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The Approach of Matching Problem in Assignment Problem

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Abstract: One of the way is to find optimum solution of matching problem is solution of assignment problem by Hungarian method. There is relation between Matching problem and Assignment problem. By applying algorithms, it gives consistency and efficiency of matching problem.

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I. INTRODUCTION

There are various applications of Matching problem in mathematics and managerial problems. Various methods are also available to find optimum solution of matching problems. Some of them are consistent and converges but others are unknown about consistency and convergent of it. We have explained solution of matching problem using assignment problem and clarify it's consistency and provided theoretical background, an in depth explanation of the algorithm.

➢ *Key Words:*

Some of key words are as follows:

- Maximum matching problem.
- Minimum Matching problem.
- Assignment problem.
- Hungarian method.
- Consistent Matching problem.
- Efficiency.

II. CONTAIN

A. *Matching Problem:*

➢ *Single Conditional Matching Problem:*

- *Definition:-*

A Bijective function (one one, on to) f: A → B satisfying following conditions is matching of elements from set A to set B.

- | A | = | B |. (Cardinality of A = cardinality of B).
- Element a_i Є A, has k^{th} preference to element b_j Є B for all i, j from 1 to n.
- If some a Є A has no preference to element b_j Є B then consider it is infinity.
- Find function f: A→B so that $\sum_1^n k$ (sum of preferences of functional values) is minimum.

➢ *Solution of Single Conditional Matching Problem:*

There are various methods to solve such matching problem, we are discuss solution of single conditional matching problem by using method for solution of assignment problem. This method gives existence of solution as well as optimality and consistency of solution.

➢ *Steps of Conversion of Single Conditional Matching Problem in to Assignment Problem and there Solution by using Method:*

- Consider given single conditional matching problem with n participants.
- Elements a_i Є A are jobs of assignment and b_j Є B are machines for all i, j from 1 to n.
- Find C_{ij} the cost of assignment problem is preference of a_i Є A to b_j Є B.
- Solve assignment problem by Hungarian method.
- If solution of assignment problem exists then single conditional matching problem is consistent problem otherwise it is not consistent single conditional matching problem.
- Allocations a_i Є A to b_j Є B are $f(a_i) = b_j$.

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- The sum of allocated C_{ij} is optimum solution of single conditional matching problem.

- *General Example: Let set A = {g_1, g_2,.....,g_n} and B = { b_1, b_2,....,b_n} be two sets with their preferences of allocations are given in following table (Table 1):*

Table showing preferences of elements in set A to elements in set B.

Table 1 Find Best Allocations of GI to BJ so that Sum of Preferences is Optimum.

A's(pref.)	**1**	**2**	**.**	**.**	**.**	**n**
g_1	b_i	b_j	.	.	.	b_n
g_2	b_i	b_j	.	.	.	b_n
.	b_i	b_j	.	.	.	b_n
.g_n	b_i	b_j	.	.	.	b_n
b_i's and b_j's are k^{th} preferences of g_i's and g_j's.						

- *Particular Example:*

Let set A = {g_1, g_2, g_3, g_4, g_5, g_6} is the set of n girls and B = {b_1, b_2, b_3, b_4, b_5, b_6} be the set of their n boyfriends. All girls wish to marry with one boyfriend and preferences of their selection are given in following table (Table 2). Match the pairs with minimum preference.

Table showing sequence of preferences of elements in set A to elements in set B.

Table 2 Find Best Allocations of GI to BJ so that Sum of Preferences is Minimum.

A's(pref.)	**1**	**2**	**3**	**4**	**5**	**6**
g_1	b_3					
g_2	b_2	b_3	b_1	b_6	b_5	
g_3	b_1	b_4	b_5	b_3	b_2	
g_4	b_1	b_5	b_3	b_6		
g_5	b_3	b_6	b_4			
g_6	b_2	b_3	b_5	b_6	b_1	
b_i's and b_j's are k^{th} preferences of g_i's and g_j's.						

- *Solution: Assignment Problem of above Minimum Matching Problem is as Follows in Table (Table 3):*

Table 3 Showing Convergent of Matching Problem to Assignment Problem.

