Modelling Human Behaviour in COVID-19 Vaccination Centres: A Discrete Event Simulation Approach

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Abstract: This study aims to improve the efficiency and effectiveness of operations at a COVID-19 vaccination centre using advanced Discrete Event Simulation (DES) techniques. With the decreasing demand for vaccinations, the focus has shifted from increasing capacity to optimizing the patient experience and operational flow. The research uses Witness Horizon, a DES tool, to create a digital twin of a vaccination centre using data collected through online surveys, interactive sessions with psychologist and past works, offering a comprehensive view of both operational and human behavioral aspects, enabling the modelling of various scenarios and addressing logistical bottlenecks and behavioral factors. The simulation results indicated that minor adjustments in patient flow, staff allocation, resource utilisation and addressing psychological barriers like vaccine anxiety, the likelihood of patients completing the vaccination process improved, significantly enhancing operational efficiency, reducing wait times and improving overall patient satisfaction. Despite limitations such as data collection and software accessibility constraints, the built model proved effective in identifying key challenges, and recommendations were made for optimizing future public health initiatives.

Keywords: *Vaccination Centres; Discrete Event Simulation; COVID-19; Witness Horizon.*

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I. INTRODUCTION

The COVID-19 pandemic dramatically transformed global healthcare systems. Vaccination centres played a critical role in containing the virus spread [1], but post-pandemic, the emphasis has shifted from capacity to optimizing the efficiency of these centres. Many patients leave vaccination centres without completing the process due to long waiting times, disorganized operations, and psychological barriers such as anxiety or fear of vaccines. This paper explores the application of Discrete Event Simulation (DES) to address these challenges.

The study's primary aim is to use simulation techniques to improve operational effectiveness and patient experience in COVID-19 vaccination centres. By leveraging Witness Horizon, a DES tool, this research provides a comprehensive analysis of vaccination centre operations and human behavioral aspects influencing the vaccination process.

The research aims to enhance the efficiency and effectiveness of COVID-19 vaccination centres, focusing on improving patient satisfaction, resource allocation, and addressing psychological barriers to vaccination. The research objectives are:

- Identify factors contributing to patients leaving vaccination centres before receiving the vaccine.
- Develop a DES model to simulate various scenarios at vaccination centres using Witness Horizon.
- Propose interventions to improve patient flow, reduce wait times, and address vaccine hesitancy.

II. LITERATURE REVIEW

Vaccination centres were set up during the COVID-19 pandemic to manage mass immunization efforts [2]. They are complex systems requiring well-coordinated operations for patient flow, resource allocation, and safety protocols [3]. Studies show that efficient vaccination centres can reduce the spread of viruses and optimize public health outcomes [4]. However, operational inefficiencies and patient dissatisfaction due to long wait times have been identified as critical problems.

Discrete Event Simulation DES has the capability of modelling complex patient flow through healthcare clinics and facilitating scenario planning by answering ‘what-if’ questions through changing patient flow patterns and service policies [5,6]. Several studies have employed DES to optimize emergency departments, outpatient clinics, and vaccination centres. By creating digital models of these operations,

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healthcare facilities can simulate various scenarios, identify bottlenecks, and test potential improvements before implementing them in practice.

Vaccine hesitancy is not a new concept and has been witnessed by people for decades [7,8]. In simple terms, vaccine hesitancy refers to a situation where people are reluctant to get vaccinated despite its availability [9]. Psychological factors, such as fear of side effects, mistrust in healthcare systems, and anxiety over needles, can lead to patients leaving vaccination centres without completing the process [10]. Understanding these behavioral patterns is essential for improving vaccination rates [11].

III. METHODOLOGY

➢ *Research Design*

This research follows a mixed-methods approach. Quantitative data were collected via online polls and surveys from individuals who attended vaccination centres, while qualitative data were gathered through interviews with psychologists and healthcare experts. Both operational and psychological data were incorporated into a Discrete Event Simulation model developed using Witness Horizon software.

➢ *Vaccination Centre Operations*

Vaccination Centres are designed to administer vaccines in a safe, efficient, and patient-friendly manner, with operational processes focused on managing large numbers of people while ensuring minimal waiting times and a high level of patient satisfaction. The centres are often set up in venues like community halls, stadiums, pharmacies, GP clinics, or pop-up facilities that can accommodate a large volume of people while maintaining physical distancing guidelines. The setup of these centres is tailored to handle varying patient flows, manage logistical challenges, and maintain the highest standards of healthcare delivery based on National Health Service (NHS) England guidelines [12-14].

Available time slots and dates for Vaccination appointments are pre-booked online. Upon arrival, patients enter the site through gates as walk-ins or by car, patients driving in drives into the car lobby where marshals take over and parks the car at the car park while the car owners proceed to the first stage of the process which is temperature check. Patients walking in walks straight through a queue to temperature check on entry. After a mandatory temperature check is done, patients with normal body temperature are made to carry on the process and move to a Sign-in queue where basic and personal details are confirmed and collected at the sign in desk. Patients whose temperatures are above normal and considered unfit for the vaccine are made to abort the process and leave through the centre's exit doors. After Sign-in, patients have the option to go on a break or use the rest room as this is only allowed at this stage of the process after which patients move into a waiting room and await availability of a Vaccination booth. Vaccination booths are allocated to patients, in the booths, personal data sheets are collected, information is given, and the Vaccines are administered, after which patients are allocated observation booths where they are being observed for immediate reactions or effects following the vaccination, this takes 15 minutes after which patients are made to exit the centre. Patients who drove in are matched with their cars and leaves through a car exit while patients who walked in exits the site through a foot exit.

