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A Sustainable Fuzzy Economic Order Quantity Model with Carbon Tax Policy using Pascal Triangular Graded Mean for Pentagonal Fuzzy Numbers

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Abstract: Preservation of environment happens by focusing on sustainable measures taken by each and every business sectors in this modern world. Discussion about certain carbon policies by researchers helps to manage supply chain system under sustainable features. Carbon tax policy is one among them which helps firms to control carbon emission. In this process of controlling and in any case of EOQ system, ambiguous nature in any parameter arises at any instant creating a path way to rectify it. So this model studied a sustainable inventory model under fuzzy perspective as a step to face uncertainty for few parameters. So this model applies Pascal Triangular Graded mean defuzzification approach for pentagonal fuzzy numbers and founds the optimal solution. A Justification is given by using numerical analysis.

Keywords: *Pascal Triangular Graded Mean Method, Carbon Tax Policy, Sustainable, Defuzzification and Pentagonal Fuzzy Number*.

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I. INTRODUCTION

Concerns about sustainability include a wide range of variables that involve societal, environmental, and financial problems. To make sure that businesses and individuals make choices that are beneficial to both current and upcoming generations, these factors are crucial. In order to guarantee the availability of resources for future generations, sustainability in the natural world concentrates on striking a balance between environmental health and human requirements. This calls for combating climate change, protecting biodiversity, using resources sensibly, and implementing sustainable practices across a range of industries, including managing waste and agriculture. And at the same time, it is essential to address pollution from a variety of sources, such as plastic trash, runoff from agriculture, and industrial pollutants. Local policy implementation is crucial for promoting sustainable behaviours, safeguarding natural resources, and addressing regional environmental concerns. So concentrating on policies like carbon tax policy is most important factor in collaboration with inventory models.

Revision of economic order quantity models with sustainability considerations under different policies is studied by Arslan et al in 2013. Analysation of the best inventory and maintenance model based on the carbon tax policy is proposed by chen et al in 2024. They develop this model to determine the best outcome and profit with the inclusion of carbon tax policy. In 2025, Rani et al examined a sustainable inventory model with the integration of eco-friendly packages, advertisements and costs factors with carbon tax policy. Thus collaboration of inventory system with carbon tax policy is performed by researchers to found the original impact.

Uncertainty in product pricing refers to difficulties in figuring out the best price because of a number of variables, such as competitors, customer behaviour, and market circumstances. This ambiguity may result in inaccurate pricing, which would impact both consumer satisfaction and profits. Both financial projections and environmental policies are impacted by uncertainties in tax rates and greenhouse gas emissions. The efficacy and impact of carbon taxes, which are intended to lower emissions by increasing the cost of fossil fuels, are uncertain. There are uncertainties in emissions data

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as well, which are impacted by things like the estimating techniques and the precision of the emission variables. Hence there is a need to solve or face this kind of uncertainties by using fuzzy methodologies.

With an analysement of pollution control, a EOQ model is developed by Bhattacharya in 2023 under fuzzy idea where the parameters are taken to be as triangular dense fuzzy number. Incorporation of fuzzy concept in a sustainable supply chain with improper production processes is discussed by Singh et al in 2025. Here they considered certain uncertain parameters as triangular fuzzy numbers. Devi et al in 2025, optimized a leakage concept sustainable supply chain model under fuzzy perspective using signed distance method for fuzzy numbers. And they integrated carbon emission cost in the total expenses of the system. The present model takes into account Pascal triangular graded mean approach in an economic order quantity model with sustainability consideration and carbon tax policy. Here parameters with vagueness are taken as pentagonal fuzzy numbers.

➢ *Definitions:*

- *Fuzzy Set:*

A fuzzy set $\tilde{A}$ defined on a Universe of discourse X may be written as a collection of ordered pairs,

$$\tilde{A} = \left\{ (x, \mu_{\tilde{A}}(x)) : x \in X, \mu_{\tilde{A}} \in [0,1] \right\}$$

Where each pair $(x, \mu_{\tilde{A}}(x))$ is called a singleton and the element $\mu_{\tilde{A}}(x)$ belongs to the interval [0,1].The function $\mu_{\tilde{A}}(x)$ is called as membership function.

