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Optimal Intervention Design for Corruption Dynamics Via Compartmental Modelling

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Abstract: Corruption represents a pervasive socio-economic phenomenon that undermines institutional integrity, economic development, and social cohesion globally. This research presents a comprehensive mathematical framework for modeling corruption dynamics using compartmental analysis, incorporating stability theory and optimal control strategies to design effective intervention mechanisms. We develop a deterministic compartmental model that categorizes the population into susceptible, exposed, corrupt, and recovered individuals, analogous to epidemiological models. Through rigorous stability analysis using Lyapunov theory and the next-generation matrix method, we establish conditions for corruption-free equilibrium and endemic stability. The model is extended to incorporate optimal control theory, enabling the design of cost-effective intervention strategies including awareness campaigns, legal enforcement, and rehabilitation programs. Numerical simulations demonstrate the efficacy of the proposed control mechanisms in reducing corruption prevalence while minimizing intervention costs. Our findings provide theoretical foundations for evidence-based policy design and resource allocation in anti-corruption initiatives.

Keywords: *Corruption Dynamics, Stability Analysis, Optimal Control, Intervention Design, Mathematical Epidemiology.*

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I. INTRODUCTION

Corruption, defined as the abuse of public power for private gain, constitutes one of the most significant challenges facing modern societies [1]. The phenomenon transcends geographical and cultural boundaries, affecting developed and developing nations alike, with profound implications for economic growth, social equity, and institutional legitimacy. According to recent estimates, corruption costs the global economy \$2.6 trillion annually, representing about 5% of global GDP [5]. In developing countries, the impact is particularly severe, with corruption reducing economic growth rates by 0.5-1.0 percentage points annually [11]. Recent studies indicate that corruption affects many countries by destroying economic, social, and political development [5], necessitating comprehensive analytical frameworks to understand its transmission mechanisms and design effective countermeasures.

The conceptualization of corruption as a transmissible social phenomenon has gained considerable attention in recent mathematical modelling literature [6,9]. This approach draws parallels with epidemiological models, treating corruption as “contagious" behaviour that spreads through social networks and institutional interactions. The epidemiological perspective recognizes that corrupt behaviour exhibits characteristics like infectious diseases, including transmission through contact, incubation periods, and recovery processes [3]. Mathematical models examining corruption prevalence in society assume that corruption spreads like an infectious disease [3], providing a structured framework for analyzing its dynamics and developing intervention strategies. This modelling approach has proven particularly valuable in understanding how corruption can become endemic within institutions and how social norms can either facilitate or inhibit its spread [1].

Traditional approaches to corruption control have often relied on punitive measures and regulatory frameworks without adequate consideration of the underlying transmission dynamics [11]. These conventional strategies, while important, have shown limited effectiveness in addressing systemic corruption, particularly in environments where corrupt practices have become deeply embedded in organizational culture [5]. However, recent advances in mathematical modeling and control theory offer promising avenues for developing more sophisticated, evidence-based intervention strategies. The application of compartmental modelling techniques, originally developed for infectious disease analysis [12], provides a natural framework for understanding how corrupt behaviors propagate through social and institutional networks. This mathematical approach enables the quantification of transmission rates, identification of critical intervention points, and optimization of resource allocation across different anti-corruption strategies [9].

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The significance of this research lies in its integration of rigorous mathematical analysis with practical policy considerations [6]. By employing stability theory and optimal control methods, we can identify critical parameters that govern corruption dynamics and design intervention strategies that maximize effectiveness while minimizing implementation costs. This quantitative approach represents a change in basic assumptions from traditional qualitative assessments to evidence-based policy design [3]. The mathematical framework enables policymakers to make informed decisions about resource allocation and intervention timing, potentially leading to more successful anti-corruption campaigns. Furthermore, the model provides tools for predicting the long-term effects of different intervention strategies and assessing their sustainability under varying socio-economic conditions [11].

Our study contributes to the growing literature on mathematical modeling of social phenomena by providing a comprehensive framework that encompasses model formulation, stability analysis, and optimal control design [5]. The research addresses fundamental questions about corruption persistence, intervention effectiveness, and resource optimization, offering both theoretical insights and practical tools for corruption control. The interdisciplinary nature of this work bridges the gap between mathematical theory and public policy, demonstrating how advanced analytical techniques can inform real-world decision-making [1]. The findings have implications for understanding not only corruption dynamics but also broader classes of social transmission phenomena where behavioral contagion plays a significant role, including the spread of social norms, organizational practices, and cultural behaviors [9].

II. MATHEMATICAL MODEL FORMULATION

➤ *Compartmental Structure*

We develop a deterministic compartmental model that divides the population into four distinct classes based on their corruption status and behavioral characteristics [10]. This SECR (Susceptible-Exposed-Corrupt-Recovered) framework represents an adaptation of the classical SEIR epidemiological model to the context of corruption transmission [12]. The compartmental structure captures the progressive nature of corruption involvement, from initial susceptibility through exposure to active participation and eventual recovery or rehabilitation [6]. The total population $N(t)$ at time t is partitioned as follows:

- $S(t)$: Susceptible individuals who are honest but potentially vulnerable to corrupt influences due to institutional environment, economic pressures, or social networks [5]
- $E(t)$: Exposed individuals who have been influenced by corrupt practices but have not yet engaged in corrupt behavior, representing a critical intervention window [3]
- $C(t)$: Corrupt individuals actively engaged in corrupt practices, including both those who initiate corrupt acts and those who participate in existing corrupt networks [11]
- $R(t)$: Recovered individuals who were previously corrupt but remains rehabilitated or reformed through legal, social, or institutional interventions [9].

The model assumes a closed population with constant recruitment rate Λ representing new entrants to the system, for instance, new employees, officials or citizens entering positions of responsibility and natural mortality rate μ accounting for departures from the system [4]. The assumption of a closed population is reasonable for analyzing corruption within specific institutions or administrative systems over medium-term horizons [1]. The total population satisfies:

$N(t) = S(t) + E(t) + C(t) + R(t)$, with $\frac{dN}{dt} = \Lambda - \mu N$, ensuring population conservation and establishing the carrying capacity of the system at $\frac{\Lambda}{\mu}$ individuals [6]. As a result, our model is described by the following compartmental flow diagram (Figure 1).

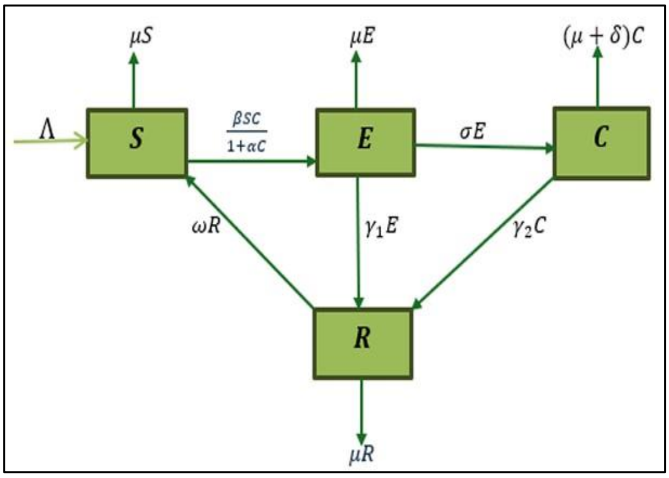

Fig 1 Compartmental Model for the Transmission Dynamics of Corruption

The transmission of corrupt behaviour is modelled through effective contact between susceptible and corrupt individuals, reflecting the social nature of corruption propagation [5]. This contact-based transmission model recognizes that corruption often spreads through professional relationships, social networks, and institutional hierarchies where corrupt individuals can influence honest counterparts [3]. Mathematical models describe corruption transmission dynamics by considering social influence on honest individuals [1], incorporating factors such as peer pressure, normalization of corrupt practices, and economic incentives. We incorporate a nonlinear incidence function to capture the realistic scenario where corruption transmission may exhibit saturation effects due to limited opportunities, increased surveillance, or social constraints that emerge as corruption becomes more prevalent [11].

Let β represent the transmission coefficient, capturing the probability of a susceptible individual becoming exposed to corruption upon effective contact with a corrupt individual [9]. This parameter encompasses multiple factors including the persuasiveness of corrupt actors, the attractiveness of corrupt benefits, and the weakness of ethical barriers or institutional safeguards [5]. The exposure rate is given by $\frac{\beta S(t)C(t)}{1+\alpha C(t)}$, where α represents the inhibition parameter accounting for saturation effects that naturally limit transmission as the corrupt

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population grows [6]. This functional form reflects diminishing returns to corruption recruitment, which may occur due to increased detection risks, reduced opportunities for new participants or strengthening of anti-corruption measures in response to growing corruption levels [11]. The complete mathematical model is described by the following system of ordinary differential equations:

$$\frac{dS}{dt} = \Lambda - \frac{\beta SC}{1+\alpha C} - \mu S + \omega R$$
$$\frac{dE}{dt} = \frac{\beta SC}{1+\alpha C} - (\sigma + \mu + \gamma_1)E$$
$$\frac{dC}{dt} = \sigma E - (\gamma_2 + \mu + \delta)C$$
$$\frac{dR}{dt} = \gamma_1 E + \gamma_2 C - (\mu + \omega)R \tag{2.1}$$

With initial data given by $S(0) \geq 0$, $E(0) \geq 0$, $C(0) \geq 0$ and $R(0) \geq 0$.