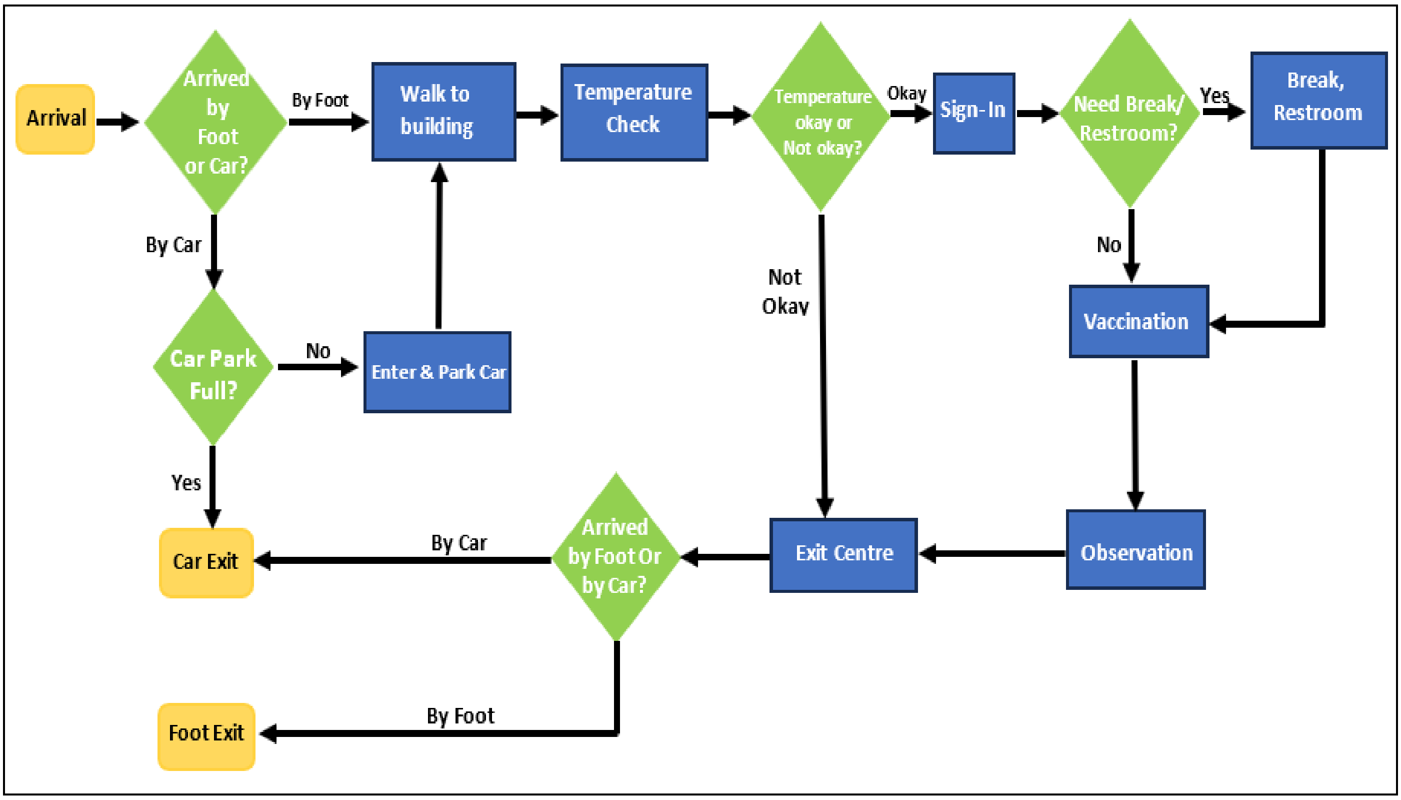

Fig 1 Process Flow Chart for Patients

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➢ *Data Collections*

This Primary Data: Online polls provided data on patient experiences, including wait times, satisfaction, and perceived organization at the centres. Polls were created and distributed across a social media platform to people who have successfully or unsuccessfully undergone a vaccination process. The questions covered key dimensions such as patient age, gender and mode of transportation distribution, facility organization, information received, general operations and activities while in the centre. Poll session questions and the results were gathered, processed and computed on the Table 1 below.

An interview with a comparative psychologist was also conducted to provide insights into the psychological barriers patients face during vaccination [15].

Secondary Data: Operational data from [16,17] previous studies and vaccination centre reports were used to simulate different scenarios. Metrics such as patient arrival times, resource availability, and service times shown in Table 1 were fed into the simulation model.

Table 1 Patient Statistical Data from Poll

Data	**Percentage (%)**	**Value in Model's Distribution Editor**
Gender Statistics (Patient Gender DATA)		
Male patients	39%	1
Female patients	58%	2
Others/ Prefer not to say	3%	3
Age Statistics (Patient Age DATA)		
Patients Aged 18 – 30	54%	1
Patients Aged 31 – 50	36%	2
Patients Aged 51 – 75	10%	3
Means of Transportation Statistics (Patient Transport DATA)		
Patients by Car	50%	1
Patients by Foot	50%	2
After Observation Statistics (After Observation DATA)		
Left the building	90%	1
Wait/ Further Redirected	10%	2
After Consultancy Statistics (After Consulatancy DATA)		
Became more confident/ agreed to be vaccinated	49% (92%)	1
Left the building unvaccinated	8%	2
There was no consultant on site	43%	
Patient Temperature Statistics (Patient Temperature DATA)		
Normal temperature	87%	1
Temperature not normal	13%	2
Use of Restroom/ Break Time Statistics		
Used the restroom/ took a break	10%	
Did not use the restroom/ take a break	90%	
Vaccination Rate		
Vaccination successful	53.7%	
Vaccination not successful	46.3%	
Reason for leaving or not carrying out the Vaccination Process		
Disorganized process/ operations	16.4%	
Long wait times/ delays	30.9%	
Anxiety concerns about the vaccine/ process	27.8%	
No clear information/ communication	6.2%	
Couldn't gain access due to Overcrowding	9.4%	
Couldn't make it for the appointment	9.3%	

➢ *Simulation Model*

Discrete Event Simulation (DES) is a powerful modelling technique used to analyse and optimize complex systems by representing them as a sequence of discrete events occurring over time [18]. In healthcare applications, DES has become an invaluable tool for improving operational efficiency, resource allocation, and patient flow in various settings, including hospitals, clinics, and vaccination centres [19]. Using the Witness Horizon DES tool, a digital twin of a COVID-19 vaccination centre was created. The model simulates the entire patient journey—from entry to exit—while considering logistical elements such as resource allocation and patient flow, and human behavioral factors like vaccine hesitancy.

Discrete Event Simulation (DES) is composed of several key elements that work together to simulate the behavior of

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complex processes and systems. The key elements of the model are labelled in table 2 and displayed in fig 2.

The test for effectiveness of this vaccination centre's model is to be measured by the following factors;

- Number of patients entered and vaccinated not to be less than 700 patients per day (the higher the better)
- Average time in centre should not exceed 30 minutes (the less the better)
- Maximum time in centre should not exceed 40 minutes (the less the better)
- Refusal due to overcrowding should not exceed 50 patients per day (the less the better)
- At the end of the 10th hour of daily operations, number of patients in the centre and number of patients in the process should be as close to zero (0) as possible.
- Daily Utilization should not be less than 70% (the higher the better).

➢ *Model Testing*

The base model for a vaccination centre was thoroughly tested through multiple simulations to evaluate its performance under varying operational conditions. The testing involved adjusting parameters like patient arrival rates, resource availability, and procedures. Key steps in the process included:

- Structural Validation: Ensured the model's flow and logic mirrored real-world processes (e.g., patient arrival, registration, vaccine administration).
- Data Input Validation: Compared input variables like service times and queue capacities with real-world data to verify consistency through statistical tests.
- Face Validation: Gathered feedback from healthcare professionals and stakeholders to confirm the model's behavior in different scenarios.
- Sensitivity Analysis: Tested how changes in key inputs affected the model's performance, this revealed that the temperature check area was highly sensitive.
- Output Validation: Compared the model's results (e.g., waiting times, success rates) with actual data, ensuring accuracy, and refined where discrepancies were found.

This iterative testing process ensured the model was accurate, reliable, and aligned with real-world operations.

