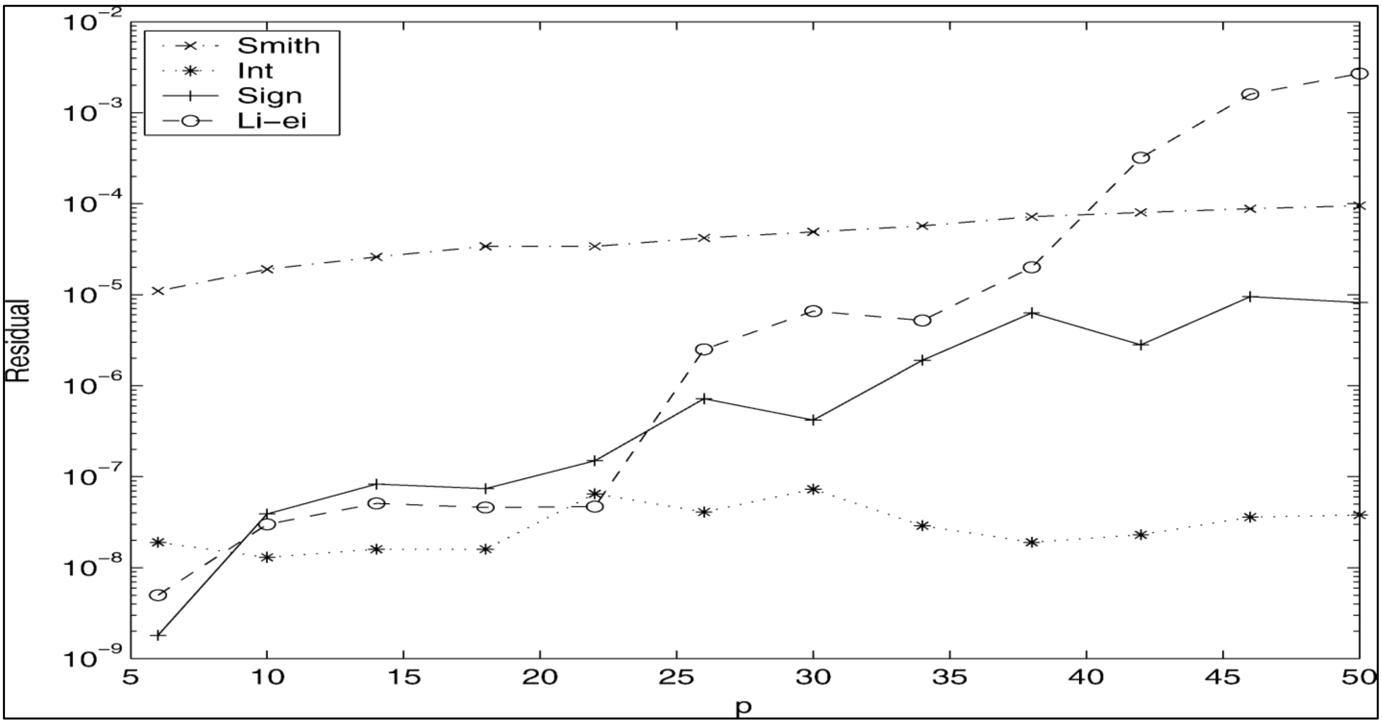

Fig 2: Infinity Norm of the Recedual Error in Computing $A^{\frac{1}{p}}$ for $\varepsilon -$ circulent matrix A.

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III. CONCLUSION AND OPEN PROBLEMS

Various equations to expressing principal p^{th} root of matrix A in different forms. It reduces calculations for numerical iterations of $A^{1/p}$ on unit circle, for finding matrix sign function of matrix of block companion. All the results are to inverting matrix Laurent polynomial to finding Wiener – Hopf factorization. Also for using iteration of fix point.

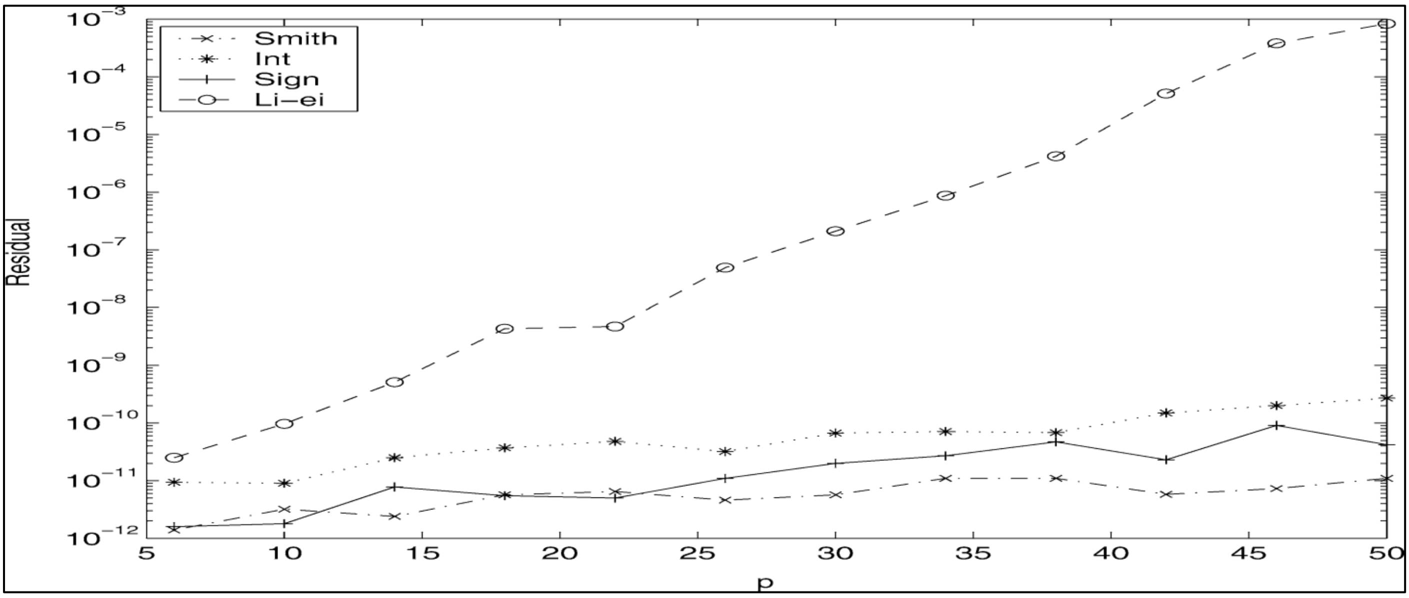

Fig 3: Infinity norm of Residual Error in Computing $A^{\frac{1}{p}}$ for Companion Matrix Corresponding with Polinomial $\prod_{i=1}^{5} (x - i)$.

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