- *Pentagonal Fuzzy Number*

A fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4, a_5)$ where $a_1 < a_2 < a_3 < a_4 < a_5$ are defined on R is called pentagonal fuzzy number if its membership function is

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & for \ \ x < a_1, \ a_5 \le x \\ \dfrac{(x-a_1)}{(a_2-a_1)}; & for \ \ a_1 \le x \le a_2 \\ \dfrac{(x-a_2)}{(a_3-a_2)}; & for \ \ a_2 \le x \le a_3 \\ 1, & for \ \ x = a_3 \\ \dfrac{(a_4-x)}{(a_4-a_3)}; & for \ \ a_3 \le x \le a_4 \\ \dfrac{(a_5-x)}{(a_5-a_4)}; & for \ \ a_5 \le x \le a_4 \end{cases}$$

II. METHODOLOGY

➢ *Pascal Triangular Graded Mean Method:*

If $\tilde{B} = (b_1, b_2, b_3, b_4, b_5)$ is a pentagonal fuzzy number, then the defuzzification formula of pascal triangular graded mean method is given by,

$$P_{TG}(\tilde{B}) = \frac{(b_1 + 4b_2 + 6b_3 + 4b_4 + b_5)}{16}.$$

➢ *Assumptions*:

- A supply chain model with carbon tax policy is taken into account.
- Demand is a constant value.
- Cost of setting up products, variable cost of greenhouse gas emissions, cost of holding items and rate of tax are found to be uncertain.

➢ *Notations:*

- *Crisp Parameters:*

F – cost of setting up products.

T – rate of tax.

E_f – fixed cost of greenhouse gas emissions.

r – Demand.

V – cost of variable.

E_v – variable cost of greenhouse gas emissions.

H – cost of holding items.

I – expense of GHG emissions for holding inventory.

Q_E – economic order quantity with green criteria.

$TC(Q_E)$ – Total cost of refillment.

- *Fuzzy Parameters:*

$\tilde{F}$ – fuzzy cost of setting up products.

$\tilde{T}$ – rate of tax under fuzzy.

$\tilde{E}_v$ – fuzzy variable cost of greenhouse gas emissions.

$\tilde{H}$ – fuzzy cost of holding items.

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$\tilde{Q}_E^*$ – fuzzy economic order quantity with green criteria.

$P_{TG}\left(T\tilde{C}\left(Q_E\right)\right)$ – fuzzy total cost of refillment.

➢ *Sustainable EOQ Model with Carbon Tax Policy:*

The total cost of the considered sustainable economic order quantity model is given by,

$$TC\left(Q_E\right)=\frac{\left(F+TE_f\right)r}{Q_E}+r\left(V+TE_v\right)+\frac{\left(H+TI\right)Q_E}{2} \quad \text{--(1)}$$

Differentiating eqn (1) with respect to Q_E and equating to zero gives the economic order quantity with green criteria and it is given by,

$$Q_E=\sqrt{\frac{2\left(F+TE_f\right)r}{\left(H+TI\right)}}\,.$$

➢ *Sustainable Fuzzy EOQ Model with Carbon Tax Policy:*

Suppose the parameters $\tilde{F}=\left(F_1,F_2,F_3,F_4,F_5\right)$, $\tilde{T}=\left(T_1,T_2,T_3,T_4,T_5\right)$ $\tilde{E}_v=\left(E_{v_1},E_{v_2},E_{v_3},E_{v_4},E_{v_5}\right)$ and $\tilde{H}=\left(H_1,H_2,H_3,H_4,H_5\right)$ are taken as pentagonal fuzzy numbers, then after converting the above parameters as pentagonal fuzzy numbers we have the total cost as,

$$T\tilde{C}\left(Q_E\right)=\frac{\left(\tilde{F}+\tilde{T}E_f\right)r}{Q_E}+r\left(V+\tilde{T}\tilde{E}_v\right)+\frac{\left(\tilde{H}+\tilde{T}I\right)Q_E}{2}$$