Table 2 Witness Element Mapping Table

Witness Element	**Description in the Model**
Entity	Patient
Queue	Car_Park, TempCheck_Que, SignIn_Que, Vaccination_Que, Vaccination_Booth, Obeservation_Booth, Vacc_Success, Vacc_Unsuccess, Return_to_Car
Activity	Site_Entry, Car_Lobby, TempCheck, SignIn, Restroom_Break, Allocate_Vaccination, Allocate_Observation, Consultancy_and_Vac, Centre_Exit_Door, Car_Exit, Foot_Exit
Attribute	PatientID, PatientsPerHour, Vacc_Capactiy, All Patient Data and Group set
State Variable	All KPIs

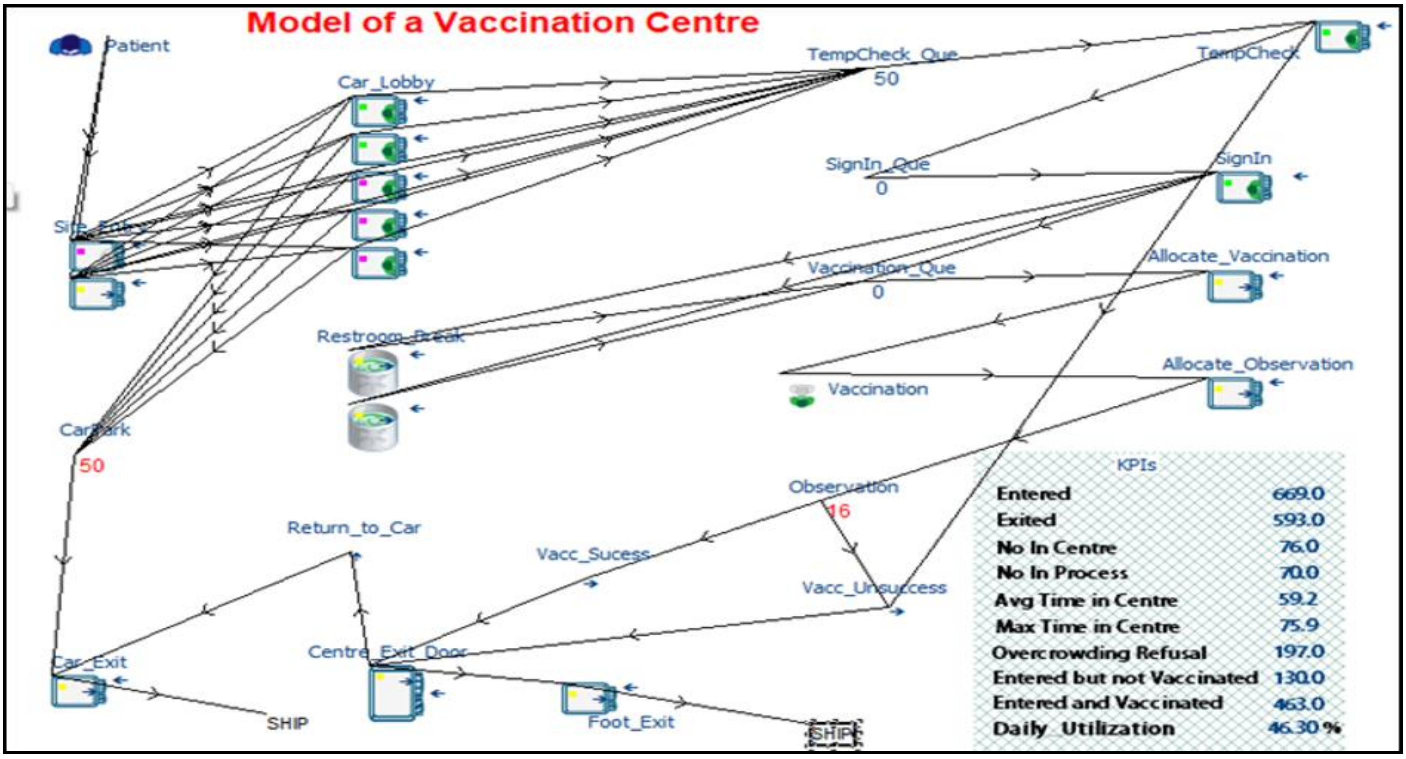

Fig 2 Base Model @ 600 Mins of Simulation

IV. RESULTS

➢ *Base Model Experiment*

The tested base model was made to run at full capacity for 10 hours under a fixed rate of 100 patients per hour (PPH) using the data set and parameters available in Table 2. above. The model is built in a way that the parameters can be easily iterated within the model at any appropriate stop point to enable for iterations.

Fig. 3. shows that at the end of the 10th hour of workday (600 minutes), 76 patients were still in the vaccination centre and 70 were still in the process of undergoing vaccination. This clearly does not represent efficient services as the patients will be forced to leave the centre unattended to.

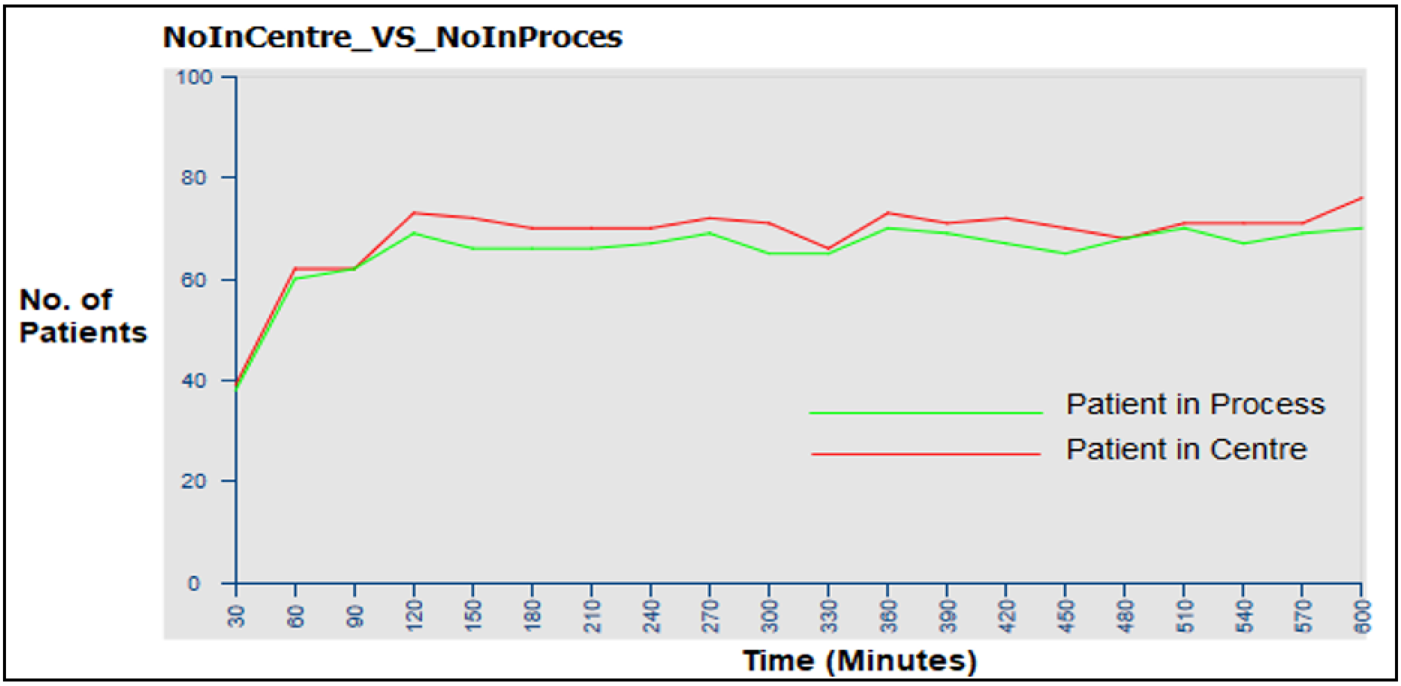

Fig 3 Patients in Centre vs. Patients in Process for Base Model.

