

Determination of some Dynamic Characteristics of Membrane Magneto Hydraulic Pushers using Mathematical Modeling

Rusudan Bitsadze¹; Simon Bitsadze²

¹Department of Mathematics; ²Department of Engineering Graphics and Engineering Mechanics,
^{1,2}Georgian Technical University, Tbilisi, Georgia

Publication Date: 2025/03/11

Abstract: The equation describing liquid pressure located between armature and core during their attraction is written for the membrane magneto hydraulic pusher discussed in the work. Resulting from solution of the boundary problem set for the mentioned equation there is obtained the time distribution of pressure in liquid. Through integration of the pressure value on the pusher's electromagnet core ring a magnitude of a displacement resistance force for an oil to be squeezed is determined, based on which the ratio between time and armature stroke is obtained. An opinion is expressed on the reasonability of reduction of electromagnet armature ring-shaped end area with the purpose of armature attraction time reduction that may be implemented, if the armature end surface will have a narrow ring-shaped projection. The conclusion on the optimal time of armature attraction is drawn.

Keywords: Pusher, Armature, Core, Pressure, Integration.

How to Cite: Rusudan Bitsadze; Simon Bitsadze. (2025). Determination of some Dynamic Characteristics of Membrane Magneto Hydraulic Pushers using Mathematical Modeling. *International Journal of Innovative Science and Research Technology*, 10(2), 1901-1904. <https://doi.org/10.5281/zenodo.14979647>.

I. INTRODUCTION

The discussed membrane magneto hydraulic pusher (MHP) includes a body performed in the form of interconnected hydrocylinders of different diameters, at that, a piston rod placed in the small-diameter hydrocylinder is performed with the opportunity of connection with actuating mechanism, while the electromagnet armature located in the large-diameter hydrocylinder situated inside the direct current electromagnet coil is rigidly coupled with a membrane rigidly fixed in the body [1-2]. The space above membrane is connected with the under-piston area of the rod through armature and core holes. The above-piston area of the rod is connected with the under-membrane area by a pipe and regulating valve. The above-membrane area also by the unilateral valve is connected with the above-piston area of the rod and under-membrane area.

During attraction of pusher's direct current electromagnet armature, a liquid placed between armature and electromagnet core is pushed out to the hole in the core center and to the crack between the armature and cylinder. At that, the force supplied for liquid expulsion will heavily depend on the thickness of the liquid layer to be pushed out, in particular, decrease in liquid layer thickness leads to the increase of the corresponding displacement force and in case of very small thickness of the liquid layer to be pushed out

this force will substantially increased. This phenomenon is resulted from dampening, i.e. reduction of impact force at the reciprocally fast approach of two surfaces.

Proceeding from the above-mentioned, one can conclude that during attraction of the membrane MHP's electromagnet armature-piston in case of small thickness of the liquid layer placed between armature-piston and core the resistance of the pushed out liquid will be quite high that, in its turn, will increase the armature attraction time, and, respectively, the rod lift time.

We set a goal of determination of the armature movement time during attraction taking the liquid displacement resistance force into account and search of those methods, which will make it possible to reduce the liquid displacement resistance force to the optimal value.

II. METHODOLOGY

The armature and core ends represent the rings with R and r radiuses (Fig. 1), that's why during armature attraction the pressure P of a liquid located between armature and core can be described by Reynolds equation, which is written in polar coordinates as follows [3]:

$$\frac{d^2 P}{d\rho^2} + \frac{1}{\rho} \frac{dP}{d\rho} = \frac{12\mu}{h^3} \cdot \frac{dh}{dt} \quad (1)$$

At that, the following boundary conditions take place:

$$\begin{cases} P|_{\rho=r} = P_o \\ P|_{\rho=R} = P_o \end{cases} \quad (2)$$

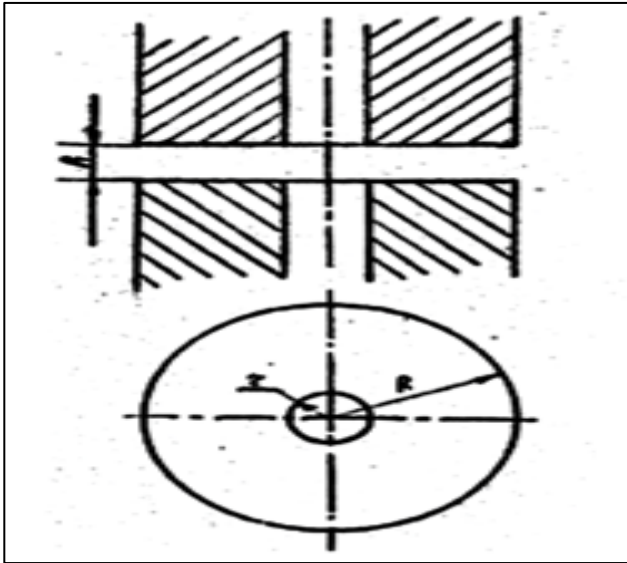


Fig 1: Armature and Core

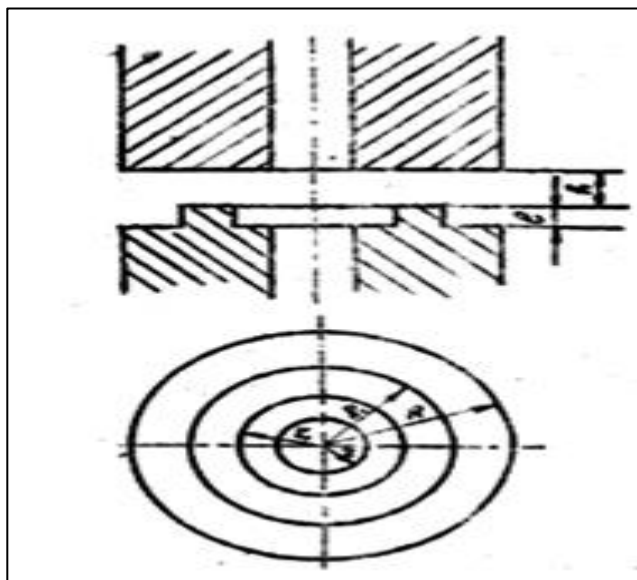


Fig 2: Armature and Core with Projection

Where μ is oil viscosity, P_o – pressure magnitude in the hydrosystem, t – movement time during attraction, ρ – polar radius, R and r – ring radiuses, h – armature stroke.

In order to solve the (1), (2) and (3) boundary problem let's rewrite the equation (1) in the form convenient for us:

$$\frac{d}{d\rho} \left(\rho \frac{dP}{d\rho} \right) = \frac{12\mu}{h^3} \cdot \frac{dh}{dt} \rho.$$

Through integration of the last equation we get that

$$\rho \frac{dP}{d\rho} = \frac{12\mu}{h^3} \cdot \frac{dh}{dt} \cdot \frac{\rho^2}{2} + c_1, \quad c_1 = \text{const.} \quad (3)$$

i.e.

$$\frac{dP}{d\rho} = \frac{12\mu}{h^3} \cdot \frac{dh}{dt} \cdot \frac{\rho}{2} + \frac{c_1}{\rho}.$$

Repeated integration will give us that

$$P = \frac{3\mu\rho^2}{h^3} \cdot \frac{dh}{dt} + c_1 \ln \rho + c_2, \quad c_2 = \text{const.} \quad (4)$$

Proceeding from (2) and (4),

$$P_o = \frac{3\mu r^2}{h^3} \cdot \frac{dh}{dt} + c_1 \ln r + c_2, \quad (5)$$

While based on (3) and (5) we have that

$$P_o = \frac{3\mu R^2}{h^3} \cdot \frac{dh}{dt} + c_1 \ln R + c_2. \quad (6)$$

Now let us to subtract (4) from the equality (6) and to calculate c_1 from the obtained equality. We will get, that

$$\frac{3}{h^3} \cdot \frac{dh}{dt} (R^2 - r^2) + c_1 (\ln R - \ln r) = 0,$$

From Which

$$c_1 = \frac{3\mu h' (R^2 - r^2)}{h^3 \ln(r/R)}. \quad (7)$$

Taking (5) and (7) into account,

$$c_2 = -\frac{3\mu R^2}{h^3} \cdot \frac{dh}{dt} - \frac{3\mu h' (R^2 - r^2)}{h^3 \ln(r/R)} \ln r + P_o. \quad (8)$$

By substituting (7) and (8) in (4), we will get

$$P = \frac{3\mu}{h^3} \cdot \frac{dh}{dt} \left[\rho^2 - r^2 + \frac{(R^2 - r^2) \ln(\rho/r)}{\ln(r/R)} \right] + P_o. \quad (9)$$

The equality (9) provides the time distribution of pressure along the radius (from r to R) in the liquid area to be displaced. In the dimensionless form the equality (4) will be written as follows

$$\frac{h^3}{\mu r^2 h'} P = 3 \left[\left(\frac{\rho}{r} \right)^2 - 1 + \frac{\left(\left(\frac{R}{r} \right)^2 - 1 \right) \ln(\rho/r)}{\ln(r/R)} \right] + \frac{h^3}{\mu r^2 h'} P_o \quad (10)$$