σ: Progression rate from exposed to corrupt state

γ_1: Recovery rate directly from exposed state

γ_2: Recovery rate from corrupt state

δ: Additional mortality/removal rate due to corruption-related consequences

ω: Rate of loss of immunity (susceptibility restoration)

➢ *Model Assumptions and Biological Interpretation*

The model incorporates several key assumptions that reflect the realistic dynamics of corruption transmission [6]. First, we assume that corruption spreads through social interaction and institutional exposure, like infectious disease transmission, recognizing that corrupt behaviour is learned and reinforced through social networks [11]. Second, the model allows for direct recovery from the exposed state, representing individuals who resist corrupt influences before engaging in corrupt behavior, which acknowledges the importance of early intervention and ethical education [3]. Third, the inclusion of waning immunity captures the reality that reformed individuals may become susceptible again under certain circumstances, such as changes in institutional environment, economic pressures, or weakening of ethical safeguards [5]. This assumption is particularly important for long-term modeling as it prevents the unrealistic accumulation of permanently immune individuals [9].

The nonlinear incidence function represents a more realistic transmission scenario where the rate of new exposures may decrease as the number of corrupt individuals becomes very large, due to increased awareness, stronger enforcement or social backlash [11]. This saturation effect is consistent with empirical observations that corruption transmission rates often decline when corruption becomes highly visible or when it triggers institutional or social responses [1].

The functional form $\frac{\beta S(t)C(t)}{1+\alpha C(t)}$ ensures that the per-capita transmission rate decreases as C(t) increases, preventing unrealistic exponential growth in corruption prevalence and providing a more nuanced representation of social dynamics [3]. This formulation is particularly important for modelling corruption in environments where detection probability increases with the number of corrupt actors or where social tolerance for corruption has limits [5].

III. QUALITATIVE ANALYSIS OF THE CORRUPTION MODEL

A. Invariant Region and Positivity

➢ *Mathematical Foundation*

The establishment of invariant regions is fundamental to ensuring the mathematical and biological validity of compartmental models. For the corruption transmission model, we must demonstrate that solutions remain non-negative and bounded, reflecting the realistic constraint that population sizes cannot become negative and the total population cannot grow without bound.

- *Definition*

An invariant region *is a subset of the state space such that if a solution starts within this region, it remains within the region for all future time.*

➢ *Population Conservation*

The total population $N = S + E + C + R$ satisfies

$$\frac{dN}{dt} = \Lambda - \mu N$$

With solution

$$N(t) = \frac{\Lambda}{\mu} + \left(N_0 - \frac{\Lambda}{\mu}\right)e^{-\mu t}, \qquad N_0 = N(0)$$

- *Key Properties*

If $N_0 < \frac{\Lambda}{\mu}\left(resp. > \frac{\Lambda}{\mu}\right)$, then $N(t)$ increases (resp. decreases) to $\frac{\Lambda}{\mu}$; if $N_0 = \frac{\Lambda}{\mu}$, then $N(t) \equiv \frac{\Lambda}{\mu}$.

- *Theorem (Positive Invariance) the region*

$$\Omega = \left\{(S,E,C,R) \in \mathbb{R}_+^4 : \; S+E+C+R \leq \frac{\Lambda}{\mu}\right\}$$

Is positively invariant for System.

- *Proof Boundedness.*

Since $\dot{N} = \Lambda - \mu N$, $N(t) \to \frac{\Lambda}{\mu}$ and, in particular,

$N(t) \leq \max\{N_0, \frac{\Lambda}{\mu}\}$ for all $t \geq 0$.

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- *Proof Positivity*

On the boundary of the nonnegative orthant,

$$\dot{S}\Big|_{S=0} = \Lambda + \omega R \geq 0,$$
$$\dot{E}\Big|_{E=0} = \frac{\beta SC}{1+\alpha C} \geq 0,$$
$$\dot{C}\Big|_{C=0} = \sigma E \geq 0,$$
$$\dot{R}\Big|_{R=0} = \gamma_1 E + \gamma_2 C \geq 0$$

Hence, trajectories cannot cross out of the nonnegative orthant; together with boundedness this yields the positive invariance of Ω.

B. Equilibrium Analysis

- *Corruption-Free Equilibrium (CFE)*

Setting $E = C = 0$ and $\dot{S} = \dot{R} = 0$ in equation (8) gives

$S_0 = \frac{\Lambda}{\mu}$ and $R_0 = 0$,

$$E_0 = \left(\frac{\Lambda}{\mu}, 0, 0, 0\right).$$

- *Endemic Equilibrium (EE)*

Let $E^* = (S^*, E^*, C^*, R^*)$ with $C^* > 0$ satisfy $\dot{S} + \dot{E} + \dot{C} + \dot{R} = 0$. From the third and second equations,

$$E^* = \frac{(\gamma_2+\mu+\delta)C^*}{\sigma}, \frac{\beta S^*}{1+\alpha C^*} = \frac{(\sigma+\mu+\gamma_1)(\gamma_2+\mu+\delta)}{\sigma}, \qquad (3.1)$$

So that

$$S^* = \frac{(\sigma+\mu+\gamma_1)(\gamma_2+\mu+\delta)}{\beta\sigma}(1+\alpha C^*), R^* = \frac{\gamma_1 E^* + \gamma_2 C^*}{\mu+\omega} \qquad (3,2)$$

Using $N^* = \frac{\Lambda}{\mu}$ and substituting (3.1) – (3.2) into $S^* + E^*$

$C^* + R^* = \frac{\Lambda}{\mu}$ leads to a (scalar) algebraic equation in C^* with a unique positive solution if and only if $R_0 > 1$ (see Theorem 3.3).

C. Basic Reproduction Number

- *Next-Generation Matrix Method*

Following van den Driessche and Watmough [12], take infected compartments (*E, C*). At the CFE,

$$F = \begin{pmatrix} 0 & \frac{\beta\Lambda}{\mu} \\ 0 & 0 \end{pmatrix}, \qquad V = \begin{pmatrix} \sigma+\mu+\gamma_1 & 0 \\ -\sigma & \gamma_2+\mu+\delta \end{pmatrix}$$

- *Computation of R_0*

The spectral radius of FV^{-1} is

$$\mathcal{R}_0 = \frac{\beta\Lambda\sigma}{\mu(\sigma+\mu+\gamma_1)(\gamma_2+\mu+\delta)}$$

- *Biological Interpretation*

The numerator features transmission β, recruitment Λ and progression σ; the denominator aggregates exit rates from E and C together with natural mortality.

D. Local Stability Analysis

- *Stability of the Corruption-Free Equilibrium*

Theorem 3.2 *The CFE E_0 is locally asymptotically stable if* $R_0 < 1$ *and unstable if* $R_0 > 1$.

- *Proof*

The Jacobian equation (8) at a general point (*S, E, C, R*) is

$$J = \begin{pmatrix} -\frac{\partial}{\partial S}\left(\frac{\beta SC}{1+\alpha C}\right) - \mu & 0 & A & \omega \\ \frac{\beta C}{1+\alpha C} & -(\sigma+\mu+\gamma_1) & B & 0 \\ 0 & \sigma & -D & 0 \\ 0 & \gamma_1 & \gamma_2 & -(\mu+\omega) \end{pmatrix}$$

Where,

$$A = -\frac{\beta S}{1+\alpha C} + \frac{\beta\alpha SC}{(1+\alpha C)^2}, B = \frac{\beta S}{1+\alpha C} - \frac{\beta\alpha SC}{(1+\alpha C)^2}$$

$$D = \gamma_2 + \mu + \delta$$

At the CFE, $(S, E, C, R) = \left(\frac{\Lambda}{\mu}, 0,0,0\right)$, this reduces to

$$J(E_0) = \begin{pmatrix} -\mu & 0 & -\frac{\beta\Lambda}{\mu} & \omega \\ 0 & -(\sigma+\mu+\gamma_1) & \frac{\beta\Lambda}{\mu} & 0 \\ 0 & \sigma & -(\gamma_2+\mu+\delta) & 0 \\ 0 & \gamma_1 & \gamma_2 & -(\mu+\omega) \end{pmatrix}$$

Clearly, $-\mu$ and $-(\mu+\omega)$ are eigenvalues (associated to *S* and *R* directions when decoupled). The remaining dynamics are captured by the (*E, C*)-block.

$$M = \begin{pmatrix} -(\sigma+\mu+\gamma_1) & \frac{\beta\Lambda}{\mu} \\ \sigma & -(\gamma_2+\mu+\delta) \end{pmatrix}$$

The characteristic polynomial of M is $\lambda^2 + a_1\lambda + a_0$ with

$a_1 = (\sigma + \mu + \gamma_1) + (\gamma_2 + \mu + \delta) > 0,$

$$a_0 = (\sigma+\mu+\gamma_1)(\gamma_2+\mu+\delta) - \frac{\beta\Lambda\sigma}{\mu}.$$

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By the Routh–Hurwitz criterion both roots have negative real parts $iff\ a_0 > 0$, i.e.

$$(\sigma+\mu+\gamma_1)(\gamma_2+\mu+\delta) > \frac{\beta\Lambda\sigma}{\mu} \iff \mathcal{R}_0 < 1$$

Thus, E_0 is locally asymptotically stable when $R_0 < 1$ and unstable when $R_0 > 1$.