Applying Pascal Triangular Graded mean method for the total cost with carbon tax policy we obtain the total cost under fuzzy as,

$$P_{TG}\left(T\tilde{C}\left(Q_E\right)\right)=\frac{1}{16}\left[\begin{array}{l}\left(\frac{\left(F_1+T_1E_f\right)r}{Q_E}+r\left(V+T_1E_{v_1}\right)+\frac{\left(H_1+T_1I\right)Q_E}{2}\right)+\\ 4\left(\frac{\left(F_2+T_2E_f\right)r}{Q_E}+r\left(V+T_2E_{v_2}\right)+\frac{\left(H_2+T_2I\right)Q_E}{2}\right)+\\ 6\left(\frac{\left(F_3+T_3E_f\right)r}{Q_E}+r\left(V+T_3E_{v_3}\right)+\frac{\left(H_3+T_3I\right)Q_E}{2}\right)+\\ 4\left(\frac{\left(F_4+T_4E_f\right)r}{Q_E}+r\left(V+T_4E_{v_4}\right)+\frac{\left(H_4+T_4I\right)Q_E}{2}\right)+\\ \left(\frac{\left(F_5+T_5E_f\right)r}{Q_E}+r\left(V+T_5E_{v_5}\right)+\frac{\left(H_5+T_5I\right)Q_E}{2}\right)\end{array}\right] \quad \text{--(2)}$$

Differentiating eqn (2) with respect to Q_E and equating to zero gives the fuzzy optimum order quantity with carbon tax policy and it is given by,

$$\tilde{Q}_E^*=\sqrt{\frac{2\left[\left(F_1+T_1E_f\right)r+4\left(F_2+T_2E_f\right)r+6\left(F_3+T_3E_f\right)r+4\left(F_4+T_4E_f\right)r+\left(F_5+T_5E_f\right)r\right]}{\left(H_1+T_1I\right)+4\left(H_2+T_2I\right)+6\left(H_3+T_3I\right)+4\left(H_4+T_4I\right)+\left(H_5+T_5I\right)}}$$

➢ *Numerical Example:*

- *Crisp Model:*

$$F=50,\ T=6,\ E_f=70,\ r=60,\ V=13,$$
$$E_v=6,\ H=3,\ I=2.$$

The economic order quantity with green criteria is, $Q_E=61.31883887.$

The total cost of refillment, $TC\left(Q_E\right)=3859.782583.$

- *Fuzzy Model:*

$$\tilde{F}=(10,20,50,55,60)\,,\ \tilde{T}=(1,3,6,7,8)\,,$$
$$\tilde{E}_v=\left(5,5.5,6,6.5,7\right),\ \tilde{H}=\left(1,2,3,4,5\right).$$

The fuzzy economic order quantity with green criteria, $\tilde{Q}_E^*=60.36585.$

The fuzzy total cost of refillment, $P_{TG}\left(T\tilde{C}\left(Q_E\right)\right)=3571.234802.$