Fig. 4. shows the maximum and average time spent by patients in the centre. The Max. time rose to its peak of about 70 minutes at around the 200th minute and stayed relatively same till the 600th minute recording 75.9 minutes at that time. Avg. time spent was 59.2 minutes at 600 minutes.

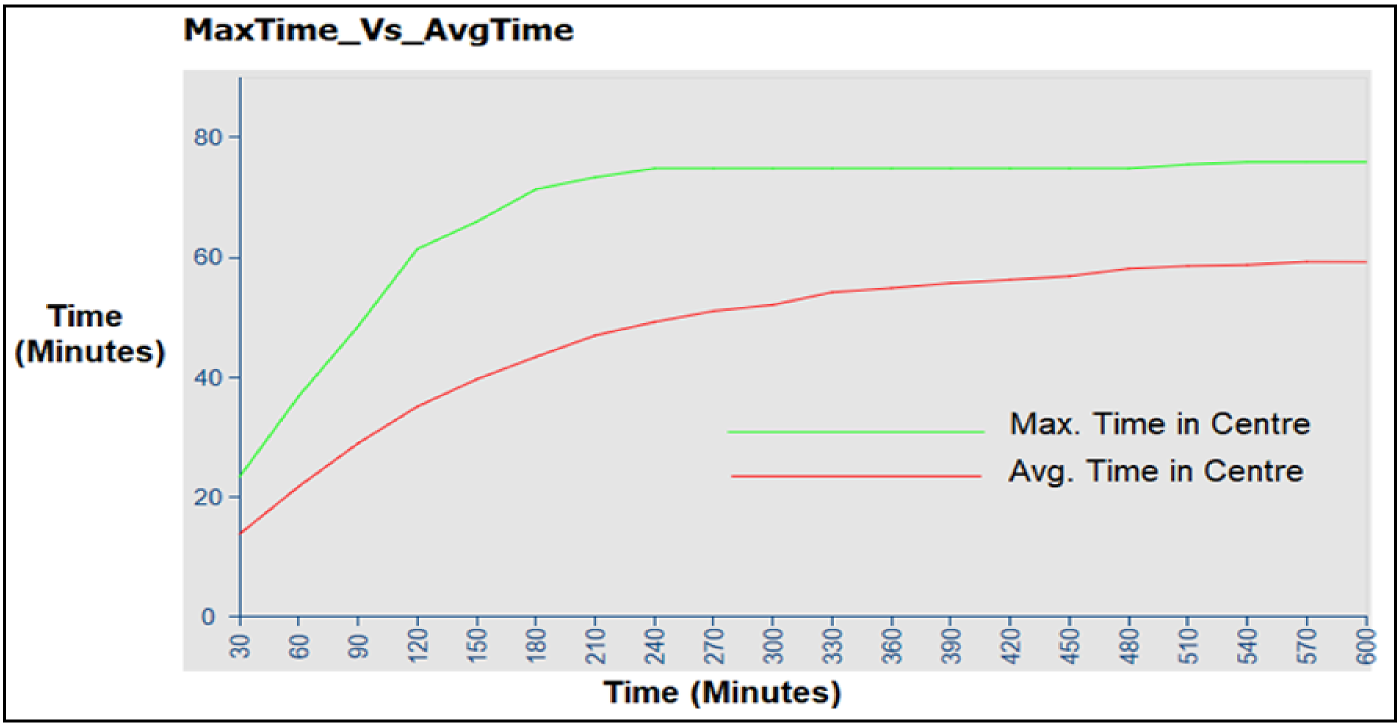

Fig 4 Max vs. Avg Time for Base Model.

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For vaccination centres of this kind, overcrowding is a thing of concern and as such considerations are made to curtail or checkmate overcrowding. To achieve this, an '…IF' rule was set in the output rule of the entity "patient" to limit entry into the car park to 30 and limit the number of patients in the centre KPIs 3 to 35. This helps to make sure the centre is not overcrowded at any given time.

The statistical data shows that patients typically would leave the vaccination centre if the perceived total processing time is 50 minutes and above, this was implemented into the model by a rule placed just before the vaccination stage, in Allocate_Vaccination activity to check the average time in the centre at that point and if its 50 minutes or more, patients are pushed to Vacc_Unsuccess to signify that they were unsuccessful with the process, else, if less than 50 minutes, they continue the process.

An activity "Consultancy_and_Vac" was introduced into the model using the "AfterCONSULTANCY" distribution gotten from the statistical date, this is to cater for patients who could not be vaccinated due to certain psychological factors like fear, anxieties or phobia.

Fig. 5 shows the dashboard containing the KPIs after all the above modifications were carried out on the model.

KPIs	
Entered	628.0
Exited	592.0
No In Centre	36.0
No In Process	36.0
Avg Time in Centre	31.8
Max Time in Centre	41.6
Overcrowding Refusal	248.0
Entered but not Vaccinated	83.0
Entered and Vaccinated	509.0
Daily_Utilization	50.90%

Fig 5 KPI Dashboard of Slightly Modified Model @600 Mins

The outcome shown in the KPIs on Fig 5. above fails to meet the KPIs aforementioned which measures the effectiveness of the Vaccination Centre's model and as such signifies that the base model experiment replication does not meet the acceptable service levels of this centre.

➢ *Parameter Refinements*

The model testing done showed that Patient Per Hour (PPH), Temp_Check and Sign_In were the most sensitive elements in the model, as change in their values brings about a significant change the outcome of the Simulation.