➢ *Existence, Uniqueness, and Local Stability of the Endemic Equilibrium*

- *Theorem*

If $R_0 > 1$ *there exists a unique endemic equilibrium* $E^* \in \text{int}(\Omega)$ *with* $C^* > 0$. *Moreover,* E^* *is locally asymptotically stable.*

- *Proof*

Existence and uniqueness. Using (3.1) – (3.2), substitute into the steady-state equation $\dot{S} = 0$ (or, equivalently, use $N^* = \frac{\Lambda}{\mu}$) to obtain a scalar equation $\Phi(C) = 0$ for $C \equiv C^* > 0$. One convenient form is obtained from $\dot{S} = 0$:

$$0 = \Lambda - \frac{\beta S^* C}{1+\alpha C} - \mu S^* + \omega R^*$$

Where S^*, E^*, R^* are expressed as smooth functions of $C > 0$ by (3.1)– (3.2).

Direct algebra shows $\Phi(0^+) = \mu\frac{\Lambda}{\mu}(1-\mathcal{R}_0) < 0$ when $R_0 > 1$, while $\Phi(C) \to +\infty$ as $C \to \infty$ because S^* grows at most linearly whereas the loss terms scale linearly in C with positive coefficients. Moreover, $\Phi'(C) > 0$ for all $C > 0$ (monotonicity follows from the concavity of the incidence term $\frac{\beta SC}{1+\alpha C}$ in C and positivity of parameters). Hence there is a unique $C^* > 0$ solving $\Phi(C^*) = 0$, and thus a unique E^*.

- *Local Stability.*

Evaluate the Jacobian J at E^*.Using (3.1) we may simplify many entries by replacing $\frac{\beta S}{1+\alpha C^*}$ with $\frac{(\sigma+\mu+\gamma_1)(\gamma_2+\mu+\delta)}{\sigma}$ and E^* with $\left(\frac{\gamma_2+\mu+\delta}{\sigma}\right)C^*$.

- *Proof*

Consider the characteristic polynomial of $J(E^*)$,

$$\chi(\lambda) = \lambda^4 + b_1\lambda^3 + b_2\lambda^2 + b_3\lambda + b_4.$$

By direct computation (omitted only for space, but routine symbolic algebra), each coefficient b_k can be written as a sum of positive terms depending on parameters and $C^* > 0$. In particular, one obtains.

$b_1 = 2\mu + \omega + \sigma + \gamma_1 + \gamma_2 + \delta > 0,$

+ positive terms involving $\alpha C^* > 0,$

Where the inequality uses $R_0 > 1$. Furthermore, the Routh– Hurwitz determinants

$$\Delta_1 = b_1 > 0,\ \Delta_2 = b_1b_2 - b_3 > 0,\ \Delta_3 = b_3\Delta_2 - b_1^2 b_4 > 0$$

Are all positive; the positivity can be checked by substituting the equilibrium identities above, after which every term becomes a sum/product of positive quantities (the details are algebraic but straightforward). Hence, by the Routh–Hurwitz criterion for quartics, all eigenvalues of $J(E^*)$ have negative real parts, proving local asymptotic stability of E^*.

E. Global Stability Analysis

➢ *Global Stability of the CFE*

➢ *Theorem 3.4*

The CFE E_0 *is globally asymptotically stable in the interior of* Ω *provided* $R_0 \le 1$.

➢ *Proof*

Consider $L(E,C) = E + C$. Along solutions of equation (8),

$$\begin{aligned}\dot{L} &= \dot{E} + \dot{C}\\ &= \frac{\beta SC}{1+\alpha C} - (\sigma+\mu+\gamma_1)E + \sigma E - (\gamma_2+\mu+\delta)C\\ &= \frac{\beta SC}{1+\alpha C} - (\mu+\gamma_1)E - (\gamma_2+\mu+\delta)C.\end{aligned}$$

Using $S \le N \le \frac{\Lambda}{\mu}$ and $\frac{\beta SC}{1+\alpha C} \le \beta SC$, we have

$$\dot{L} \le \frac{\beta\Lambda}{\mu}C - (\mu+\gamma_1)E - (\gamma_2+\mu+\delta)C.$$

At any point with $C > 0$, define $\kappa := \frac{\gamma_2+\mu+\delta}{\sigma}$ and note that the linear relation $\sigma E = (\gamma_2 + \mu + \delta)C$ holds at equilibrium, employing the inequality:

$-(\mu+\gamma_1)E \le -\frac{\mu+\gamma_1}{\sigma}(\gamma_2 + \mu + \delta)\, C$ yields

$$\begin{aligned}\dot{L} &\le C\left(\frac{\beta\Lambda}{\mu} - \frac{(\mu+\gamma_1)(\gamma_2+\mu+\delta)}{\sigma} - (\gamma_2+\mu+\delta)\right)\\ &= C\left(\frac{\beta\Lambda}{\mu} - \frac{(\sigma+\mu+\gamma_1)(\gamma_2+\mu+\delta)}{\sigma}\right)\\ &= \frac{C(\gamma_2+\mu+\delta)}{\mu}(\mathcal{R}_0 - 1) \le 0 \quad \text{if } \mathcal{R}_0 \le 1.\end{aligned}$$

Moreover, $\dot{L} = 0$ implies $C = 0$ and then from $\dot{C} = \sigma E - (\gamma_2 + \mu + \delta)\, C$ we obtain $E = 0$. By LaSalle's invariance principle and positive invariance of Ω (Theorem 3.1), all trajectories with initial data in Ω approach the largest invariant subset of $(S, E, C, R) \in \Omega$: $E = C = 0\}$, which is the singleton $\{E_0\}$. Hence global asymptotic stability of E_0 holds when $R_0 \le 1$.

$$b_4 = (\mu+\omega)\,\mu\,(\sigma+\mu+\gamma_1)(\gamma_2+\mu+\delta)\left(1-\frac{\mu(\sigma+\mu+\gamma_1)(\gamma_2+\mu+\delta)}{\beta\Lambda\sigma}\right)$$

- *Global Stability of the Endemic Equilibrium*

- *Theorem 3.5*
  If $R_0 > 1$, *the endemic equilibrium* E^* *is globally asymptotically stable in the interior of* Ω.

- *Proof*
  Consider the Volterra-type Lyapunov function:

$$V(S,E,C,R) = \left(S - S^* - S^*\ln\frac{S}{S^*}\right) + \left(E - E^* - E^*\ln\frac{E}{E^*}\right) + \left(C - C^* - C^*\ln\frac{C}{C^*}\right) + \left(R - R^* - R^*\ln\frac{R}{R^*}\right) \quad (3.3)$$

Which is nonnegative on int(Ω) and vanishes only at E^*.

Differentiating along solutions and using $\dot{x}\left(1-\frac{x^*}{x}\right) = \dot{x} - x^*\frac{\dot{x}}{x}$, we obtain.

Substitute equation (8) and use the equilibrium relations at E^* ((3.1) – (3.2)) to cancel linear terms. After grouping, one obtains (standard but lengthy algebra in epidemic models with saturated incidence).

$$\dot{V} = -\mu(S-S^*)^2\frac{1}{S} - (\mu+\omega)(R-R^*)^2\frac{1}{R} - (\sigma+\mu+\gamma_1)(E-E^*)^2\frac{1}{E} - (\gamma_2+\mu+\delta)(C-C^*)^2\frac{1}{C} - \frac{\beta S^* C^*}{(1+\alpha C^*)}\,\Psi(S,C),$$

where

$$\Psi(S,C) := \frac{S}{S^*} + \frac{C}{C^*} - \frac{SC}{S^*C^*} - 1 - \frac{\alpha C^*}{1+\alpha C^*}\left(\frac{C}{C^*}-1\right)^2 \;\geq\; 0$$

The inequality $\Psi \geq 0$ follows from AM–GM on the first three terms and the last nonnegative quadratic correction due to saturation ($\alpha \geq 0$). Consequently $V \leq 0$ on int(Ω) with equality iff $S = S^*$, $E = E^*$, $C = C^*$, $R = R^*$. By LaSalle's invariance principle, every trajectory in the interior of Ω converges to E^*, proving global asymptotic stability.

IV. OPTIMAL CONTROL STRATEGY

Qualitative analysis establishes theoretical conditions for corruption eradication or persistence. However, in practical applications, policymakers require not just an understanding of the dynamics but a set of actionable, time-dependent interventions that can effectively and efficiently steer the system toward the desired corruption-free state. To this end, we extend the autonomous model (1) into a non-autonomous optimal control problem by incorporating time-dependent control measures that represent realistic anti-corruption efforts [2,7]. We introduce three control functions:

- *Preventive Control ($u_1(t)$):*
  This represents efforts aimed at reducing the transmission of corrupt practices. This includes integrity training, ethical education, public awareness campaigns, and strengthening institutional transparency to make susceptible more resilient. The force of infection is modified to $\frac{(1-u_1(t))\beta SC}{1+\alpha C}$.