A's(pref.)	**b_1**	**b_2**	**b_3**	**b_4**	**b_5**	**b_6**
g_1	∞	∞	1	∞	∞	∞
g_2	3	1	2	∞	5	4
g_3	1	5	4	2	3	∞
g_4	1	∞	3	∞	2	4
g_5	∞	∞	1	3	∞	2
g_6	5	1	2	∞	3	4

Solution of it by **Hungarian Method is** red cells are allocated cells in table (Table 4) :

Table 4 Showing solution of assignment problem by Hungarian Method.

A's(pref.)	**b_1**	**b_2**	**b_3**	**b_4**	**b_5**	**b_6**
g_1	∞	∞	1	∞	∞	∞
g_2	3	1	2	∞	5	4
g_3	1	5	4	2	3	∞
g_4	1	∞	3	∞	2	4
g_5	∞	∞	1	3	∞	2
g_6	5	1	2	∞	3	4

Hence g_1 marry with b_3 with preference 1,

g_2 marry with b_2 with preference 1,

g_3 marry with b_4 with preference 2,

g_4 marry with b_1 with preference 1,

g_5 marry with b_6 with preference 2,

g_6 marry with b_5 with preference 3.

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Total preferences for chosen pairs = 10.

B. Double Conditional Matching Problem:

➢ *Definition:-*

A bijective function (one one, on to) f: A → B satisfying following conditions is matching of elements from set A to set B.

- | A | = | B |. (Cardinality of A = cardinality of B).
- Element a_i Є A, has k_i^{th} preference to element b_j Є B for all i, j from 1 to n.
- Similarly b_j Є B has t_j^{th} preference to element a_i Є A for all i, j from 1 to n.
- If some a_i Є A has no preference to element b_j Є B then consider it is infinity and If some b_j Є B has no preference to element a_i Є A then consider it is infinity

Find function f: A→B so that $\sum_{i,j=1}^{n} t_j + k_i$ (sum of preferences of functional values) is minimum.

- *Solution of Double Conditional Matching Problem:*

Steps of conversion of single conditional matching problem in to assignment problem and there solution by using Hungarian method:

- Consider given single conditional matching problem with n participants.
- Elements a_i Є A are jobs of assignment and b_j Є B are machines for all i, j from 1 to n.
- Find $C_{ij} = k_i + t_j$ the cost of assignment problem. Where k_i is preference of a_i Є A to b_j Є B and t_j is preference of b_j Є A to a_i Є B .
- Solve assignment problem by Hungarian method.
- If solution of assignment problem exists then single conditional matching problem is consistent problem otherwise it is not consistent single conditional matching problem.
- Allocations a_i Є A to b_j Є B are $f(a_i) = b_j$.
- The sum of allocated C_{ij} is optimum solution of single conditional matching problem.

➢ *General Example:*

Let set A = {g_1, g_2,…..,g_n} and B = { b_1, b_2,….,b_n} be two sets with their preferences of allocations are given in following tables (Table 5 and Table 6):

Preferences of A to B in table (Table 5).

Table 5 Showing Preferences of Elements in set A to Elements in set B

A's(pref.)	**1**	**2**	**.**	**.**	**.**	**n**
g_1	b_i	b_j	.	.	.	b_n
g_2	b_i	b_j	.	.	.	b_n
.	b_i	b_j	.	.	.	b_n
.	b_i	b_j	.	.	.	b_n
.	b_i	b_j	.	.	.	b_n
g_n	b_i	b_j	.	.	.	b_n
b_i's and b_j's are k^{th} preferences of g_i's and g_j's.						

Preferences of A to B in table. (Table 6)

Table 6 Showing Preferences of Elements in set B to Elements in set A.

B's(pref.)	**1**	**2**	**.**	**.**	**.**	**n**
b_1	g_i	g_j	.	.	.	g_n
b_2	g_i	g_j	.	.	.	g_n
.	g_i	g_j	.	.	.	g_n
.	g_i	g_j	.	.	.	g_n
.	g_i	g_j	.	.	.	g_n
b_n	g_i	g_j	.	.	.	g_n
g_i's and g_j's are t^{th} preferences of b_i's and b_j's.						