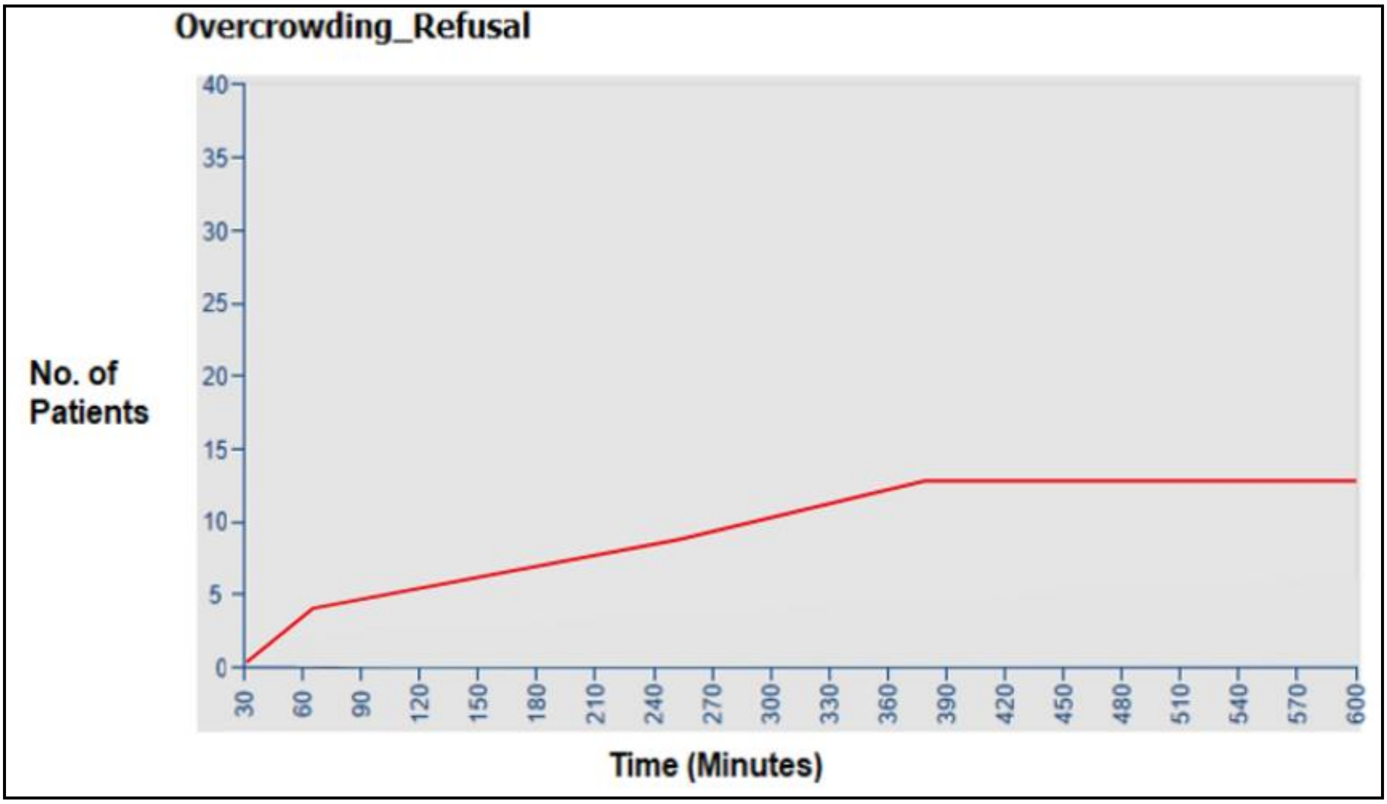

Fig 6 Overcrowding Refusals for 85 PPH, 1 Sign-in Desk and 2 Temp_Check Booths

If the patient per hour (PPH) is reduced to 85, there will be no significant change in the simulation results, but further increasing the Temperature Check (Temp_Check) booths to 2 brought about a 18.2% increase in the centre's daily utilisation and a drastic reduction in the overcrowding numbers to 13 as shown in Fig 6.

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The entry and exit chart had a better look as the lines parallelly aligned better as seen on Fig. 7 which signifies a good flow in the system and also shows that more patients were allowed to enter the system and potentially increasing the number of patients likely to get vaccinated.

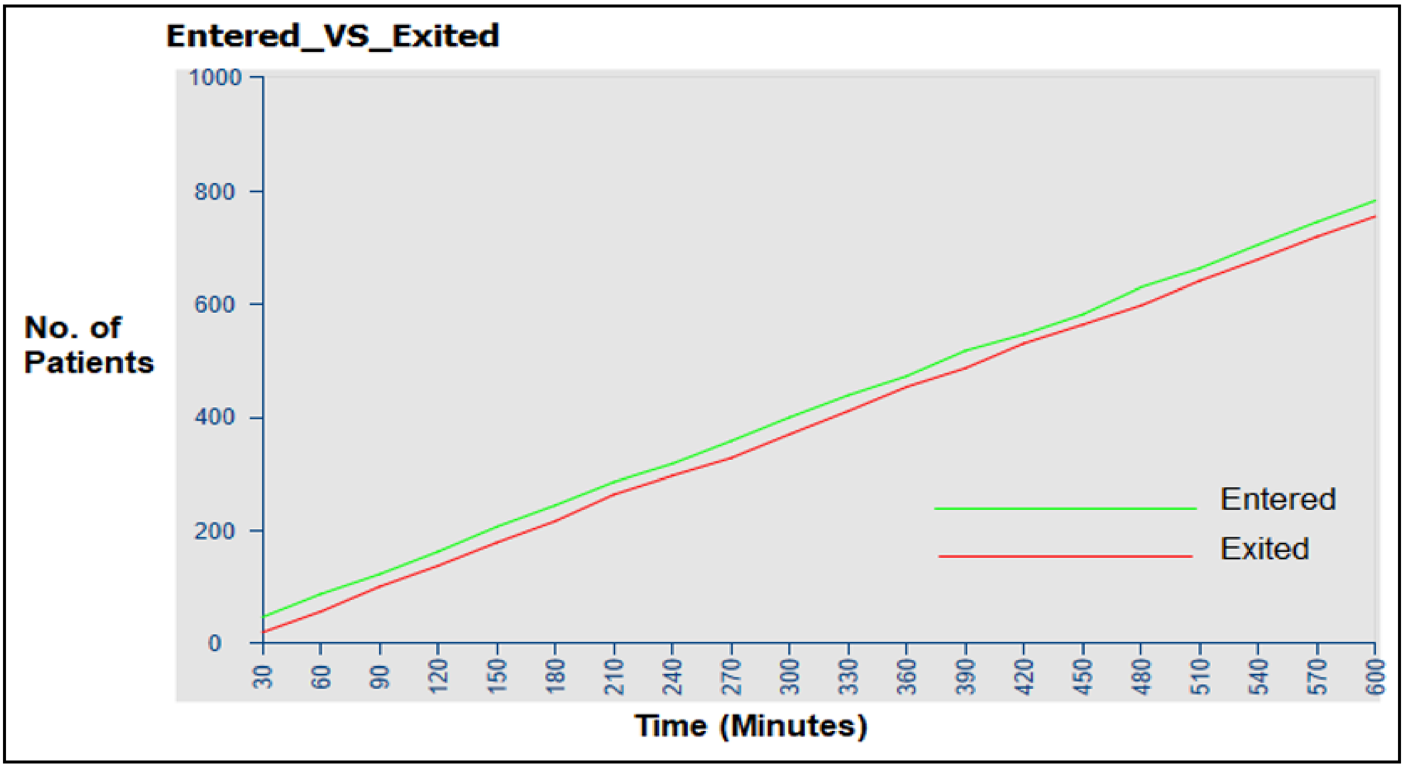

Fig 7 Patients Entered Vs. Patients Exited for 85 PPH, 1 Sign-In & 2 Temp_Checks

Fig. 8 shows that at the end of the 10 hours of simulation, there were still number of patients in centre and number of patients in process which does not necessarily align with the accepted service levels.