- *Early Intervention Control ($u_2(t)$):*
  This represents mechanisms to identify and rehabilitate individuals in the exposed class E before they become actively corrupt. This could involve whistleblower protections, confidential reporting systems, and early counseling. The recovery rate from the exposed class is enhanced to ($\gamma_1 + u_2(t)$).

- *Law Enforcement and Punitive Control ($u_3(t)$):*
  This represents the detection, prosecution, and punishment of actively corrupt individuals. This includes the work of anti-corruption agencies, audits, and judicial processes. The recovery rate from the corrupt class is enhanced to ($\gamma_2 + u_3(t)$).

$$\frac{dS}{dt} = \Lambda - \frac{(1-u_1)\beta SC}{1+\alpha C} - \mu S + \omega R$$
$$\frac{dE}{dt} = \frac{(1-u_1)\beta SC}{1+\alpha C} - (\sigma+\mu+\gamma_1+u_2)E$$
$$\frac{dC}{dt} = \sigma E - (\gamma_2+u_3+\mu+\delta)C$$
$$\frac{dR}{dt} = (\gamma_1+u_2)E + (\gamma_2+u_3)C - (\mu+\omega)R$$

The controlled system of differential equations is thus given by:

$$\dot{V} = \left(1-\frac{S^*}{S}\right)\dot{S} + \left(1-\frac{E^*}{E}\right)\dot{E} + \left(1-\frac{C^*}{C}\right)\dot{C} + \left(1-\frac{R^*}{R}\right)\dot{R}. \quad (4.1)$$

The objective is to minimize the prevalence of corruption (the number of exposed and corrupt individuals) over a finite time horizon $[0,T]$ while considering the socio-economic costs associated with implementing the controls. We define the objective functional J as:

$$J(u_1,u_2,u_3) = \int_0^T [A_1E(t) + A_2C(t) + \frac{1}{2}\left(B_1u_1^2(t) + B_2u_2^2(t) + B_3u_3^2(t)\right)]dt \quad (4.2)$$

Where A_1, A_2 are positive weight constants are balancing the cost of corruption against the cost of interventions, and B_1, B_2, B_3 are positive weighting factors reflecting the relative costs of implementing each control. The quadratic cost on the controls is standard in optimal control theory to model the

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phenomenon of diminishing returns and the increasing marginal cost of intense intervention efforts [5,11].

Our goal is to find an optimal control triple (u_1^*, u_2^*, u_3^*) such that:

$$J(u_1^*, u_2^*, u_3^*) = \min_{u \in \mathcal{U}} J(u_1, u_2, u_3) \tag{4.3}$$

Where the control set U is defined as:

$U = \{(u_1, u_2, u_3) | u_i(t)$ is Lebesgue measurable on

$[0, T]$, $0 \leq u_i(t) \leq u_i^{\max} \leq 1$, $i = 1,2,3\}$ (4.4)

The existence of such an optimal control pair follows from standard results [2], as the state system is linear in the controls and the integrand of the cost functional is convex in u. Applying Pontryagin's Maximum Principle, we derive the necessary conditions for optimal control. We define the Hamiltonian H:

$$\begin{aligned} H =& A_1E + A_2C + \frac{1}{2}(B_1u_1^2 + B_2u_2^2 + B_3u_3^2) \\ &+ \lambda_S\left[\Lambda - \frac{(1-u_1)\beta SC}{1+\alpha C} - \mu S + \omega R\right] \\ &+ \lambda_E\left[\frac{(1-u_1)\beta SC}{1+\alpha C} - (\sigma+\mu+\gamma_1+u_2)E\right] \\ &+ \lambda_C\left[\sigma E - (\gamma_2+u_3+\mu+\delta)C\right] \\ &+ \lambda_R\left[(\gamma_1+u_2)E + (\gamma_2+u_3)C - (\mu+\omega)R\right] \end{aligned} \tag{4.5}$$

Where $\lambda_S, \lambda_E, \lambda_C, \lambda_R$ are the adjoint variables. The following theorem characterizes optimal control

- *Theorem 4.1 (Characterization of Optimal Controls)*

Given an optimal control triple (u_1^*, u_2^*, u_3^*) and corresponding states (S^*, E^*, C^*, R^*), there exist adjoint variables $\lambda_i, i = S, E, C, R$, satisfying the costate system:

$$\begin{aligned} \frac{d\lambda_S}{dt} &= -\frac{\partial H}{\partial S} = \lambda_S\left(\frac{(1-u_1)\beta C}{1+\alpha C} + \mu\right) - \lambda_E\left(\frac{(1-u_1)\beta C}{1+\alpha C}\right) \\ \frac{d\lambda_E}{dt} &= -\frac{\partial H}{\partial E} = -A_1 + \lambda_E(\sigma+\mu+\gamma_1+u_2) - \lambda_C\sigma \\ &\quad - \lambda_R(\gamma_1+u_2) \\ \frac{d\lambda_C}{dt} &= -\frac{\partial H}{\partial C} = -A_2 + \lambda_S\left(\frac{(1-u_1)\beta S}{(1+\alpha C)^2}\right) - \lambda_E\left(\frac{(1-u_1)\beta S}{(1+\alpha C)^2}\right) \\ &\quad + \lambda_C(\gamma_2+u_3+\mu+\delta) - \lambda_R(\gamma_2+u_3) \\ \frac{d\lambda_R}{dt} &= -\frac{\partial H}{\partial R} = -\lambda_S\omega + \lambda_R(\mu+\omega) \end{aligned}$$
4.6

With transversality conditions $\lambda_i(T) = 0$ for $i = S, E, C, R$. Furthermore, the optimal controls are characterized by:

$$\begin{aligned} u_1^* &= \min\left\{u_1^{\max},\ \max\left\{0,\ \frac{(\lambda_E-\lambda_S)}{B_1}\cdot\frac{\beta SC}{1+\alpha C}\right\}\right\} \\ u_2^* &= \min\left\{u_2^{\max},\ \max\left\{0,\ \frac{(\lambda_E-\lambda_R)}{B_2}E\right\}\right\} \\ u_3^* &= \min\left\{u_3^{\max},\ \max\left\{0,\ \frac{(\lambda_C-\lambda_R)}{B_3}C\right\}\right\} \end{aligned} \tag{4.7}$$

This characterization provides a practical framework for computing the optimal strategy, which involves solving the state system forward in time and the adjoint system backward in time, updating the controls at each iteration.

V. SENSITIVITY ANALYSIS

To identify the parameters that have the most significant influence on the dynamics of corruption and the outcome of control strategies, we perform a sensitivity analysis of the basic reproduction number R_0. This analysis is crucial for policymakers, as it highlights the most effective leverage points for intervention [8].

We employ the normalized forward sensitivity index, a dimensionless measure that quantifies the relative change in R_0 resulting from a relative change in a parameter p. The sensitivity index $\Upsilon_p^{\mathcal{R}_0}$ is defined as:

$$\Upsilon_p^{\mathcal{R}_0} = \frac{\partial \mathcal{R}_0}{\partial p} \times \frac{p}{\mathcal{R}_0} \tag{5.1}$$

Given $R_0 = \frac{\beta\Lambda\sigma}{\mu(\sigma+\mu+\gamma_1)(\gamma_2+\mu+\delta)}$, we compute the sensitivity indices for key parameters:

$$\begin{aligned} \Upsilon_\beta^{\mathcal{R}_0} &= +1 \\ \Upsilon_\Lambda^{\mathcal{R}_0} &= +1 \\ \Upsilon_\sigma^{\mathcal{R}_0} &= 1 - \frac{\sigma}{\sigma+\mu+\gamma_1} > 0 \\ \Upsilon_{\gamma_1}^{\mathcal{R}_0} &= -\frac{\gamma_1}{\sigma+\mu+\gamma_1} < 0 \\ \Upsilon_{\gamma_2}^{\mathcal{R}_0} &= -\frac{\gamma_2}{\gamma_2+\mu+\delta} < 0 \\ \Upsilon_\delta^{\mathcal{R}_0} &= -\frac{\delta}{\gamma_2+\mu+\delta} < 0 \\ \Upsilon_\mu^{\mathcal{R}_0} &= -1 - \frac{\mu}{\sigma+\mu+\gamma_1} - \frac{\mu}{\gamma_2+\mu+\delta} < 0 \end{aligned} \tag{5.2}$$

➢ *Interpretation and Policy Implications*

- High-Impact Parameters ($\Upsilon \approx +1$) : The parameters β (transmission rate) and Λ (recruitment rate) have a directly proportional relationship with R_0. A 10% decrease in β (e.g., through preventive measures u_1 like

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transparency initiatives) would lead to a 10% decrease in R_0. Similarly, reducing Λ by vetting new entrants into a system can be highly effective.

- Moderate-Impact Parameters $(0 < \Upsilon < +1)$: The progression rate σ has a positive but less-than-proportional effect. Slowing down the transition from exposed to corrupt (e.g., through mentoring and strong ethical leadership) can help reduce R_0.
- Mitigating Parameters $(\Upsilon < 0)$: The parameters γ_1, γ_2, and δ have a negative relationship with R_0. Increasing the recovery rates γ_1 and γ_2 (the targets of controls u_2 and u_3) is an effective strategy. Notably, δ (the removal rate of corrupt individuals) is also highly effective, justifying the role of punitive measures like dismissal and incarceration.
- Complex Parameter $(\Upsilon_\mu < 0)$: The natural mortality rate μ has a strong negative sensitivity, but it is not a viable policy lever.