The oil resistance to be pushed out when the armature approaches the core will be calculated via pressure integration over the ring:

$$F = \int_0^{2\pi} \int_r^R P(\rho) \rho d\rho dt \quad (11)$$

By virtue of (11)

$$F = 2\pi \int_r^R \left\{ \frac{3\mu}{h^3} \cdot \frac{dh}{dt} [(\rho^2 - r^2)\rho + \frac{(R^2 - r^2)}{\ln(R/r)} \cdot \ln(\rho/r) \cdot \rho] + P_o \rho \right\} d\rho$$

taking into account that

$$\begin{aligned} \int \rho \ln \frac{\rho}{r} d\rho &= \frac{1}{2} \ln \frac{\rho}{r} \rho^2 - \frac{1}{2} \int \rho^2 \cdot \frac{r}{\rho} \cdot \frac{1}{r} d\rho = \\ &= \frac{1}{2} \ln(\rho/r) \rho^2 - \frac{1}{4} \rho^2, \end{aligned}$$

we will finally get

$$F = \frac{3\pi\mu h' r^4}{2h^3} \left\{ \left(\frac{R}{r} \right)^4 - 1 - \frac{\left[\left(\frac{R}{r} \right)^2 - 1 \right]^2}{\ln(r/R)} \right\} + \pi P_o (R^2 - r^2),$$

or else, in dimensionless form

$$\begin{aligned} \frac{h^3}{\mu r^4 h'} F &= -\frac{3\pi}{2} \left\{ \frac{R^4}{r^4} - 1 - \frac{\left[\left(\frac{R}{r} \right)^2 - 1 \right]^2}{\ln(R/r)} \right\} + \\ &+ \frac{h^3 \pi P_o}{\mu r^2 h'} \left[\left(\frac{R}{r} \right)^2 - 1 \right]. \end{aligned} \quad (12)$$

The armature attraction force for quite small h with a satisfactory accuracy may be regarded as a constant. Now, if we rewrite (12) in the following form

$$\begin{aligned} \frac{dt}{dh} &= \frac{3\pi\mu r^2}{2} \times \\ &\times \frac{\left\{ \left(\frac{R}{r} \right)^4 - 1 - \left[\left(\frac{R}{r} \right)^2 - 1 \right] / \ln(R/r) \right\}}{-\frac{F}{r^2} + P_o \left[\left(\frac{R}{r} \right)^2 - 1 \right]} \cdot \frac{1}{h^3} \end{aligned}$$

and then integrate it, we will get a ratio between time and armature stroke:

$$\begin{aligned} \int_{t_o}^{t_1} dt &= \int_{h_o}^{h_1} \frac{3\pi\mu r^2}{2} \times \\ &\times \frac{\left\{ \left(\frac{R}{r} \right)^4 - 1 - \left[\left(\frac{R}{r} \right)^2 - 1 \right]^2 / \ln(R/r) \right\}}{-\frac{F}{r^2} + P_o \left[\left(\frac{R}{r} \right)^2 - 1 \right]} \times \\ &\times \frac{1}{h^3} dh, \end{aligned}$$

When changing time from t_o to t_1 the armature will shift from h_o to h_1 .

Thus,

$$\begin{aligned} \Delta t &= t_1 - t_o = \\ &= \frac{3\pi\mu \left[\frac{1}{h_1^2} - \frac{1}{h_o^2} \right] \cdot \left[R^4 - r^4 - \left((R^2 - r^2) / \ln(R/r) \right)^2 \right]}{4 \left[F - P_o \pi (R^2 - r^2) \right]} \end{aligned} \quad (13)$$

As h_o we have to take the thickness, in case of which the resistance to displacement becomes so substantial that a decelerated motion of armature starts.

III. CONCLUSIONS

It is reasonable to reduce the armature attraction time via reduction of area S of oil squeezing-out ring that may be accomplished if the armature end surface will have a narrow ring-shaped projection with height l and radiuses r , R (Fig. 2). It is obvious that when $S \rightarrow 0$, then $\Delta t \rightarrow t_{min}$. Though, reduction of S will cause inadmissible increase of bearing stress and compression stress on the contact surface of projection and core end, especially during the anticipated impact.

That is why the time, in case of which the bearing and compression stresses are equal to their permissible values, has to be regarded as an optimum approach time.

REFERENCES

- [1]. S.G. Bitsadze and O.S. Ezikashvili. (1977). Magneto hydraulic pusher. Authorship certificate №582188, USSR.
- [2]. S. Bitsadze and R. Bitsadze (2022). Membrane magneto hydraulic pusher. Patent of invention P 2022 7337 B.
- [3]. N. Petrov, O. Reynolds, A. Zommerfeld, A. Miguel, N. Zhukovsky, S. Chaplygin. (1934). Hydrodynamic theory of lubrication, STTI.