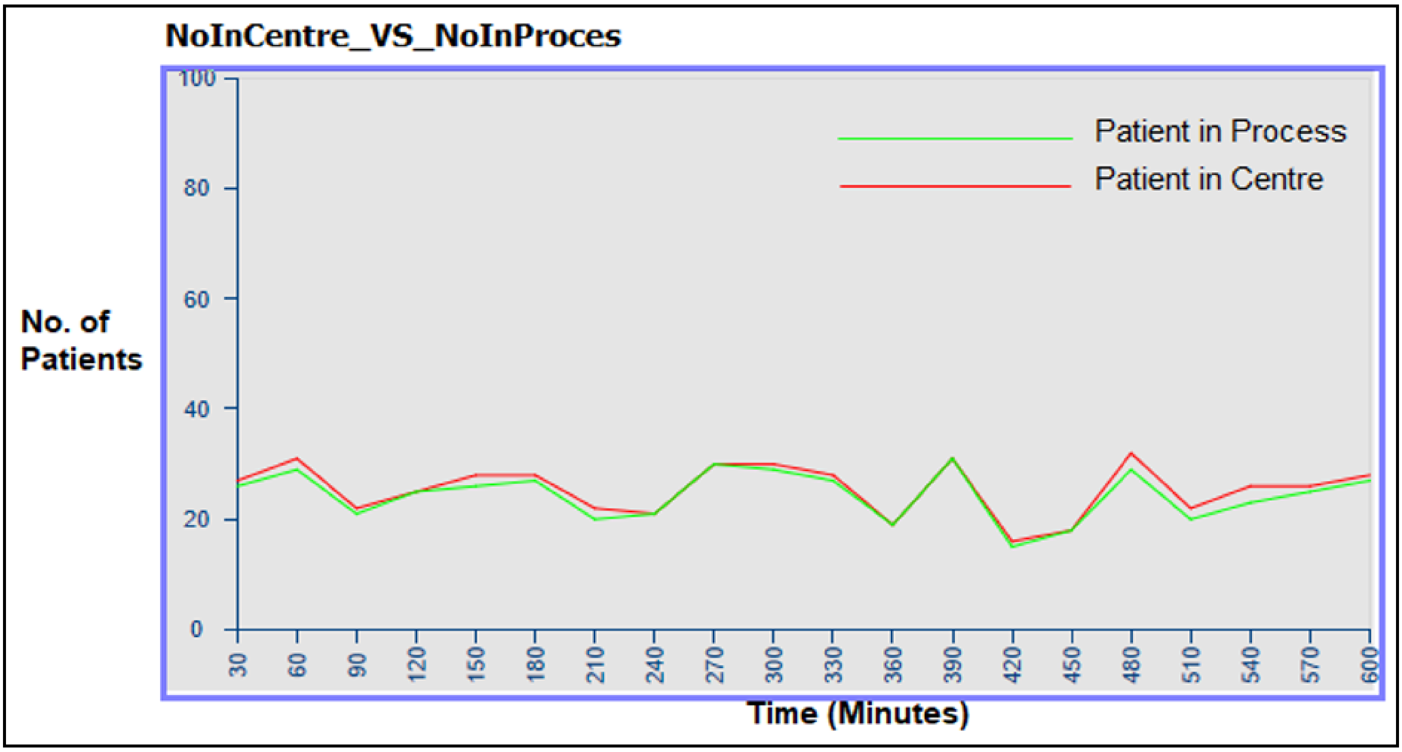

Fig 8 Patients in Process vs. Patients in Centre for 85 PPH, 1 Sign-in and 1 Temp_Check.

If the Patient Per Hour is increased back to 90 and quantity of Temp_Check is left at 2, although, the daily utilization increased to 71.60%, however, the overcrowding refusals as shown on Fig. 9 increased to 31.

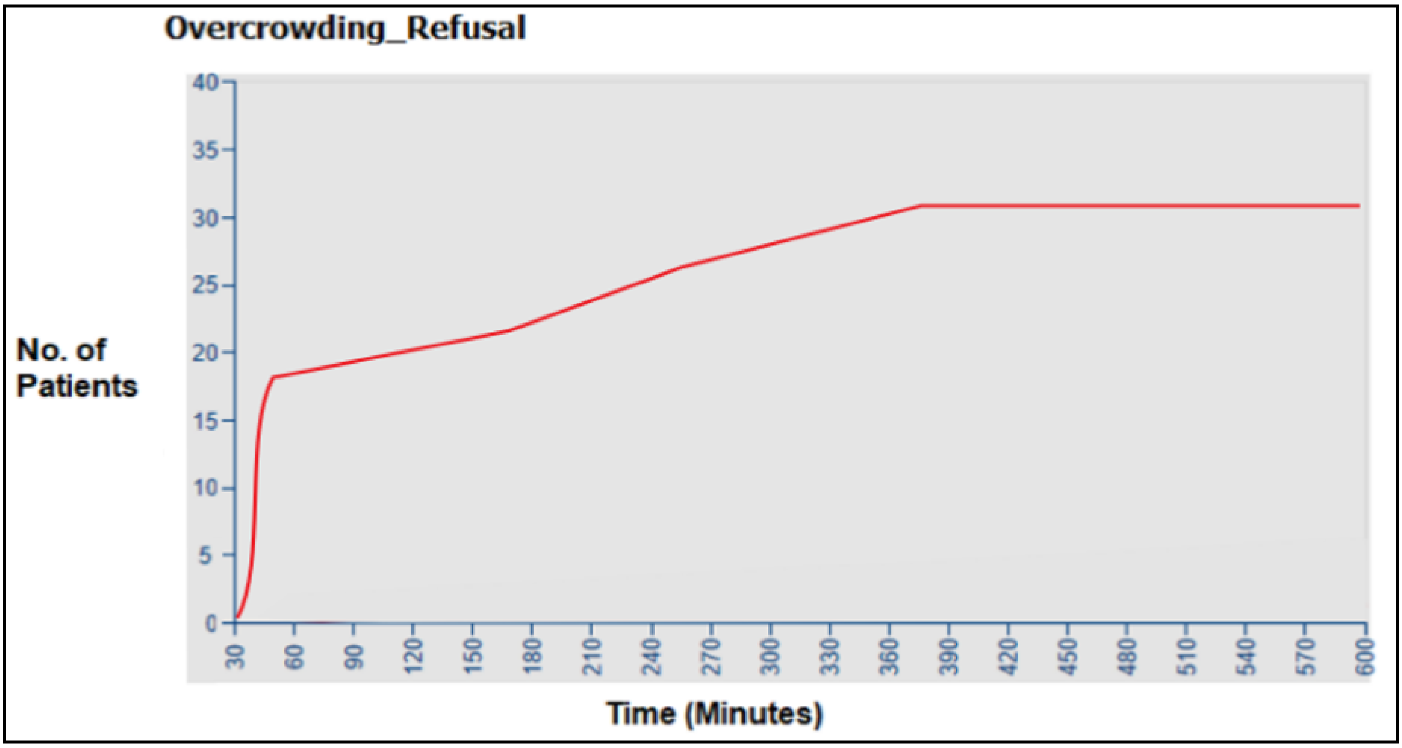

Fig 9 Overcrowing Refusals for 90 PPH, 1 Sign-in and 1 Temp_Check.

Fig. 10 shows that if the PPH is left at 90, Temp_Check booths is left at 2 but Sign-in desk is increased to 2, the Avg. and Max. times are at reduced values of 19.5 and 24.4 minutes respectively.