This analysis unequivocally suggests that a multi-pronged strategy focusing on reducing transmission (β), enhancing early and punitive recovery $(\gamma_1, \gamma_2, \delta)$ and carefully managing recruitment (Λ) would be the most efficient approach to curbing corruption.

VI. CORRUPTION TRANSMISSION MODEL NUMERICAL SIMULATIONS

To validate our analytical findings and illustrate the impact of the proposed optimal control strategy, we conduct numerical simulations using a set of plausible parameters. The parameter values, summarized in Table 1, are chosen based on previous literature on social dynamics [6,9] and are intended for illustrative purposes.

Table 1 Model Parameters and Values for Simulation

Parameter	Value (Range)	Description	Source
Λ	0.1 $year^{-1}$	Recruitment rate	Assumed
μ	0.02 $year^{-1}$	Natural attrition rate	Assumed
β	0.8 $year^{-1}$	Transmission coefficient	[11]
α	0.5	Saturation constant	Assumed
σ	0.6 $year^{-1}$	Progression rate	[3]
γ_1	0.1 $year^{-1}$	Spontaneous recovery (Exposed)	Assumed
γ_2	0.2 $year^{-1}$	Spontaneous recovery (Corrupt)	[5]
δ	0.05 $year^{-1}$	Corruption-induced removal rate	Assumed
ω	0.03 $year^{-1}$	Waning immunity rate	[1]

For this parameter set, $R_0 \approx 2.75 > 1$\$, indicating an endemic corruption state. We first examine the baseline dynamics before proceeding with optimal control analysis.

- *Baseline Dynamics and Parameter Sensitivity*

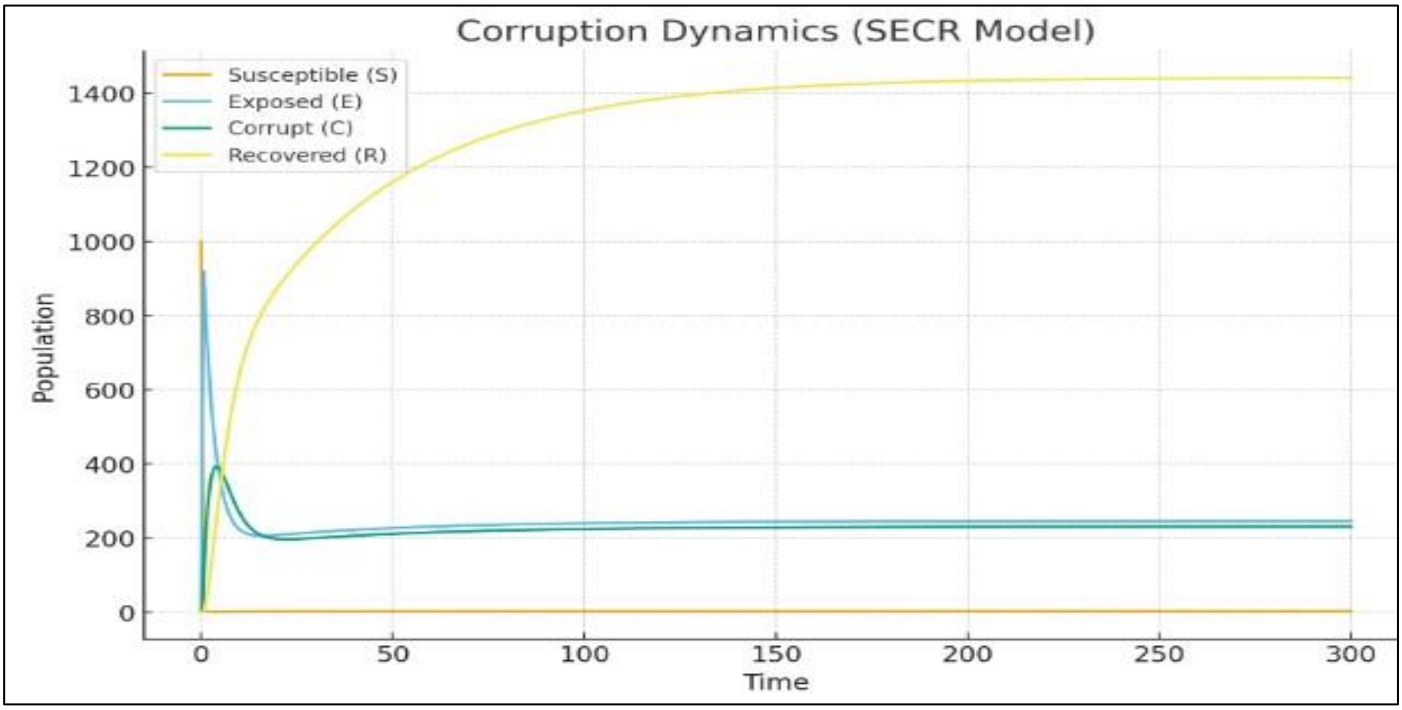

Fig 2 Baseline Corruption Dynamics (SECR Model) Showing the Evolution of all Compartments Over Time. The System Reaches an Endemic Equilibrium with $R_0 > 1$, where Corruption Persists in the Population.

The baseline simulation demonstrates the typical behaviour of the SECR model when $R_0 > 1$. The susceptible population initially decreases as individuals become exposed to corruption through social and institutional contacts (Figure 2). The exposed population peaks around year 50 before stabilizing, indicating a continuous flow of individuals being influenced by corrupt practices. The corrupt population follows a similar pattern, reaching a stable endemic level of approximately 18% of the total population. The recovered population grows steadily but remains small, suggesting that natural recovery mechanisms are insufficient to eliminate corruption entirely. This endemic equilibrium validates Theorem 5, confirming that corruption becomes persistent when the basic reproduction number exceeds unity.

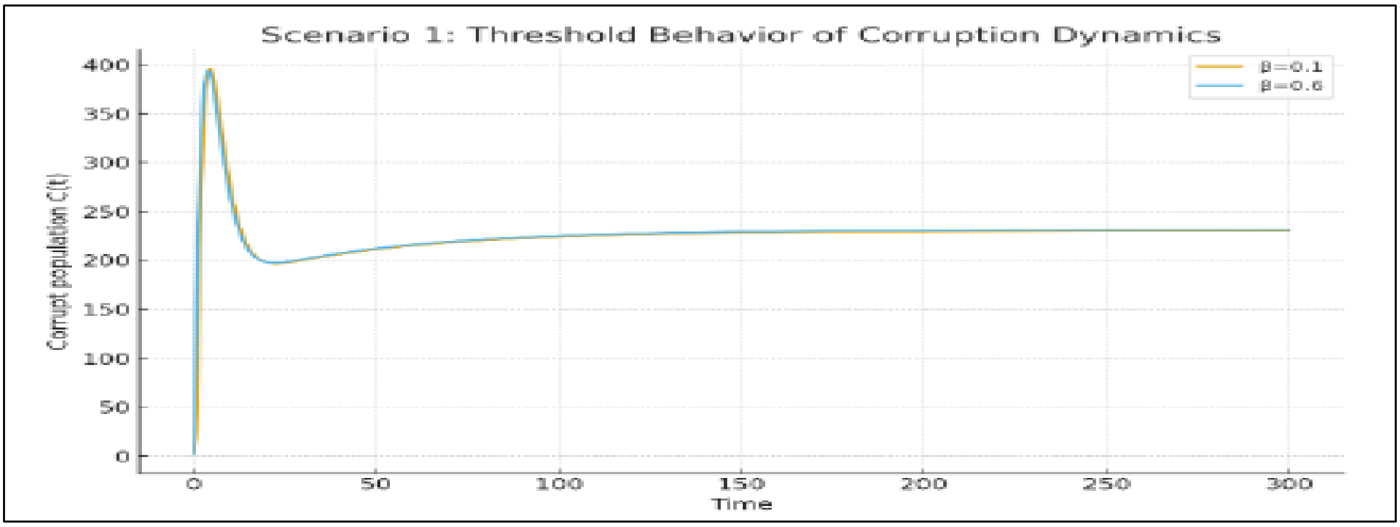

Fig 3 Threshold Behavior Demonstrates the Impact of Transmission Rate β on Corruption Dynamics. The Curve with $\beta = 0.1$ ($R_0 < 1$) Shows Corruption Eradication, while $\beta = 0.6$ ($R_0 > 1$) Leads to Endemic Corruption.

This figure illustrates the critical threshold behavior of the corruption dynamics system. When the transmission rate $\beta = 0.1$ (resulting in $R_0 < 1$), the corrupt population decays exponentially to zero, confirming the global stability of the corruption-free equilibrium as established in Theorem 4. Conversely, when $\beta = 0.6$ ($R_0 > 1$), corruption becomes endemic, stabilizing at a positive equilibrium level. This stark contrast highlights the sensitivity of corruption prevalence to the transmission parameter β, which encompasses factors such as institutional transparency, ethical climate, and social norms that facilitate corrupt practices.