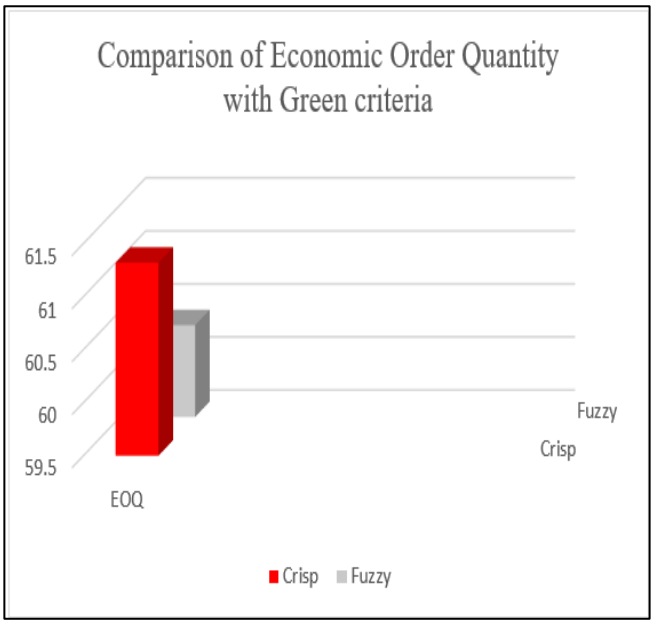

Fig 1 Comparison of Economic Order Quantity with Green criteria

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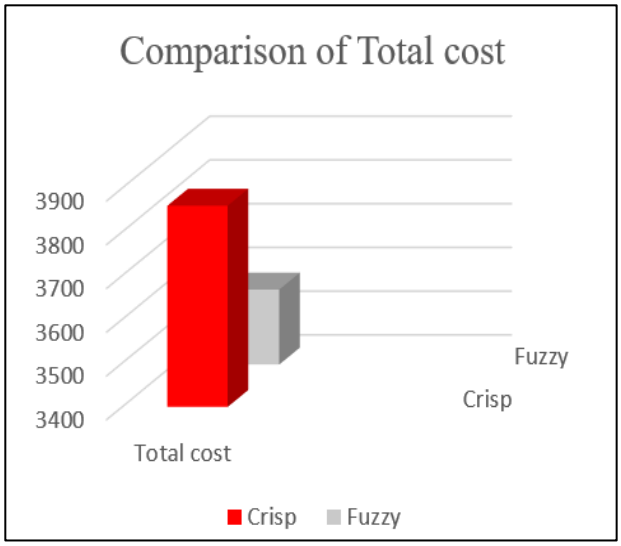

Fig 2 Comparison of Total cost

III. CONCLUSION

In the developing world with technological advancements, factors like changing climatic conditions and customer preferences in the manufactural areas makes an unpredictable state resulting in the search for a proper initiative to face such uncertainty. Since fuzzy is the best tool to rectify the issue, implementation of this fuzzy approach will help sectors to get a clear outcome by the analyzation of specific parameters with vague conditions. This present model has addressed about the above criteria in the green inventory model with carbon tax policy and sustainability considerations by using Pascal triangular graded mean method for pentagonal fuzzy numbers. The optimal solutions in terms of economic order quantity and total cost of refillment has been determined for crisp and fuzzy senses where the outcome yield through fuzzy approach yields reduced value than the outcome obtained through crisp idea. Also the comparison of results has been expressed diagrammatically to show the differences. Hence this model will be useful for firms to remove ambiguity among few uncertain parameters and meanwhile to create a sustainable environment.

REFERENCES

[1]. Arslan, M. C., & Turkay, M. (2013). EOQ revisited with sustainability considerations. *Foundations of Computing and Decision Sciences*, *38*(4), 223-249.

[2]. Chen, W. J., Lu, C. J., Hsu, P. T., & Yang, C. T. (2024). Research on an optimal maintenance and inventory model based on carbon tax policy. *Processes*, *12*(3), 599.

[3]. Rani, N., Maheshwari, S., & Sharma, M. K. (2025). A sustainable inventory model: integrating eco-packaging, advertising, and pricing strategies under carbon tax policies. *International Journal of System Assurance Engineering and Management*, 1-18.

[4]. Bhattacharya, P. P., Bhattacharya, K., De, S. K., Nayak, P. K., Joardar, S., & Das, K. (2023). Solution of a Pollution-Based Economic Order Quantity Model Under Triangular Dense Fuzzy Environment. *Optimization Techniques in Engineering: Advances and Applications*, 345-362.

[5]. Singh, S. R., & Zaidi, U. (2025). Incorporating Fuzzy Learning Approach into a Two-Echelon Sustainable Supply Chain with Imperfect Production Under Environmental and Social Sustainability. *Process Integration and Optimization for Sustainability*, *9*(1), 167-195.

[6]. Devi, N. R., Kuber Singh, M., & Malemnganbi, H. (2025). Optimizing a Sustainable Leakage Inventory Model Under Fuzzy Environment. In *Sustainable Inventory Management: Perspectives from India* (pp. 241-267). Singapore: Springer Nature Singapore.