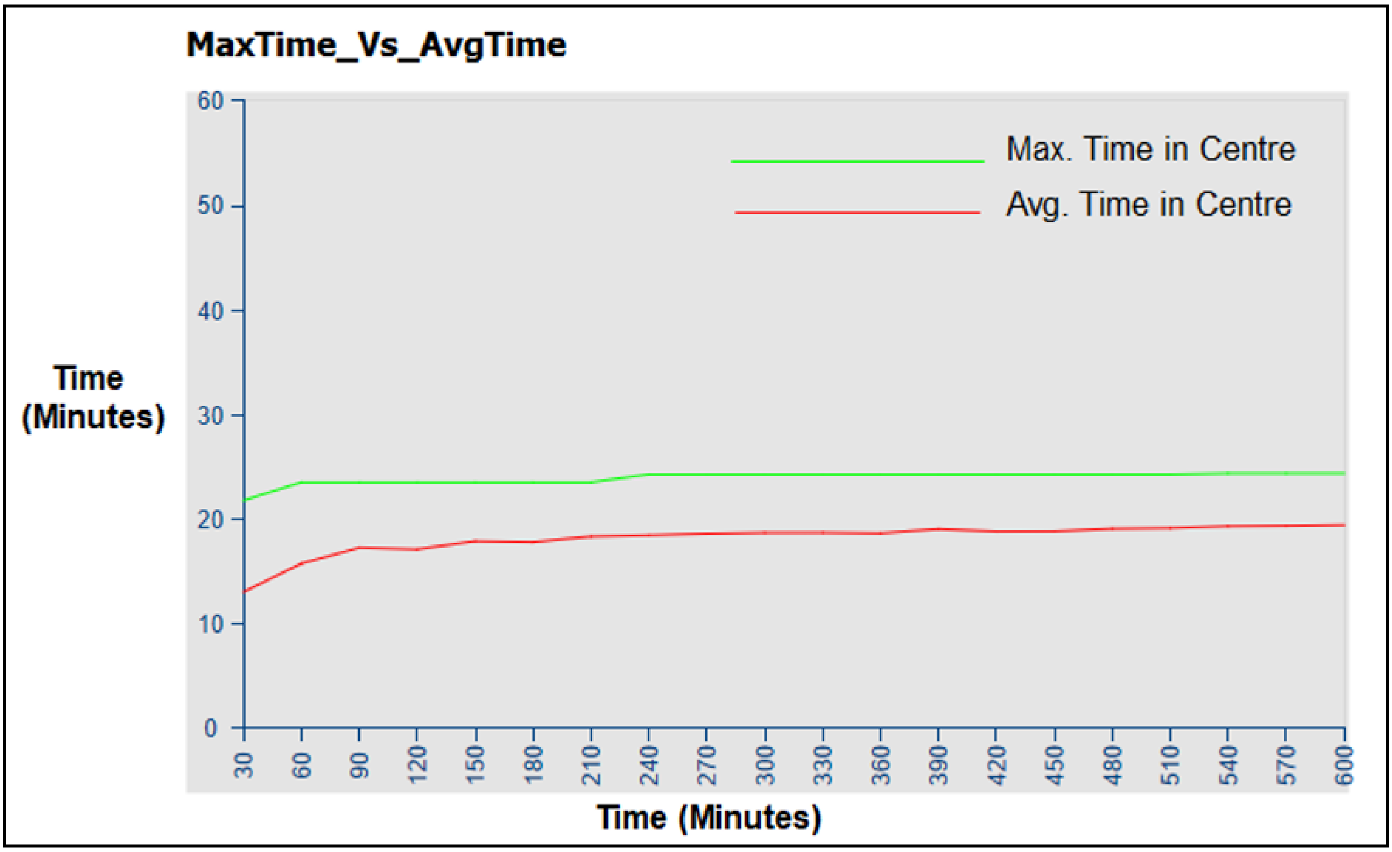

Fig 10 Max. Time vs. Avg. Time for 90 PPH, 2 Sign-in Desk, and 2 Temp_Check Booths.

If we decide to increase the Patients Per Hour value to 100, leaving the quantity of Temp_Check at 2 and quantity of sign_in desk at 2, Fig. 11 below shows that the overcrowding refusal increased to 53 and Fig. 12. Shows the Patience in Centre and Process statistics and yet again, the figures does not gravitate to zero as patients were still found to be in the centre and process at the end of 600 minutes.

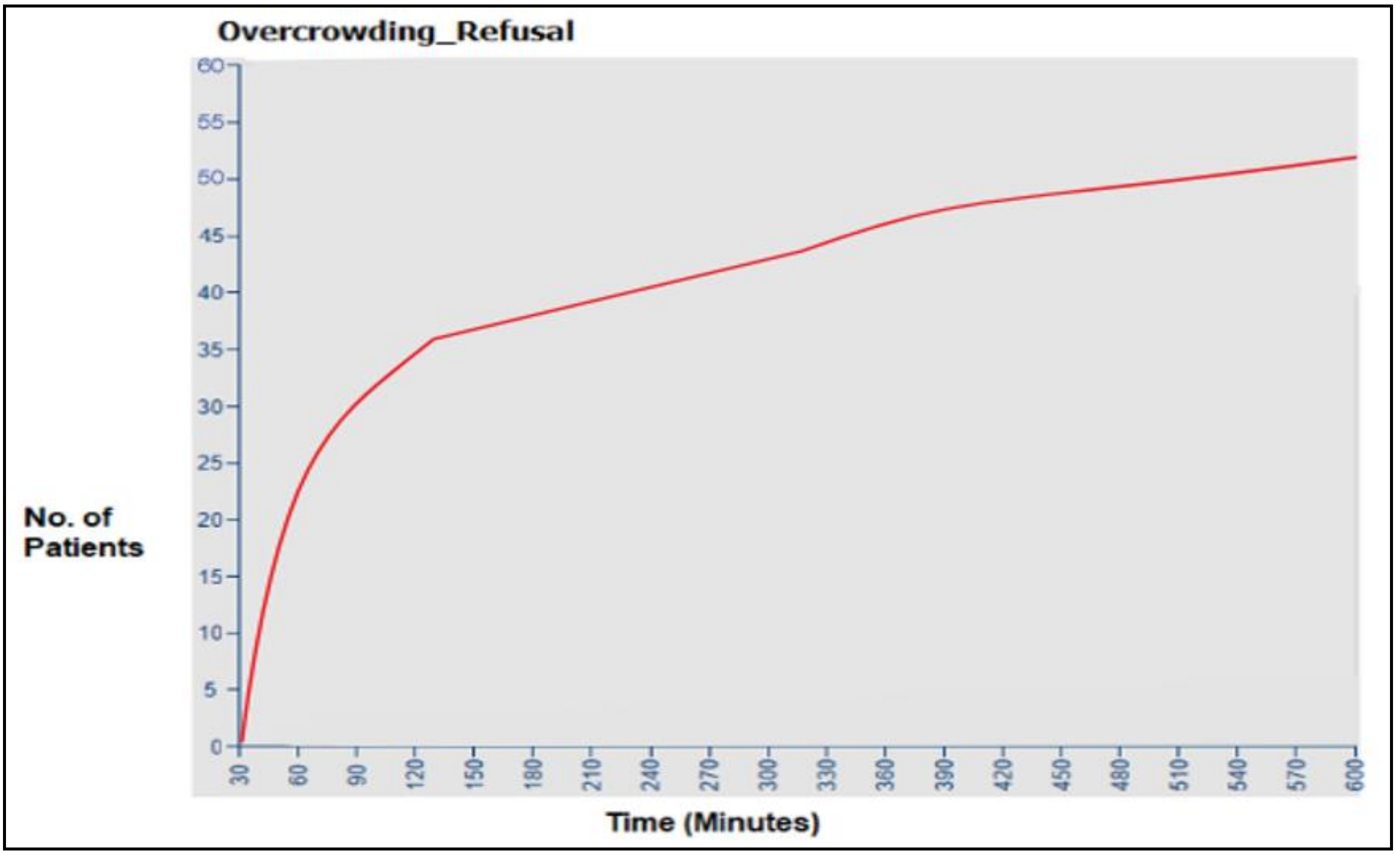

Fig 11 Overcrowding Refusals for 100 PPH, 2 Sign-in Desk, and 2 Temp_Check Booths