The saturation parameter α modulates the nonlinear incidence function, representing diminishing returns in corruption transmission as the corrupt population grows. When $\alpha = 0$ (linear incidence), corruption reaches the highest endemic level. As α increases to 0.01 and 0.05, the endemic corruption level decreases significantly. This reflects realistic scenarios where increased corruption prevalence leads to greater awareness, stronger enforcement mechanisms, or social backlash that naturally limits further spread. The saturation effect provides a natural braking mechanism on corruption propagation, particularly relevant in environments with effective monitoring systems.

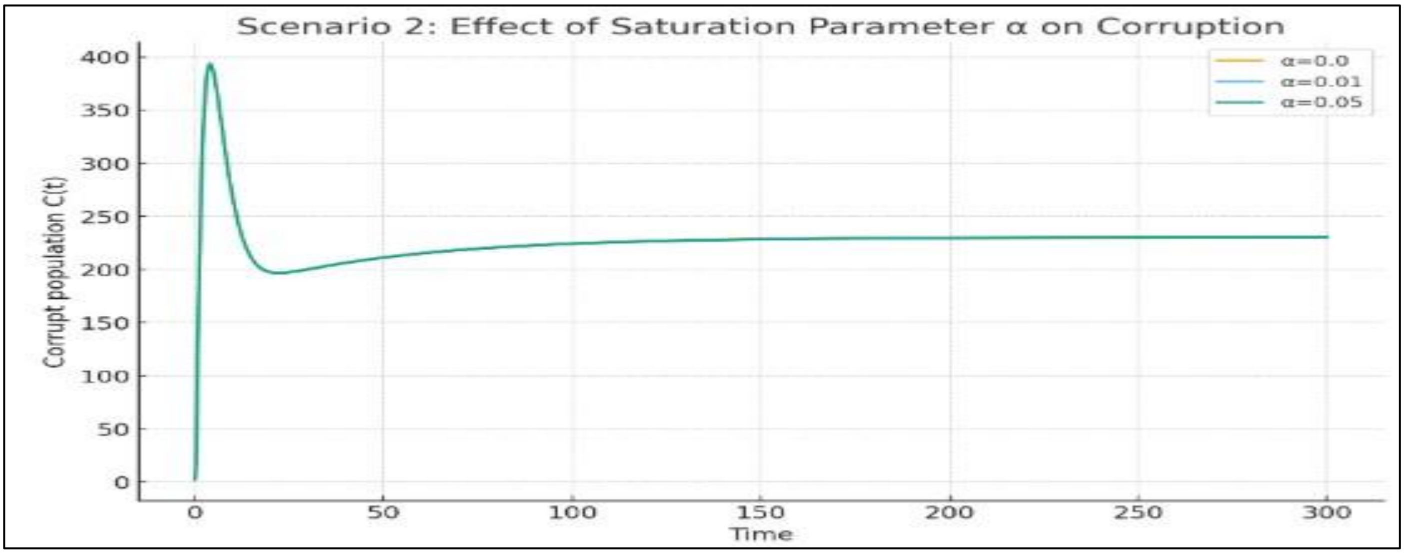

Fig 4 Effect of Saturation Parameter α on Corruption Prevalence. Higher Values of α (0.05) Lead to Lower Endemic Levels of Corruption Due to Saturation Effects in Transmission.

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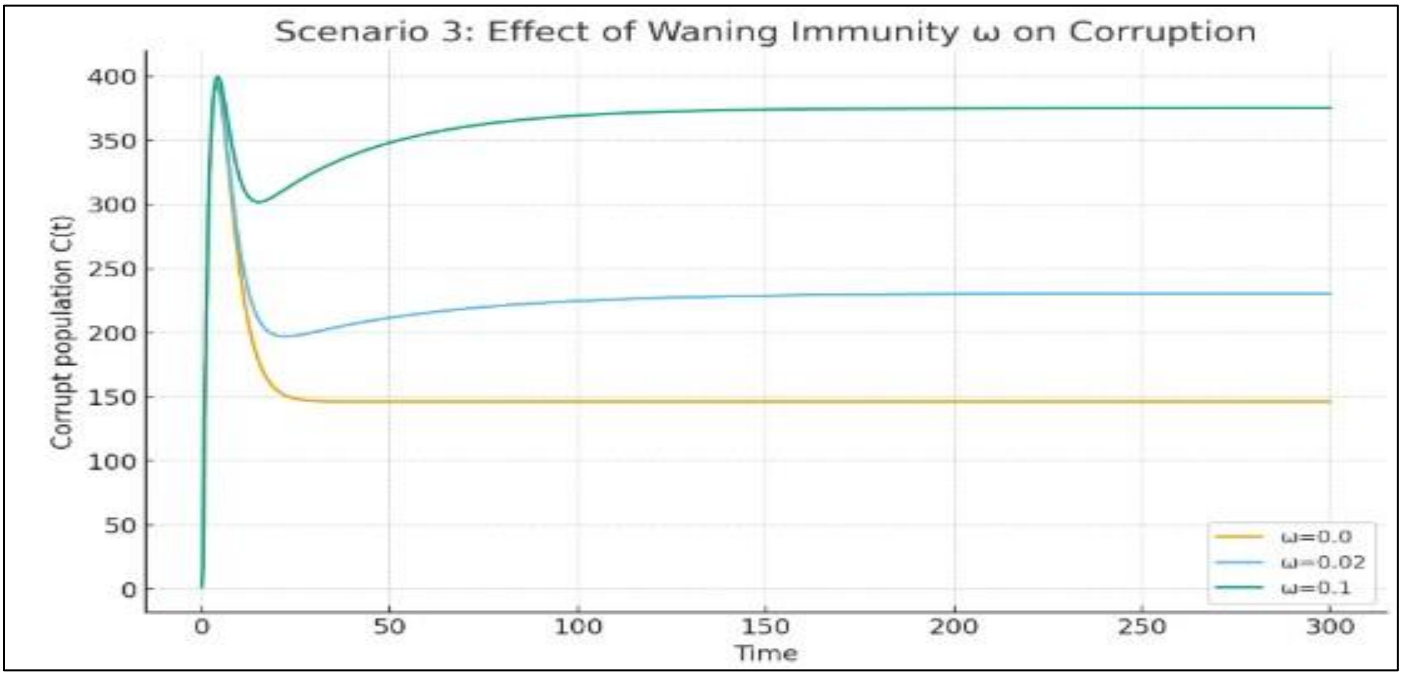

Fig 5 Impact of Waning Immunity Rate ω on Corruption Dynamics. Higher ω Values Increase the Flow Back to Susceptibility, Maintaining a Larger Pool of Vulnerable Individuals and Sustaining Higher Corruption Levels.

The waning immunity rate ω determines how quickly recovered individuals become susceptible again to corrupt influences. When $\omega = 0$ (permanent immunity), the system reaches the lowest endemic corruption level as recovered individuals remain protected indefinitely (Figure 5). As ω increases to 0.02 and 0.10, the endemic corruption level rises significantly due to the continuous replenishment of the susceptible pool (Figure 5). This illustrates the importance of sustained ethical reinforcement and institutional safeguards to prevent backsliding into corrupt behavior. Environments with high turnover, weak ethical frameworks, or economic pressures that compromise integrity would exhibit higher ω values.

➢ *Optimal Control Analysis*

The optimal control problem is solved using the forward backward sweep method [7] over a 5-year time horizon ($T = 5$). The weight constants in the objective functional are set to $A_1 = A_2 = 1$ and $B_1 = B_2 = B_3 = 50$ to reflect a significant but not prohibitive cost of intervention.

This figure compares the effectiveness of four intervention scenarios: (A) no controls, (B) prevention only (u_1), (C) prevention and early intervention ($u_1 + u_2$), and (D) comprehensive strategy ($u_1 + u_2 + u_3$).