Find Best Allocations of g_i to b_j so that sum of Preferences is Optimum.

➢ *Particular Example:*

Let set A = {g_1, g_2, g_3, g_4, g_5, g_6} is the set of n girls and B = { b_1, b_2, b_3,b_4, b_5, b_6} be the set of their n boyfriends. All girls wish to marry with one boyfriend and preferences of their selection are given in following first table as well as all boys wish to marry with one girlfriend and preferences of their selection are given in following second table. Match the pairs with minimum preference in tables (Table 7 and Table 8).

Preferences of Girls with Boys in table (Table 7)

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Table 7 showing Sequence of Preferences of Girl to Boys as a Boyfriends.

A's(pref.)	**1**	**2**	**3**	**4**	**5**	**6**
g_1	b_3					
g_2	b_2	b_3	b_1	b_6	b_5	
g_3	b_1	b_4	b_5	b_3	b_2	
g_4	b_1	b_5	b_3	b_6		
g_5	b_3	b_6	b_4			
g_6	b_2	b_3	b_5	b_6	b_1	
g_i's and g_j's are k^{th} preferences of b_i's and b_j's.						

Preferences of Boys with Girls in table. (Table 8)

Table 8 Showing Sequence of Preferences of Boy to Girls as a Girlfriends.

B's(pref.)	**1**	**2**	**3**	**4**	**5**	**6**
b_1	g_3	g_6	g_2	g_5		
b_2	g_3	g_2	g_1	g_6	g_5	
b_3	g_2	g_4	g_1	g_3	g_5	
b_4	g_6	g_5	g_2	g_3		
b_5	g_3	g_6	g_4			
b_6	g_4	g_3	g_5	g_6	g_1	
b_i's and b_j's are t^{th} preferences of g_i's and g_j's.						

Find best allocations of g_i to b_j so that sum of preferences is minimum.

- *Solution:*

Assignment problem of above minimum matching problem is as follows in table (Table 9):

Table 9 Showing Convergent of Matching Problem to Assignment Problem.

A's(pref.)	**b_1**	**b_2**	**b_3**	**b_4**	**b_5**	**b_6**
g_1	∞	∞	1	∞	∞	∞
g_2	3	1	2	∞	5	4
g_3	1	5	4	2	3	∞
g_4	1	∞	3	∞	2	4
g_5	∞	∞	1	3	∞	2
g_6	5	1	2	∞	3	4

Solution of it by **Hungarian Method is** red cells are allocated cells in table (Table 10) :

Table 10 Showing Solution of Assignment Problem by Hungarian Method.

A's(pref.)	**b_1**	**b_2**	**b_3**	**b_4**	**b_5**	**b_6**
g_1	∞	∞	2	∞	∞	∞
g_2	6	3	3	∞	10	8
g_3	4	6	8	4	8	∞
g_4	∞	∞	7	∞	4	5
g_5	∞	∞	2	∞	∞	4
g_6	10	∞	4	∞	6	8

Hence g_1 marry with b_3 with preference 2,

g_2 marry with b_2 with preference 3,

g_3 marry with b_4 with preference 4,

g_4 marry with b_5 with preference 4,

g_5 marry with b_6 with preference 4,

g_6 marry with b_1 with preference 10.

Total preferences for chosen pairs = 27.

➢ *Time Complexity and Efficiency:*

Hungarian method is very efficient method to find optimum solution of matching problem. It can also used for single conditional and double conditional matching problem. This method also verify for existence of solution as well as it's consistency.

➢ *Application in Matching Assignment Problems:*

- Job Scheduling:
- Transportation Problems:
- Student-College Admission:
- Healthcare Resource Allocation:

- Sports Team Formation:
- Supply Chain Management:

III. CONCLUSION

The Hungarian method provides an efficient solution to matching assignment problems in various domains. Its ability to solve the assignment problem in polynomial time makes it a valuable tool in operations research and combinatorial optimization.

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