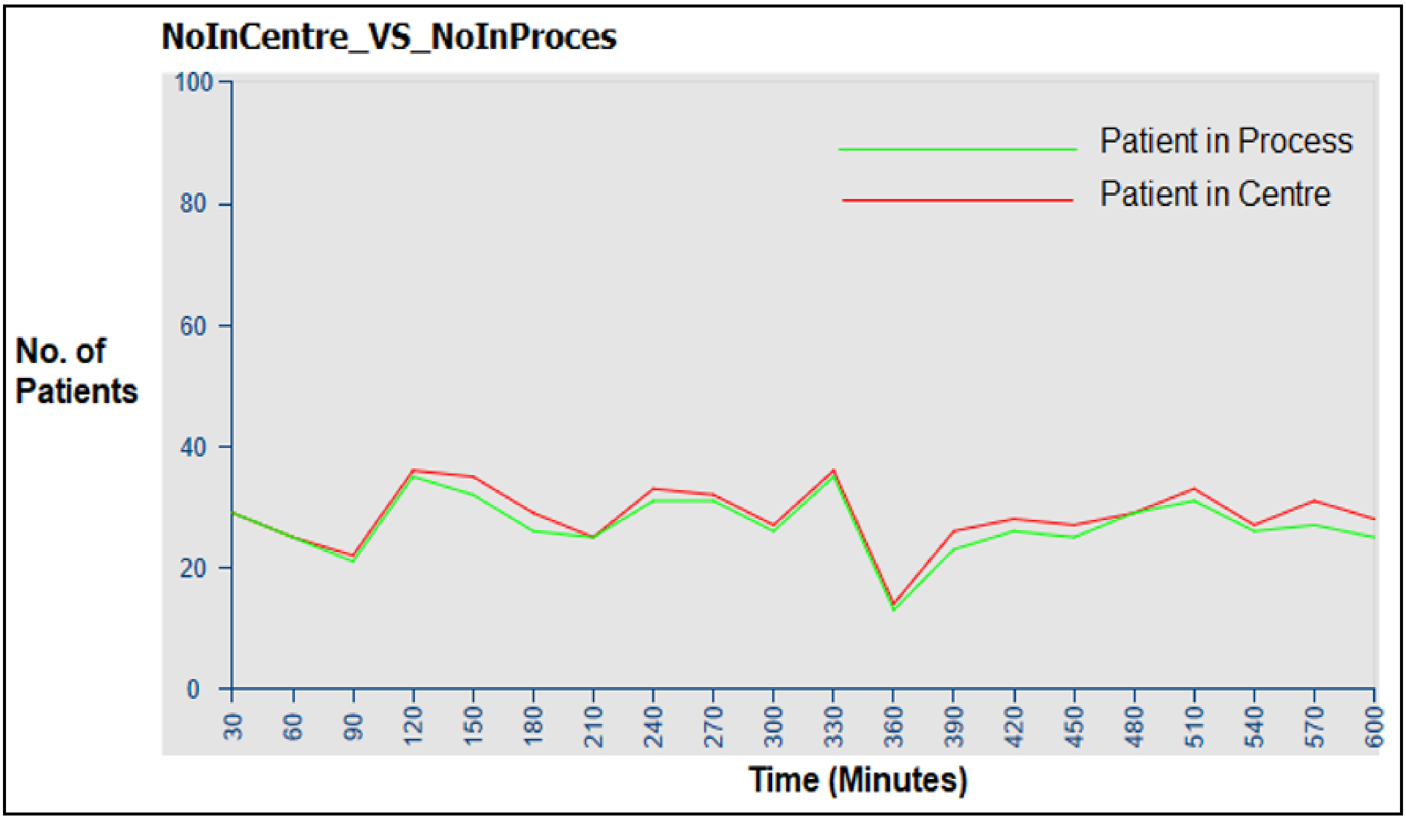

Fig 12 Patients in Centre vs. Patients in Process for 100 PPH, 2 Sign-in Desk, and 2 Temp_Check Booths

To achieve a better and more vast experimentation, more scenarios as shown in the results in Table 3. were simulated by variating the parameters of three most sensitive elements, Patient Per Hour, Temp. Check and Sign In. A total of 12 scenarios were recorded on the table and comparisons were made to ascertain the best scenario that represents a better and more effective service.

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Table 3 Experimentation/Scenario Test Results

Scenario Name	(Patient Per Hour) PPH	Temp Check	Sign-In	Entered	Exited	No. In Centre	No. In Process	Avg. Time (mins)	Max. Time (mins)	Overcrowding refusal	Un-successful Vaccination	Successful Vaccination	Daily Utilization (%)
1	100	1	2	628	593	35	35	32.4	44.2	350	83	510	51.00
2	90	1	2	628	593	35	32	31.9	42.7	249	83	510	51.00
3	85	1	2	627	593	34	32	31.8	41.2	204	83	510	51.00
4	100	2	2	926	898	28	25	18.4	25.3	53	119	779	77.90
5	90	2	2	860	832	28	26	18.7	23.4	17	106	726	72.60
6	85	2	2	820	794	26	24	19.0	24.2	11	102	692	69.20
7	100	1	1	628	593	35	35	32.4	44.2	350	83	510	51.00
8	90	1	1	628	592	36	34	31.9	41.5	248	83	509	50.90
9	85	1	1	627	592	35	34	31.5	41.7	204	83	509	50.90
10	100	2	1	906	879	27	25	19.5	27.3	72	113	766	76.60
11	90	2	1	850	821	29	27	19.8	27.4	27	105	716	71.60
12	85	2	1	819	792	27	25	19.4	27.3	12	101	691	69.10

V. DISCUSSION AND CONCLUSION

This study demonstrated how Discrete Event Simulation (DES) can be applied not only to the logistical optimisation of vaccination centres but also to the modelling of human behavior. While operational adjustments such as increasing temperature check booths or reallocating sign-in staff reduced waiting times and overcrowding, the most significant insights came from simulating behavioral factors such as anxiety, perceived waiting time, and vaccine hesitancy.

Unlike resource-based processes, human behavior is inherently subjective and unpredictable. Patients' decisions to exit early due to fear, mistrust, or perceived delays cannot be forecast with complete certainty. However, by categorising behavioral tendencies through demographic data (e.g., age, gender, mode of arrival) and incorporating them into the DES model, this research provided a more realistic picture of vaccination centre dynamics.

The unique contribution of this work lies in showing that simulation can bridge the gap between predictable operational flows and unpredictable human responses. This behaviour-centred modelling approach represents a step toward more human-focused healthcare simulations, highlighting that any effective optimisation must consider not just resources and throughput but also the emotional and psychological experiences of patients.

Future studies could expand this approach by integrating hybrid simulation techniques (e.g., agent-based models or machine learning) to capture more nuanced behavioral interactions and adapt dynamically to real-time patient responses. By acknowledging unpredictability as an essential component of healthcare modelling, this study underscores the need for patient-centred digital twins in the design and evaluation of future healthcare facilities.

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