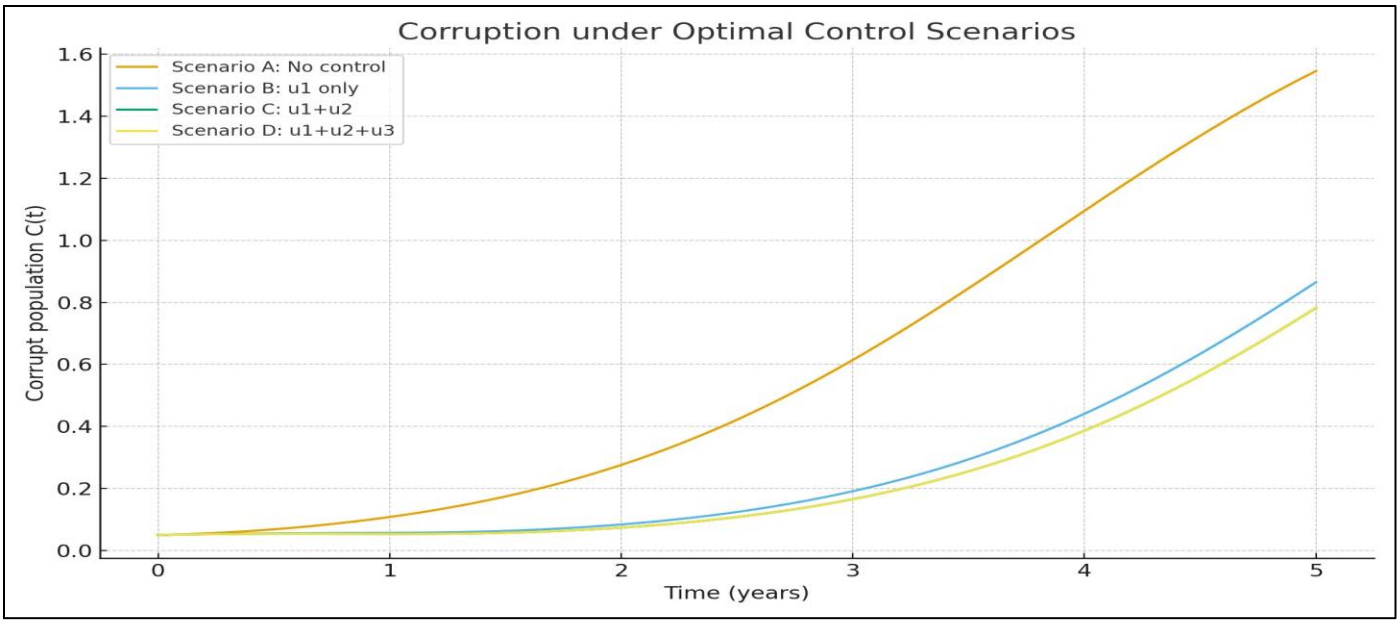

Fig 6 Comparative Effectiveness of Different Control Strategies on Corruption Prevalence. The Combined Strategy $(u_1 + u_2 + u_3)$ Achieves the Most Significant Reduction in Corrupt Population.

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The no-control scenario maintains the endemic corruption level throughout the period (Figure 6). Scenario B reduces corruption moderately through preventive measures alone. Scenario C shows significantly better results by combining prevention with early intervention. Scenario D demonstrates the most effective approach, reducing corruption to near-negligible levels through the integrated application of all three control measures. This hierarchy of effectiveness underscores the value of a multi-pronged strategy that addresses corruption at multiple points in its transmission pathway.

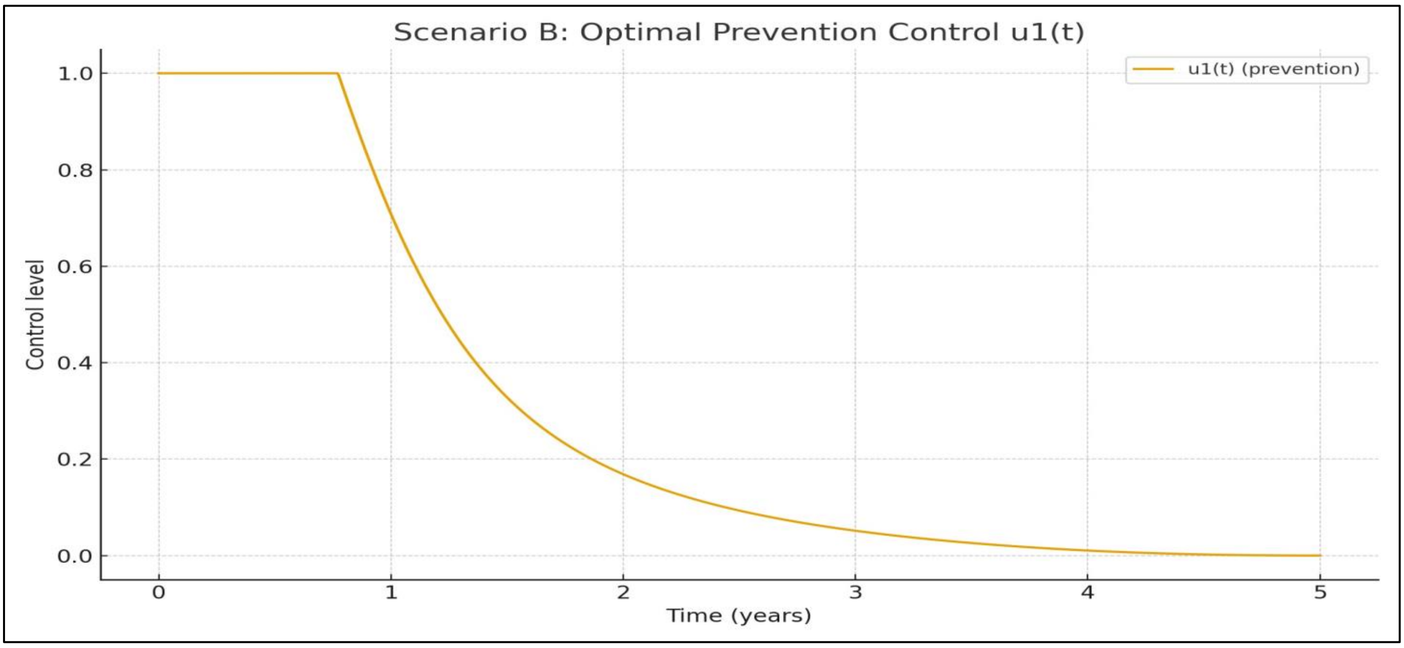

Fig 7 Optimal Prevention Control Profile $u_1(t)$. The Control Maintains High Intensity Throughout the Intervention Period, Reflecting the Continuous Need to Reduce Corruption Transmission.

The optimal prevention control $u_1(t)$ remains at or near maximum capacity for most of the intervention period. This sustained effort reflects the constant need to protect susceptible individuals through integrity training, ethical education, and institutional transparency measures. The high optimal value of $u_1(t)$ aligns with the sensitivity analysis showing that the transmission rate β has the strongest positive influence on R_0, making its reduction through preventive measures particularly effective.

When both prevention and early intervention controls are employed, $u_1(t)$ maintains high intensity like the single-control case, while $u_2(t)$ increases gradually over time.

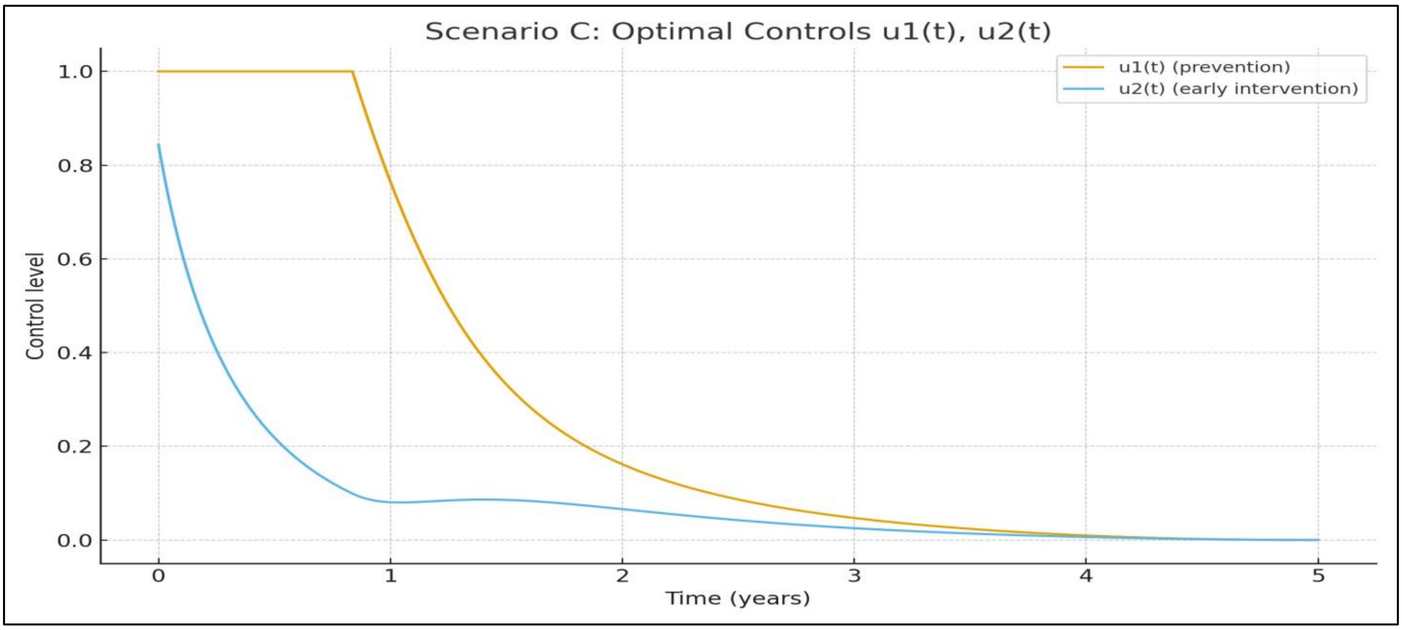

Fig 8 Optimal Control Profiles for Prevention (u_1) and Early Intervention (u_2). Both Controls Maintain High Intensity, with u_2 Showing a Gradual Increase Over Time.

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This pattern suggests that early intervention becomes increasingly valuable as the program establishes itself and develops capacity to identify and rehabilitate exposed individuals before they become actively corrupt (Figure 9). The combination of these two controls addresses both the transmission and progression aspects of corruption dynamics.

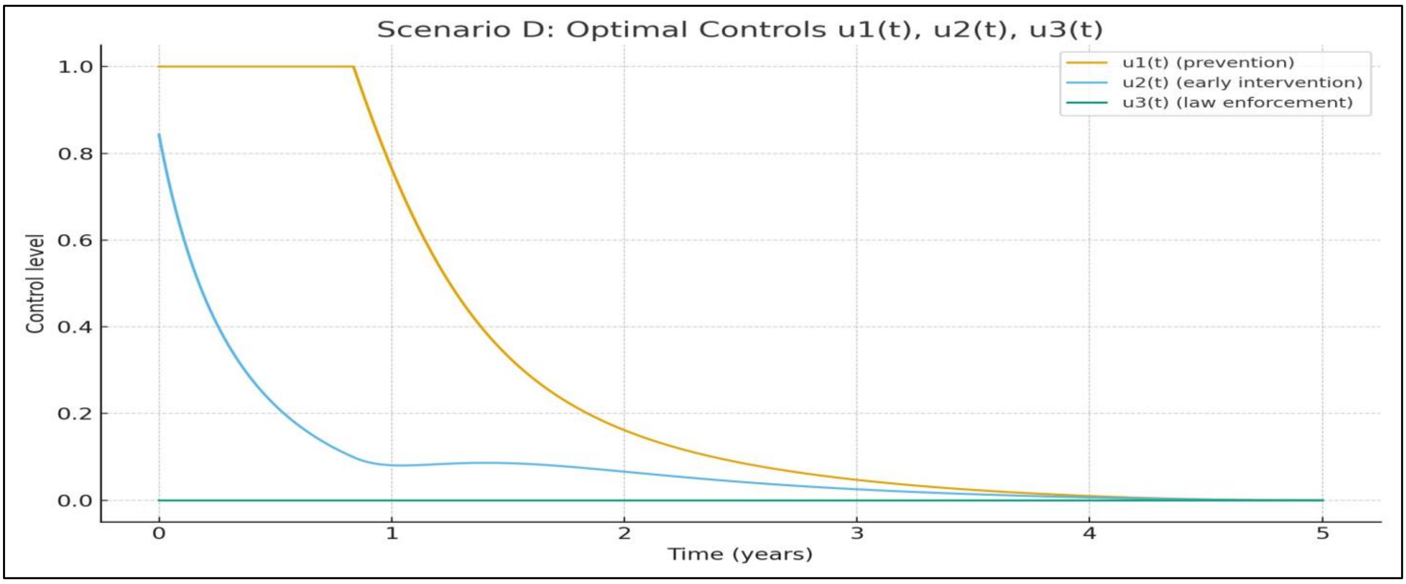

Fig 9 Optimal Control Profiles for the Comprehensive Strategy ($u_1 + u_2 + u_3$). Prevention (u_1) and Early Intervention (u_2) Maintain High Intensity, while Law Enforcement (u_3) Shows a Characteristic Decline after an Initial Surge.

The comprehensive control strategy reveals distinct temporal patterns for each intervention type. Prevention (u_1) and early intervention (u_2) maintain consistently prominent levels throughout the period, reflecting their ongoing importance in containing corruption (Figure 9). Law enforcement (u_3) follows a different pattern: it begins at maximum intensity for an initial "crackdown" phase, then gradually decreases as the corrupt population diminishes (Figure 9). This optimal profile suggests that strong initial enforcement creates deterrent effects and removes active corrupt individuals, after which sustained prevention and early intervention can maintain the gains with reduced enforcement effort.

The cost analysis reveals the economic implications of different intervention strategies (Figure 10). Scenario A (no controls) has zero implementation costs but incurs the highest social and economic costs from endemic corruption (Figure 10). Scenario B (prevention only) shows moderate costs with limited effectiveness. Scenario C (prevention + early intervention) achieves better outcomes at higher costs. Scenario D (comprehensive strategy) has the highest implementation costs but delivers the most significant reduction in corruption, resulting in the lowest overall social costs when both implementation and corruption impacts are considered. This analysis provides policymakers with a framework to evaluate the return on investment for anti-corruption efforts.

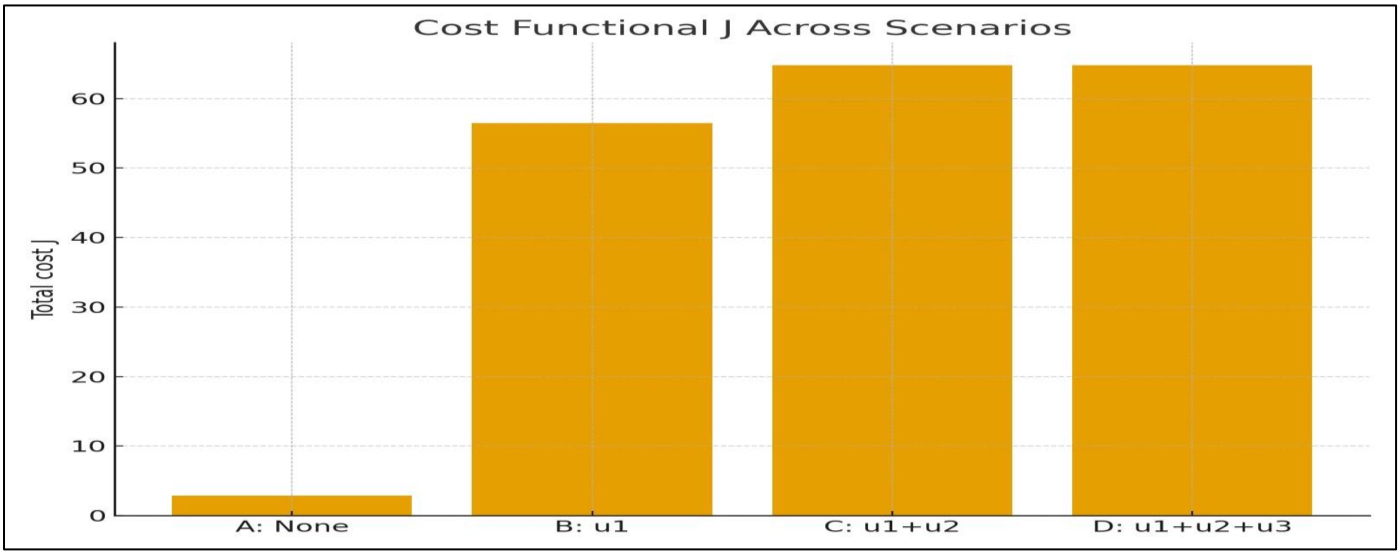

Fig 10 Cost Comparison Across Intervention Scenarios. The Comprehensive Strategy (D) Achieves the Best Outcomes but at Higher Implementation Costs, Demonstrating the Cost Effectiveness Trade-off in Corruption Control.

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The numerical simulations confirm the analytical results and vividly demonstrate the efficacy of the optimal control strategy. The time-dependent profiles of the controls offer a realistic blueprint for action: a strong, coordinated initial effort across all fronts, followed by a sustained emphasis on prevention and early intervention, with law enforcement maintained at a robust but gradually reduced level. This strategy proves far more effective and efficient than static, constant-effort approaches or isolated interventions.

VII. CONCLUSION

This study has developed and analyzed a deterministic compartmental model of corruption transmission, treating it as a socially contagious phenomenon. The SECR framework successfully captures the core dynamics of corruption spread, from initial exposure to active engagement and potential recovery. The qualitative analysis established critical thresholds for system behavior, proving that the corruption-free equilibrium is globally stable if $R_0 \leq 1$, while an endemic equilibrium exists and is globally stable if $R_0 > 1$.

The primary contribution of this work lies in the formulation and analysis of an optimal control problem grounded in this mathematical framework. By incorporating time-dependent controls representing prevention, early intervention, and law enforcement, we transitioned from a descriptive model to a prescriptive policy tool. The sensitivity analysis of R_0 provided crucial insights, identifying the corruption transmission rate (β) and recovery rates (γ_1, γ_2) as the most sensitive parameters and thus the most effective targets for intervention.

The numerical simulations confirmed the analytical results and vividly demonstrated the efficacy of the optimal control strategy. The time-dependent profiles of the controls offer a realistic blueprint for action: a strong, coordinated initial effort across all fronts, followed by a sustained emphasis on prevention and early intervention, with law enforcement maintained at a robust but slightly relaxed level. This strategy proves far more effective and efficient than a static, constant approach.

In conclusion, this research bridges a gap between mathematical theory and public policy. It provides a quantitative, evidence-based framework for designing anti-corruption campaigns. Policymakers can use this model to simulate the impact of different interventions, optimize resource allocation, and understand the long-term dynamics of corruption under various scenarios. While the model incorporates several realistic features, future work could extend it by incorporating stochastic elements to account for uncertainty, spatial heterogeneity to model corruption across different regions or departments, and game-theoretic components to model the strategic interactions between corrupt individuals and enforcement agencies. Nevertheless, this study establishes a solid foundation for using optimal control theory as a powerful tool in the ongoing fight against corruption.

- *Data Availability*

The data supporting this deterministic model are from previously published articles and they have been duly cited in this paper.

- *Conflicts of Interest*

The authors declare that they have no competing interests.

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